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OPTIMIZATION OF DRILLED SHAFT GROUP SPACING

Interim Report

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16\. Abstract

The report presents a summary of findings from an assessment of the technical literature, experience of engineers, and unpublished reports on lateral loads on pile groups. Specific interest is adopted in the design methods for drilled shafts, in particular of drilled shafts installed under similar conditions to those common in Arizona. These conditions were determined through a file search of as-built drawings for ADOT abutments supported on drilled shafts. Nine design methods were identified and are summarized herein. Based on a survey of practice, the most important of these appear to be the group reduction factor, the modulus of subgrade reaction reduction, and the p-multiplier. Each of these methods was compared to the other. A number of states, including Arizona, were found to interpret the factors presented in AASHTO 4.6.5.6.1.4 as group reduction factors, however, an evaluation of the apparent source documents indicates that they were intended for use as modulus of subgrade reaction reduction factors. Factors available in the literature for all methods were found not to represent Arizona soil or structural conditions well, so a series of finite element models and field load tests are recommended.

Appendices B, C, D, E, F, G, H, I are available from the Arizona Transportation Research Center upon request.

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METRIC (SI*) CONVERSION FACTORS

APPROXIMATE CONVERSIONS TO SI UNITS

Symbol	When You Know	Multiply By	To Find	Symbol
		LENGTH		
in	inches	2.54	centimeters	cm
ft	feet	0.3048	meters	m
yd	yards	0.914	meters	m
mi	miles	1.61	kilometers	km
		AREA		
in^2	square inches	6.452	centimeters squared	cm^2
ft^2	square feet	0.0929	meters squared	m^2
yd^2	square yards	0.836	meters squared	m^2
mi^2	square miles	2.59	kilometers squared	km^2
ac	acres	0.395	hectares	ha
		MASS (weight)		
oz	ounces	28.35	grams	g
lb	pounds	0.454	kilograms	kg
T	short tons (2000 lb)	0.907	megagrams	Mg
		VOLUME		
fl oz	fluid ounces	29.57	millimeters	mL
gal	gallons	3.785	liters	L
ft^3	cubic feet	0.0328	meters cubed	m^3
yd^3	cubic yards	0.765	meters cubed	m^3

Note: Volumes greater than 1000 L shall be shown in m^3.

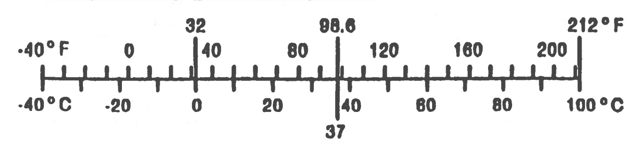
Symbol	When You Know	Multiply By	To Find	Symbol
		TEMPERATURE (exact)		
°F	Fahrenheit temperature	5/9 (after subtracting 32)	Celsius temperature	°C

These factors conform to the requirement of FHWA Order 5190.1A

*SI is the symbol for the International System of Measurements

APPROXIMATE CONVERSIONS TO SI UNITS

Symbol	When You Know	Multiply By	To Find	Symbol
		LENGTH		
mm	millimeters	0.039	inches	in
m	meters	3.28	feet	ft
yd	meters	1.09	yards	yd
km	kilometers	0.621	miles	mi
		AREA		
mm^2	millimeters squared	0.0016	square inches	in^2
m^2	meters squared	10.764	square feet	ft^2
yd^2	kilometers squared	0.39	square miles	mi^2
ha	hectares (10,000 m^2)	2.53	acres	ac
		MASS (weight)		
g	grams	0.0353	ounces	oz
kg	kilograms	2.205	pounds	lb
Mg	megagrams (1000 kg)	1.103	short tons	T
		VOLUME		
mL	millimeters	0.034	fluid ounces	fl oz
L	liters	0.264	gallons	gal
m^3	meters cubed	35.315	cubic feet	ft^3
m^3	meters cubed	1.308	cubic yards	yd^3
		TEMPERATURE (exact)		
°C	Celsius temperature	9/5 (then add 32)	Fahrenheit temperature	°F



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Executive Summary

The objective of this report was to develop a deeper understanding of laterally loaded pile group behavior. This understanding was to be developed through an extensive literature review and interviews with practitioners in the consulting and Department of Transportation (DOT) communities and prominent researchers. Information gathered form this phase was augmented with an evaluation of the conditions of use of laterally loaded pile groups in Arizona transportation projects. The project consist of 10 Tasks, namely:

Task 1: Kickoff Meeting
Task 2: Information Review
Task 3: Define Current Usage
Task 4: Evaluate Analytical Approaches
Task 5: Interim Report
Task 6: Analytical Analysis
Task 7: Design Field Load Tests
Task 8: Reports
Task 9: Executive Presentation
Task 10: Technical Presentation

This report is submitted in accordance with the agreed scope of work for Task 5, and is intended to summarize the results of our efforts on Tasks 1-4. The report specifically covers the objectives of summarizing the state of the practice in laterally loaded pile group design and analysis, describing in detail analytical approaches and describing common uses of pile groups under lateral load in Arizona. Based on these results, recommendations are provided for completing the analysis and design of field testing portions of this study, Tasks 6 and 7.

Summary of Literature and Practice

This chapter consisted of a number of subtasks. The first subtask involved reviewing information from a number of sources: technical literature, unpublished reports, ongoing research, and experience of engineers. The second subtask involved determining how common analytical methods compare with each other for Arizona conditions. The remaining subtask involved determining what constitutive models might be appropriate for finite element modeling and selecting an appropriate code.

Information Review

Technical Literature

The review of the technical literature included both results of load tests and experiments on groups of laterally loaded piles and methods of analysis. The results of the load tests are too numerous to explain here, however, the general trend was that full-scale tests showed less interaction between piles within the group than model tests. There were 9 various methods of analysis for laterally loaded pile groups found in the literature. These methods were as follows:

1. Elastic analysis
2. Hybrid analysis
3. Group reduction factor design
4. Coefficient of lateral subgrade reaction reduction
5. p-multiplier design
6. Load and resistance factor design (LRFD)
7. Group amplification procedure
8. Strain wedge method
9. Finite element modeling

Of these procedures the most commonly used are the group reduction factor method, coefficient of lateral subgrade reaction reduction, and p-multiplier design. The group reduction factor method involves developing a load-deflection curve for a single pile. The engineer then selects an allowable deflection and then enters the curve and attains the allowable load. This load is then multiplied by the group reduction factor to obtain the allowable load for a pile in the pile group. The coefficient of lateral subgrade reaction reduction method reduces the coefficient of lateral subgrade reaction, which is then used in the design calculations for the pile group. The p-multiplier approach uses a p-multiplier that is applied to the p-axis of the p-y curve. The analysis is then performed with the new p-y curve and a new load-deflection curve is developed.

Experience of Engineers

In an effort to tap the experience of engineers both locally and nationally a survey was conducted on each level. There were three areas of emphasis for the surveys: pile group design procedure, deflection criteria, and how are drilled shafts designed. As stated previously, there were two geographical areas of interest for the survey: local and nationally.

Locally an effort was made to contact both geotechnical and structural engineers who have been involved in the design of laterally loaded pile groups. From the surveys it was clear that Maricopa County area consultants follow the ADOT recommendation, which is to use the AASHTO recommendations, and the reduction factor is applied to the capacity of the single pile. The static deflection is usually kept less than 1 [in]. The common way to design laterally loaded piles is to use p-y based computer programs, COM624 and LPILE.

On a national scale all the departments of transportation were contacted and a survey conducted. When contacting the various state DOTs both structural and geotechnical engineers were contacted depending on whom was the most knowledgeable about the design procedure. The survey results from the state DOTs for design procedure are presented below:

Analysis Method	AASHTO			P-multiplier	Elastic
	15			12	1
How Factors Applied	Capacity	Soil Prop.	Don't Know		
	12	3	0		

The results of allowable deflections for the various state DOTs at both static and seismic levels are presented below:

Static				Seismic				
< 1"	> 1"	Not Sure	Varies	< 2"	> 2"	Not Sure	Not Considered	Varies
19	3	2	21	7	2	7	17	12

The common way to design laterally loaded piles is to use p-y based computer programs, COM624 and LPILE.

Unpublished Reports and Ongoing Research

An effort was made by the researchers to find unpublished reports on laterally loaded pile groups. The main groups contacted were Maricopa County engineers, state department of transportation engineers around the country, and prominent researchers in the field of deep foundations. These inquiries pointed the researchers towards very few unpublished reports.

With regards to ongoing research the researchers were able to obtain a copy of the interim report of NCHRP 24-9 titled *Static and Dynamic Lateral Loading of Pile Groups* (O'Neill et al., 1997). The report provided some insight, however, there were no results presented in the report.

During the technical literature review the procedure outlined in the *AASHTO 1996 Standard Specifications for Highway Bridges* was reviewed. As ADOT and others have interpreted AASHTO, the specification for groups of laterally loaded drilled shafts (Sec. 4.6.5.6.1.4) states that the "effects of group action for in-line CTC (center-to-center) < 8B may be considered using ....the ratio of lateral resistance of shaft in group to single shaft." There is no other guidance on how to apply these ratios to the group action problem. Many have interpreted the word "resistance" in its common usage elsewhere in AASHTO as implying force, and accordingly developed an understanding that the values given for lateral load resistance are ratios of forces. The source in AASHTO is the Canadian Geotechnical Society (1985). This was interpreted to mean the Canadian Foundation Engineering Manual (CFEM) which is published by the Canadian Geotechnical Society. This reference recommends applying the reduction factors to the coefficient of lateral subgrade reaction rather than reducing the lateral force capacity of the single pile. This method was further substantiated when examining the origin of the CFEM recommendations, in Davisson (1970). It is clear that the source referenced in the AASHTO specifications has a different interpretation than that in use by most states.

Comparison of Analytical Methods

The second subtask, comparing analytical approaches, was performed for a typical drilled shaft group under Arizona Conditions. The analytical approaches that were compared were the group reduction method, coefficient of lateral subgrade reaction reduction method and p-multiplier method. The comparison of these 3 methods to Arizona conditions is very dependent on the pile slenderness ratio (L/T). There are three categories of L/T: rigid, (L/T < 2); intermediate, (2 < L/T < 4); and flexible (L/T >4). The results are difficult to summarize but the general trends of efficiency are presented below.

Pile Slenderness	Group Reduction Factor		Coefficient of Lateral Subgrade Reaction		P-multipliers	
	Fixed#	Free	Fixed	Free	Fixed	Free
Rigid	Same*	Same	Same	Same	Same	Same
Intermediate	Same	Same	Eff. > Coeff.	Eff. > Coeff.	Eff. > p-mult.	Same
Flexible	Same	Same	Eff. > Coeff.	Eff. > Coeff.	Eff. > p-mult.	Eff. > p-mult.

Head fixity condition

* Same means that the efficiency was equal to the reduction factor used in the procedure

For example a 42 in drilled shaft, 30 ft long was used at a cemented silt site, which is categorized as an intermediate pile. Using a coefficient of lateral subgrade reaction reduction of 0.25 results in an efficiency of 0.42. The entries in the table represent a comparison of the reduction factor used in the procedure to the overall efficiency of the group, where efficiency is defined as the ratio of loads used to compare the load in the pile group divided by the load in a single pile times the number of piles in the group, at the same deflection. In equation form efficiency is as follows:

$$E = \left(\frac{P_{group}}{P_{single} \times N}\right)_{\Delta} = \left(\frac{P_{group/pile}}{P_{single}}\right)_{\Delta}$$

where:

E = efficiency
P_{group} = the load carried by the pile group
P_{single} = the load carried by the single pile
$P_{group/pile}$ = the average load carried by each pile in the group
N = number of piles in the group
Δ = the deflection at which the loads were compared

Finite Element Modeling

The final subtask was selection of constitutive models and programs for analysis of a laterally loaded drilled shaft group. In an effort to select an appropriate finite element code 96 programs were examined. The criteria for these programs were element types, material models, operating system, pre-processing, mesh generation, and solution methods. From the search of finite element programs three programs were selected for use: ABAQAS, MSC/NASTRAN, and ADINA. These three programs were selected as possibilities with the emphasis being on using ABAQAS.

There are several commercially available programs, such as GROUP and FLPIER that are available to solve the laterally loaded pile group problem. However, these programs and others like them will not be studied by the researchers. The reason being that these programs are based on empiricism. They assume the behavior of the soil through p-y curves. In order to perform a detailed analysis of the problem it is necessary to be able to analyze from scratch. This means without any preconceived notion of the solution. It is important to note that the FEM work is mainly research work and the subsequent recommendations will not require the use of FEM.

Summary of Historic Use

The objective of this task was to determine typical applications for which laterally loaded drilled shaft groups are use in Arizona and to develop an understanding of the performance of these drilled shaft groups in the field. The results of the first subtask are presented first. It was determined that the primary use for drilled shaft groups in Arizona was in bridge abutments. The ADOT files were searched and the plans reviewed for abutments founded on drilled shafts. There were 120 abutments that were found to be located on groups of drilled shafts. From the plan review the diameter, length, group geometry, center-to-center spacing, and soil type were obtained. The following table provides the ranges of values that represent Arizona conditions.

Parameter	Maximum Range
Length	31-40 ft
Diameter	36 in
Center-to-center spacing	3-3.5 [S/D]
Group Geometry	
In-line vs. Staggered	Staggered
Number of piles in group	11-20
Soil Type	Sandy-clay w/ cementation and SGC
Pile Cap	
In Contact with the Soil (Cap friction)	Almost All
Below Soil Surface (Passive Resistance)	Almost All

The second subtask was determining how groups of drilled shafts are performing in the field. Field visits and inspections were conducted by WTI. To date WTI has inspected 53 abutments. The results of the site visits were that there is no significant movement or damage of the abutments, although development of such movements would be a function of construction sequencing, time, and other variables.

Data Gaps

The soil conditions and drilled shaft geometry commonly used in Arizona were compared to those identified in the literature. Eight projects were identified in the literature for which 11 load tests were performed. The ranges in Arizona conditions that were not well represented in the literature are presented in the following table.

Parameter	Data Gaps
Length	41-50 ft
Diameter	30 and 36 in
Center-to-center spacing	3.5-4 [S/D]
Group Geometry	
In-line vs. Staggered	Staggered
Number of piles in group	> 10 shafts
Soil Type	Clayey-silt w/ cementation and SGC
Pile Cap	
In Contact with the Soil (Cap friction)	2 cases in literature
Below Soil Surface (Passive Resistance)	1 case I literature

These data gaps are of interest because if it is thought that a full-scale load test is necessary than it should be designed to fill in as many gaps as possible. If a test was to be conducted the most important aspect to test would be the case of stagger due the absence of any data in the drilled shaft literature.

Conclusions

From the development of the interim report there were three important conclusions. The first conclusion deals with the AASHTO Specifications (1996) for closely spaced laterally loaded drilled shafts. As previously stated, the AASHTO specifications state that the "effects of group action for in-line CTC (center-to-center) < 8B may be considered using ....the ratio of lateral resistance of shaft in group to single shaft." Many states including Arizona implement the AASHTO specifications as load reduction factors. The source cited in the AASHTO specifications (CGS, 1985) and Davisson (1970) recommend using the reduction factors to reduce the modulus of subgrade reaction. It is clear that the sources referenced in the AASHTO specifications have a different interpretation than most states that are using the AASHTO specifications.

The second conclusion is that field load tests should be conducted to fill in the data gaps. Based on the comparison of the literature data and the data from Arizona uses it is clear that some data gaps exist, specifically with regards to soil type. Arizona conditions have a large percentage of sites where the soil conditions are predominantly either sand, gravel, and cobbles, or cemented fine-grained material. To fill in these data gaps it is necessary to conduct a full-scale field load test. Such tests are beyond the scope of this work. However, the design of a field load test will be undertaken as part of this project.

The third conclusion is that a finite element analysis should be conducted in the second phase of the project. This conclusion is based on the fact that currently there are no recommendations that fit Arizona conditions and that full-scale load tests are too costly to perform parametric studies. In an effort to develop group reduction factors for Arizona conditions a FE analysis is to be conducted. The model will first be calibrated to actual full-scale tests for conditions that best match Arizona conditions. Once the model has been calibrated the analysis will be expanded to general conditions which are representative of Arizona transportation applications.

The presentation of the final report will be done in two parts. Part 1 will include the research results along with any necessary background information on the laterally loaded pile group problem. Part 2 of the final report will be written specifically for use by practitioners. It will provide practitioners with step-by-step guidelines as to how to implement the results of the current research into practice with examples. This document will be used in all technical presentations. If appropriate the procedure will separate the effect of the pile and pile cap, and the format will allow the use of COM624P or other programs. The design procedure will also include deformation (deflection), criteria.

1.0 Introduction

This report is submitted to the Arizona Department of Transportation in accordance with the requirements of Project Number T99-13-00060. This project was started in March of 1999, with the objective of developing a deeper understanding of laterally loaded pile group behavior. This understanding was to be developed through literature review and interviews with practitioners in the consulting and DOT communities and prominent researchers. This information was to be augmented with an evaluation of the conditions of use of laterally loaded pile groups in Arizona transportation projects. The project consists of 10 Tasks, namely:

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This report is submitted in accordance with the agreed scope of work for Task 5, and is intended to summarize the results of our efforts on Tasks 1-4. The report specifically addresses the objectives of summarizing the state of the practice in laterally loaded pile group design and analysis, describing in detail analytical approaches (including the finite element approach), and describing common uses of pile groups under lateral load in Arizona. Based on these results, recommendations are provided for completing the analysis and design of field testing portions of this study, Tasks 6 and 7.

The report consists of a number of chapters. Following this introductory chapter is Chapter 2, Information Review. This chapter includes a review of the archival literature and a number of directed reports relating to laterally loaded pile group behavior and design procedures. A summary of a number of analytical methods, including example calculations, is provided. These methods are then normalized for comparison using typical Arizona soil conditions and pile geometries. An important analytical method, the finite element approach, is summarized, along with our findings regarding potential programs for use in Task 6. Finally, a summary of the state of the practice of laterally loaded pile group design procedures in Arizona and in other state DOTs is provided.

Chapter 3, Summary of Historic Use, provides a description of the conditions of use of laterally loaded pile groups in Arizona transportation applications. This information was obtained by review of ADOT files to identify relevant projects, which were found to number 120. Project plans were reviewed to obtain geometric characteristics, and then the projects were visited to assess their current conditions. Where possible, the design methods used for these projects were identified. The soil and drilled shaft conditions common in Arizona are compared to those identified in the literature in Chapter 4, Data Gaps. This process of comparison will allow for more focused field and analytical analysis in the balance of this study, without duplicating those results already available in the literature.

Finally, the report is summarized in Chapter 5, Summary and Recommendations. Recommendations for completing Tasks 6 and 7 are found here.

The researchers would like to acknowledge the Technical Advisory Committee for this project, chaired by Mr. Frank McCullagh of ADOT. Other members include Mr. Doug Alexander, Mr. Gene Hansen, Mr. J.J. Liu, and Mr. Shafi Hasan of ADOT, Mr. Kamel Alqalam of FHWA, and Mr. Dan Heller of TY Lin. In addition, we are grateful to the Steering Committee members, including Mr. Keith Dahlen of AGRA Earth & Environmental; Mr. Kenneth Ricker of Ricker, Atkinson, McBee and Associates; Mr. Randolph Marwig of Western Technologies; Inc., Mr. Robert Turton of HDR; and Mr. Dwaine Sergent of Kleinfelder.

The researchers are also grateful to the staff of ADOT records, specifically Mr. Sunil Athalye, Ms. Wendy Fields and Ms. Marcia Hurley who helped the researchers with finding projects and pulling files. We are also grateful to Professor Michael O'Neill of the University of Houston, Professor Kyle Rollins of Brigham Young University, Professor Dan Brown of Auburn University, and Professor Gary Norris of the University of Nevada, Reno, who provided useful comments and suggestions and agreed to be interviewed for this work.

2.0 Summary of Literature and Practice

2.1 Literature Review

This section contains a review of the available literature relating to drilled shaft and driven pile group performance under lateral load. The objectives of this section are to present a summary of methods of analysis commonly undertaken, to present values for empirical factors commonly used in those methods, and to compare those methods from a design standpoint. A number of variables are important to the laterally loaded pile group problem, and these variables and their effects will be summarized. Finally, a historical context for research related to the question will be presented.

2.1.1 Methods of Analysis

The prediction of the response to lateral load of a single pile is somewhat difficult. The problem is complicated dramatically by the combination of piles into a group. Overlapping stresses from one pile influence the response of the soil ahead of other piles. Soil-structure interaction becomes pronounced in some cases, but not in all cases. Due to the complexities of the problem, simplifications are usually adopted which transform the problem to some function of the single pile problem. What follows will be a summary of the methods that have been encountered in the literature. More emphasis will be given to those methods believed to be commonly used by DOTs or consultants based on survey data to be presented later in this report.

2.1.2 Single Pile Problem

The group problem is often addressed by reference or scaling based on the single pile response. In general, there are 4 potential solution methods for the single pile problem:

1. p-y curve analysis
2. Elastic analysis
3. Field load testing
4. Modulus of subgrade reaction

2.1.2.1 p-y Curve Analysis

The use of p-y curves in obtaining a solution to the problem of a single pile under lateral load is fairly straightforward analysis. The solution is primarily based upon beam on elastic foundation (BEF) theory by Hentenyi (1946), in which the foundation is modeled as a Winkler medium. In a Winkler medium, the pressure in the foundation is proportional at every point to the deflection occurring at that point, and is independent of the pressure or deflections occurring elsewhere in the foundation. This relationship between pressure and deflection implies a lack of continuity in the foundation. The behavior of the foundation is as if it is composed of rows of closely spaced but independent springs. The differential equation is derived below using the following assumptions:

1. The pile is straight with uniform cross-section of homogenous material, and is described by its stiffness *EI*.
2. Pile has longitudinal plane of symmetry
3. Pile stays in the elastic range
4. Pile properties are the same in tension and compression
5. Pile is only subjected to static loads
6. Deformation due to shearing stresses are small
7. Transverse deformations of the pile are small

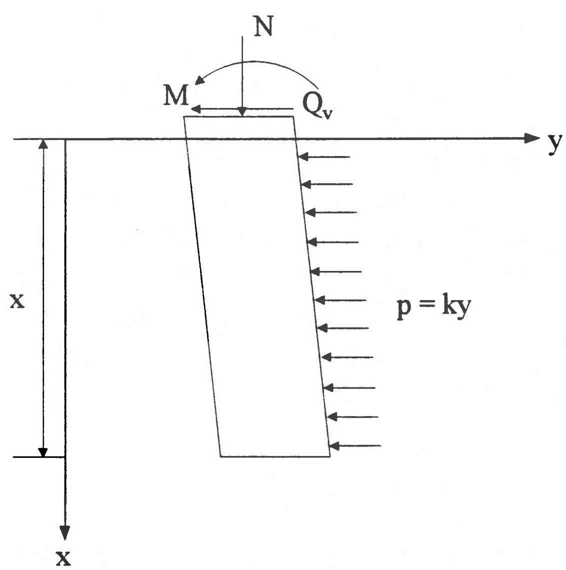


Figure 2.1: Beam on Elastic Foundation (Symbols defined on Page 6)

Hetenyi (1946) gave the derivation of the differential equation for the beam-column on a Winkler foundation. He assumed that a beam on an elastic foundation is subjected to horizontal loading and vertical compressive forces *N* acting at the centroid of the end cross-section of the beam Figure 2.1.

If an infinitely small element, bounded by two horizontals a distance *dx* apart, is cut out of this bar (see Figure 2.2), the equilibrium moments (ignoring second order terms) leads to the equation

$$\sum M_A = (M + dM) - M - Ndy - Q_v dx = 0 \qquad \text{Equation 2.1}$$

or

$$\frac{dM}{dx} - N\frac{dy}{dx} - Q_v = 0 \qquad \text{Equation 2.2}$$

Differentiating Equation 2.2 with respect to *x*, the following equation is obtained

$$\frac{d^2M}{dx^2} - N\frac{d^2y}{dx^2} - \frac{dQ_v}{dx} = 0 \qquad \text{Equation 2.3}$$

Prior to continuing it is important to note the following identities:

$$\frac{d^2M}{dx^2} = -EI\frac{d^4y}{dx^4}$$

$$\frac{dQ_v}{dx} = p$$

$$p = bk_0y = ky$$

where:

b = width of the beam
k_0 = coefficient of lateral subgrade reaction
y = deflection
k = bk_0 = modulus of subgrade reaction

Substituting in the appropriate identities; Equation 2.3 becomes:

$$EI\frac{d^4y}{dx^4} + N\frac{d^2y}{dx^2} + ky = 0 \qquad \text{Equation 2.4}$$

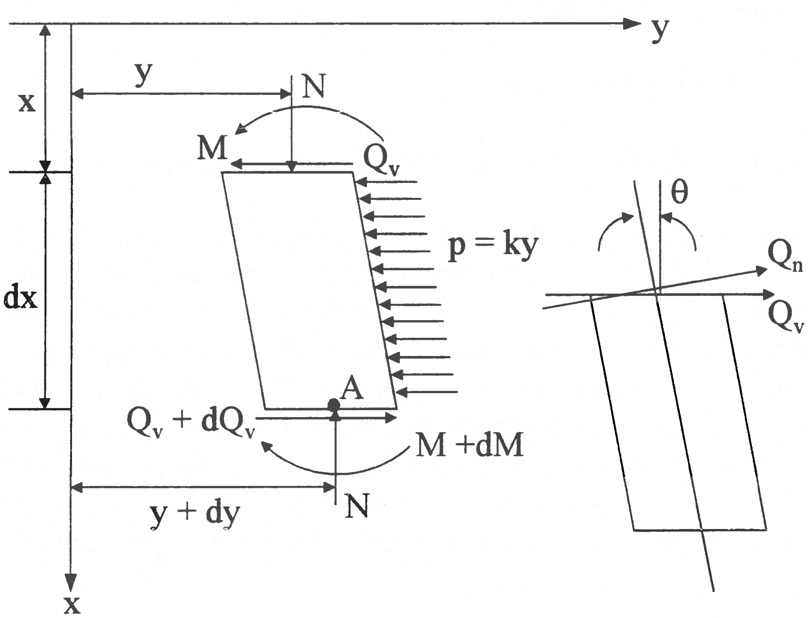


Figure 2.2: Element from Beam-Column (After Hetenyi, 1946)

The direction of the shearing force is shown in Figure 2.2. The shearing force in the plane normal to the deflection line can be obtained from the following:

$$Q_n = Q_v \cos\theta - N \sin\theta \qquad \text{Equation 2.5}$$

Since θ is usually small, one can assume the small angle relationships: cos θ = 1 and sin θ = tan θ = dy/dx. Then Equation 2.5 becomes:

$$Q_n = Q_V - N\frac{dy}{dx} \qquad \text{Equation 2.6}$$

A summary of equations that are used in analyzing piles under lateral load are:

$$EI\frac{d^4y}{dx^4} + N\frac{d^2y}{dx^2} = -p \qquad \text{Equation 2.7}$$

$$EI\frac{d^3y}{dx^3} + N\frac{dy}{dx} = -Q_v \qquad \text{Equation 2.8}$$

$$-EI\frac{d^2y}{dx^2} = M \qquad \text{Equation 2.9}$$

$$\frac{dy}{dx} = \theta \qquad \text{Equation 2.10}$$

where:

p = soil reaction per unit length
Q = shear in the pile
M = bending moment in the pile
N = axial load
θ = slope of the elastic curve defined by the axis of the pile

The p-y curve analysis solves the beam on elastic foundation (Equations 2.7-2.10) using a Winkler medium as the soil model. P-y curves describe the soil as a non-linear spring to characterize the force-deformation characteristics of the soil. The secant of the p-y curve is equal to the soil modulus, E_s. The modulus is essentially only a computation device (used to solve Equation 2.7) which varies with depth and pile deflection, and is not a unique soil property. An example of a family of p-y curve is shown in Figure 2.3. The p-y curves relate soil resistance (p) to pile deflection (y) at various depths below the ground surface. In general the p-y curves are non-linear and depend on several parameters of the pile and soil. These parameters include the diameter of the pile, depth along the pile, and the shear strength and unit weight of the soil.

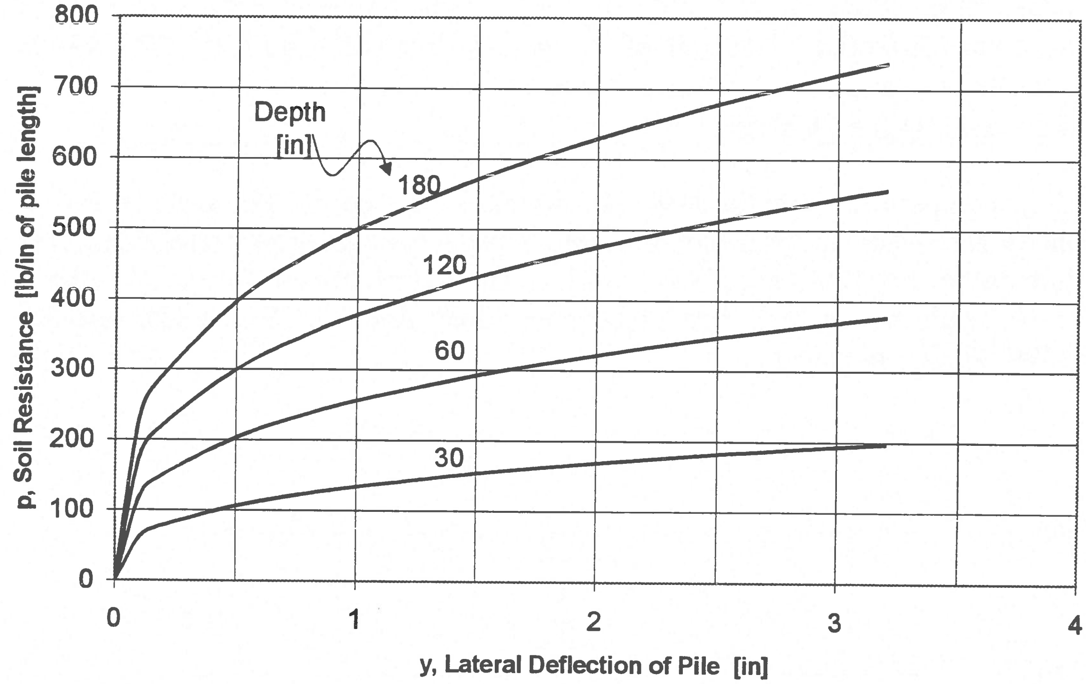


Figure 2.3: Example p-y curve

The curves may be developed from site specific soil test data (see 2.1.2.2) or from relationships developed from full-scale field load tests reported in the literature.

The secant of the p-y curve, E_s, is used as input into Equation 2.7 (in terms of p) which is then solved for y at various depths, x. This process is repeated until compatibility between the pile and soil is obtained. The analysis is then repeated for several load cases to obtain a load-deflection curve for the single pile. Reese and Matlock (1956) developed a hand solution using a p-y analysis based on E_s linearly increasing with depth. Matlock and Reese (1960) later improved upon this method when they developed a model that could handle a fixed form of variation of E_s with depth. Matlock and Reese (1961) again improved the p-y model developing a procedure that can handle arbitrary variations of modulus with depth. The hand solution for this procedure is iterative and very lengthy. However, the method has been programmed and is widely available in a number of programs. The most widely used are COM624 and LPILE.

An example is performed to provide insight into the procedure. The pile that is used for this example will be used in all further examples, so that comparisons between methods can be made. The characteristics of the pile and the soil were taken from a laterally loaded pile group at the intersection of Warner Road and the Price Freeway and are presented in Table 2.1.

Table 2.1: Pile and Soil Properties for Example

• Diameter (D) = 42 in	• Length (L) = 30 ft
• Modulus of Elasticity (E_p) = $4(10)^6$ psi	• Undrained Shear Strength (S_u)= 1 ksf
• Moment of Inertia (I_p) = $1.53(10)^5$ in^4	• Relative Stiffness Factor (T) = 134 in
• Coefficient of Variation of Subgrade Reaction (n_h)= 13.9 pci	
• Soil Modulus (E_s) = 2800 psi	

An analysis was performed on the single pile described previously. The analysis was performed using the computer program COM624 using p-y curves generated using the procedure outlined by Matlock (1970). The load-deflection curve in Figure 2.4 is the result of the single pile analysis. From this curve a deflection of 0.38 in at a horizontal load of 100 [kips] is obtained.

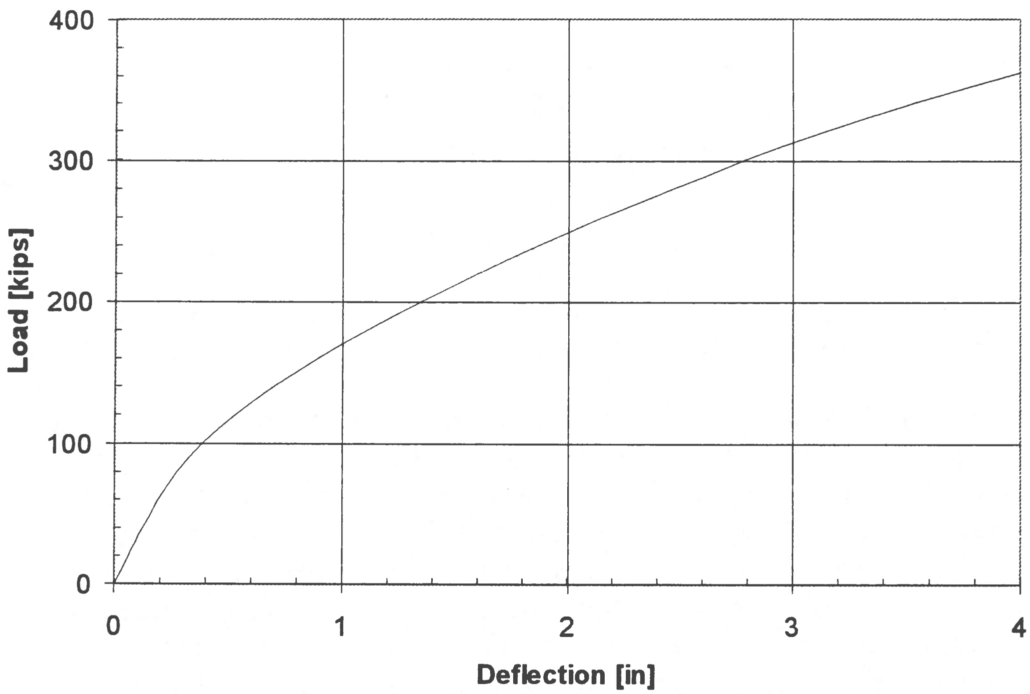


Figure 2.4: Example Load-Deflection Curve

2.1.2.2 Field Load Testing

The most straightforward approach to single pile design is to perform a field load test. A field load test would consist of installing a pile at the site with adequate instrumentation to measure the load and deflection. Once the data is obtained a load-deflection curve can be drawn for use in design. The engineer would enter the load-deflection curve with either a specified load or an allowable deflection and read off the corresponding deflection or load respectively. If the test is done at full scale, the results can be used directly in design.

To allow for the translation of the results to any scale, the engineer needs to instrument the pile with strain gauges prior to loading to measure the bending moment. This allows the engineer to determine the magnitude of the moment along the length of the pile. Graphical differentiation of the moment curve produces a shear diagram, which can be differentiated again to produce a soil reaction curve (or p-y curve). The soil reaction curve could then be used to estimate the behavior of other piles of different sizes as described in section 2.1.2.1. To obtain a complete curve of deflection with depth the engineer can double integrate the moment curve. This allows the designer to determine the point of fixity of the pile.

As an alternative to the installation of strain gauges, it is possible to simply measure the load and deflection of the pile head, as described above, and use trial and error to find a set of p-y curves which produces a good match to the measured load-deflection curve. Then COM624 can be used to calculate the shear and moment for design.

It is important to note that when a field load test is performed it is still necessary to perform a geotechnical investigation to determine the spatial heterogeneity of the site. If the site is relatively uniform, a single test may be sufficient to characterize the site.

2.1.2.3 Elastic Analysis

Another approach to the single pile problem is to model the foundation as an elastic continuum rather than isolated springs. An elastic continuum is an ideal, elastic, homogeneous, isotropic mass having constant elastic parameters E and ν_s (where E is the modulus of elasticity of the soil and ν_s is Poisson's ratio of the soil). It represents the case of complete continuity of the foundation. The most widely used solution for this foundation type for laterally loaded piles was done by Poulos (1971a). The displacements within the soil were evaluated using Mindlin's equation for horizontal displacement due to a horizontal load within a semi-infinite mass. In this model the soil is represented as an ideal elastic half space. Since it is an elastic continuum, possible local yielding between the piles and the soil is not taken into account. The pile used in Poulos' (1971a) analysis is assumed to be a thin rectangular vertical strip of width d, length L, and constant flexibility E_pI_p. The development of the relevant equations is lengthy and will not be repeated here. Conveniently, Poulos developed a hand solution to the single pile problem using charts of influence factors. Equations and influence factors were developed to solve for the displacement, moment, and rotation of a single pile for both free and fixed head conditions.

To illustrate Poulos' procedure, an example is presented of a fixed head pile subjected to horizontal load. The remaining cases are fully described in Poulos (1971a) and will not be discussed here. The pile and soil properties were presented in Table 2.1. The equation for the deflection of a fixed head single pile at the ground surface is as follows:

$$\rho = I_{\rho F} \frac{H}{E_s L} \qquad \text{Equation 2.11}$$

where:

H = Applied horizontal load
E_s = Soil Modulus
L = Length of the pile
$I_{\rho F}$ = Displacement influence factor for fixed-head pile subjected to lateral load, where:

$I_{\rho F}$ = function of K_R, pile flexibility factor and L/D, pile slenderness:

$$K_R = \frac{E_p I_p}{E_s L^4} \qquad \text{Equation 2.12}$$

where:

E_p = Young's Modulus of the pile
I_p = Moment of Inertia of the pile

From the example pile

$$K_R = \frac{\left(4x10^6 \ \frac{\text{lb}}{\text{in}^2}\right)\left(1.53x10^5 \ \text{in}^4\right)}{\left(2800 \ \frac{\text{lb}}{\text{in}^2}\right)\left((30 \ \text{ft})\left(\frac{12 \ \text{in}}{1 \ \text{ft}}\right)\right)^4} = 1.30x10^{-2}$$

$$\frac{L}{D} = \frac{30 \ \text{ft}}{42 \ \text{in}\left(\frac{1 \ \text{ft}}{12 \ \text{in}}\right)} = 8.6$$

Taking $L/D = 8.6 \approx 10$, because Poulos' charts begin with an L/D ratio of 10, Figure 2.5 reveals a value of $I_{\rho F} = 2.2$. Applying a horizontal load (H) of 100 [kips] results in a displacement, ρ, given by:

$$\rho = 2.2\left(\frac{100 \ \text{kips}\left(\frac{1000 \ \text{lb}}{1 \ \text{kip}}\right)}{\left(2800 \ \frac{\text{lb}}{\text{in}^2}\right)\left(30 \ \text{ft}\left(\frac{12 \ \text{in}}{1 \ \text{ft}}\right)\right)}\right) = 0.22 \ \text{in}$$

Note that this is substantially less than the previous estimate using p-y curves, 0.38 in, indicating the importance of the non-linear effects.

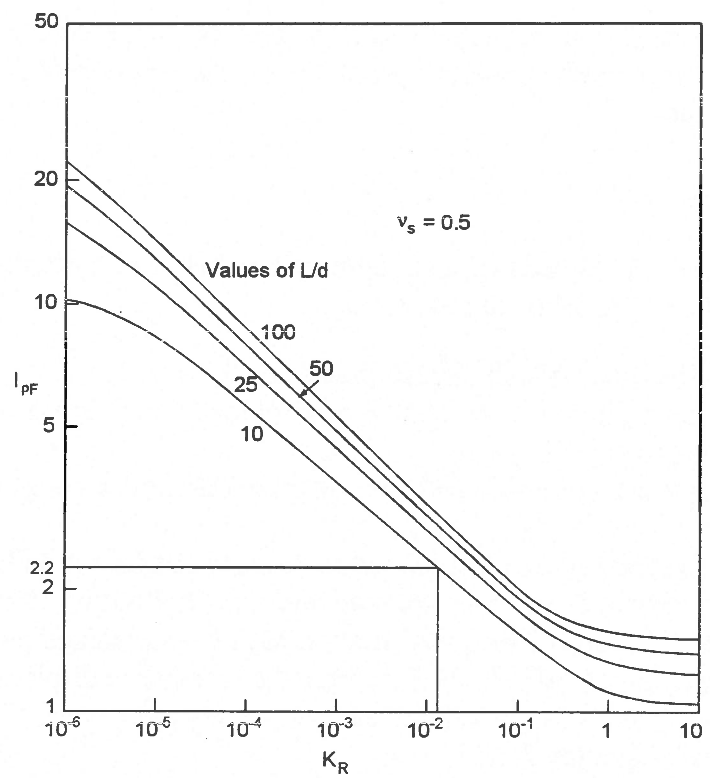


Figure 2.5: Influence Factors I_{pF}- Fixed Head Pile (From Poulos 1971a)

2.1.2.4 Modulus of Subgrade Reaction

The modulus of subgrade reaction is another method to solve the single pile problem. However, it is difficult to separate it from the pile group method, so it will be presented in that section (2.2.4).

2.2 Description of Analytical Approaches

There are a number of analytical methods used for the problem of a group of laterally loaded piles. The list of analytical methods encountered during the literature search includes the following:

1. Elastic analysis
2. Hybrid analysis
3. Group reduction factor design
4. P-multiplier design
5. Modulus of subgrade reaction reduction
6. Load-and-Resistance Factor Design philosophy (LRFD)
7. Group Amplification Procedure
8. Strain Wedge (SW) Method
9. Finite Element Modeling (FEM)

These methods will be described individually so as to develop an understanding of each method. Once this is accomplished, we will present a comparison, where possible, of the methods and their results.

2.2.1 Elastic Analysis

The elastic analysis method has been applied to pile group behavior by Poulos (1971b). Solutions are available for the following three cases:

1. A free-head pile group with all displacements equal
2. A free-head pile group in which equal horizontal load and or moment is applied to each pile in the group.
3. A fixed-head pile group in which all piles displace the same amount

Of these three cases, the third is the most related to Arizona conditions and is the only one described here. The remaining two cases are fully described in Poulos (1971b). Approximate solutions for displacement and rotation have been obtained by assuming that the principle of superposition holds. In other words, the increase in displacement of a pile due to all the surrounding piles can be calculated by summing the increase in displacement due to each pile in turn (Equation 2.13).

$$a = a_2 + a_3 + a_4 \qquad \text{Equation 2.13}$$

where:

a_2, a_3, a_4 are the values of the interaction factors for pile 1 due to piles 2, 3, and 4. Poulos provides figures of interaction factors, which are presented in Appendix B. The interaction factor is a function of spacing between piles, angle between the piles or departure angle (β)(Figure 2.6), and K_R (Equation 2.12). Once the interaction factors have been determined the displacement of the group can be calculated. The ratio of the displacement of the group, ρ_g, to the displacement, ρ_1, of a single pile carrying the same load as a pile in the group is:

$$\frac{\rho_g}{\rho_1} = 1 + a \qquad \text{Equation 2.14}$$

Figure 2.6 also shows the influence of adjacent piles on deflection. In Figure 2.6 a_1 is the deflection due to the load on pile 1, a_2 is the additional deflection of pile 1 due to the load on pile 2 and so on.

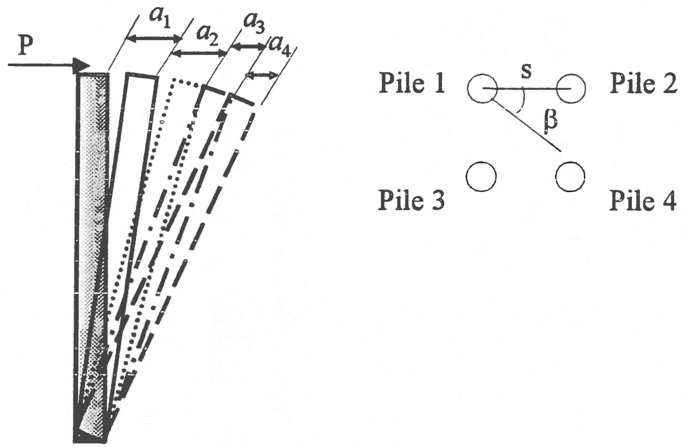


Figure 2.6: Increased Deflection of Pile due to Elastic Interaction of Piles in the Group

The general theory is now applied to the specific case of a fixed head pile group in which all the piles have the same deflection. The equation to determine the displacement of any pile k in the group with a fixed head subject to a horizontal load by superposition is:

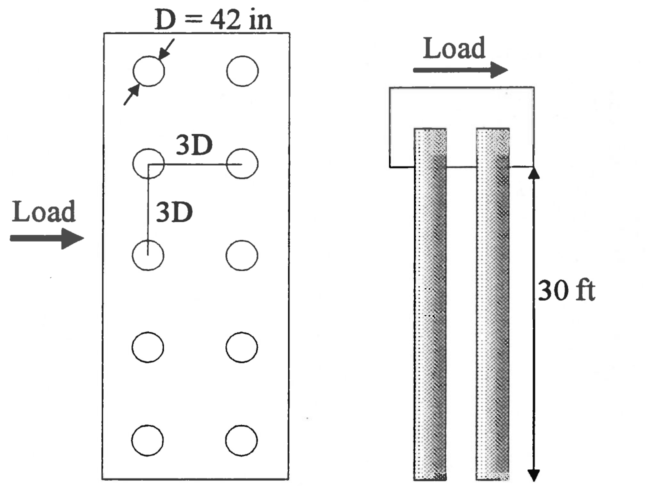
$$\rho_k = \overline{\rho}_F \left(\sum_{\substack{j=1 \\ j \neq k}}^{m} H_j \alpha_{\rho Fkj} + H_k \right) \qquad \text{Equation 2.15}$$

where:

$\overline{\rho}_F$ = the displacement of a single fixed-head pile due to a unit horizontal load
H_j = the load on pile j
$\alpha_{\rho Fkj}$ = the value of $\alpha_{\rho F}$ for two piles corresponding to the spacing between piles k and j and the angle β between the direction of loading and the line joining the centers of the piles k and j.

The theoretical value for the unit reference displacement, $\overline{\rho}_F$, for a single pile was previously described in the single pile section. The only difference from the single pile analysis is that the value of H is 1 (i.e. load applied is 1 to obtain $\overline{\rho}_F$.

A 10 pile group was analyzed having the same pile properties as the one analyzed for the single pile and having a group geometry as shown in Figure 2.7. For an average horizontal load per pile of 100 [kips] the resulting deflection is 0.67 in. In order to obtain a group reduction factor based on load, it is necessary to determine the amount of load the group can resist at a deflection level of 0.22 in (the deflection at a load of 100 kips from elastic analysis). At a deflection level of 0.22 in the load carried by the group was 328 [kips] or 32.8 [kips/pile]. This results in a reduction factor of approximately 33%. The step by step procedure is lengthy and, therefore, presented in Appendix B.



(a) Plan View (b) Elevation View

Figure 2.7: 10 Pile Group used in Example Analysis

The previous example was performed using the hand procedure as described in Poulos (1971b). This method has been added to the programs PIGLET (Randolph, 1980) and DEFPIG (Poulos, 1980). Some modifications that were made to the method include eliminating the need to assume uniform elastic properties. However, the method still doesn't consider the non-linear behavior of the soil.

2.2.1.1 Advantages and Disadvantages

The primary advantage of the elastic method is the ease in understanding the theory behind the method and the ability to follow a hand calculation. The elastic formulation does not lend itself to consideration of non-linear behavior so that it is less appropriate for higher deflection levels, and there is some doubt as to whether pile-soil interaction is finally accounted for, given that measured reduction factors for full-scale field tests greatly exceed values calculated with this method.

2.2.2 Hybrid Analysis

The hybrid analysis combines the theory of soil nonlinearity for a single pile analysis along with an elastic interaction analysis. The most widely used hybrid analysis was developed by Focht and Koch (1973). Simply stated, the analytical method is a solution of:

$$y_G = y_s + y_g \qquad \text{Equation 2.16}$$

where:

y_G = total group deflection at load P per pile
y_s = single pile deflection at load P per pile
y_g = additional deflection due to group effects

The theory behind this method is that the displacement of an individual pile subjected to lateral load is large enough to create plastic strain. Due to the presence of plastic strain, y_s is calculated using a non-linear p-y curve method. The additional displacement of a single pile due to the interaction with other piles in the group is assumed to be much smaller, and is therefore calculated using an elastic analysis. The group effects are calculated using an equation similar to Poulos' (1971b) approach. The only modification is replacing the understood value of unity preceding the shear load, H_k, acting on pile k by a relative stiffness factor, R, defined as y_s divided by ρ. Here y_s is the displacement of the single pile using nonlinear or p-y analysis, and ρ is the displacement of a single pile using elastic analysis.

$$\rho_k = \overline{\rho}_F \left(\sum_{\substack{j=1 \\ j \neq k}}^{m} H_j \alpha_{\rho Fkj} + RH_k \right) \qquad \text{Equation 2.17}$$

Equation 2.17 states that the total deflection of pile k is the "plastic" deformation, y_s, plus the integrated elastic effect of loads on all other piles in the group, which has been defined as y_g.

An example calculation was performed using the same pile group used in the elastic analysis (Figure 2.7). For an average lateral load per pile of 100 [kips], the resulting deflection is 0.84 in. In order to obtain group efficiency based on load it is necessary to determine the amount of load the group can resist at a deflection level of 0.38 in (deflection of a single pile with a 100 [kip] load using nonlinear analysis). At a deflection level of 0.38 in the load carried by the group was 454 [kips] or 45.4 [kips/pile]. This results in a group reduction factor of (45.4/100), or approximately 45%. Note that this value is derived for a higher deflection than that used for the elastic analysis in the previous section. As for the elastic approach, the step by step hand procedure is lengthy and is presented in Appendix C.

2.2.2.1 Advantages and Disadvantages

The hybrid analysis is an improvement over the elastic method, in that it accounts for the nonlinearity of the soil stress-strain relationship when analyzing a single pile. However, it does not account for nonlinear soil effects when dealing with the pile groups. The hand procedure is long and time consuming, and to the authors' knowledge a commercial program for this procedure is not available. Although the reduction factor of 45% calculated with this method approaches the lower bound of lab-measured reduction factors for small scale model tests (Figure 2.15), it is still far below the reduction factors measured for full scale pile groups in the field.

2.2.3 Group Reduction Factor Design

The group reduction factor design method requires, as input, a load-deflection curve for the single pile. The most common method of obtaining a load-deflection curve is to use p-y curve analysis. The engineer enters the load-deflection curve so developed at an acceptable level of deflection to obtain the single pile capacity. The single pile capacity is then multiplied by a reduction factor to obtain the lateral load capacity of each pile in the group at the deflection under consideration. In the literature, this reduction factor is often called efficiency. Many engineers and regulatory agencies appear to apply the factors specified by AASHTO as group reduction factors.

A brief example will serve to illustrate the group reduction factor design procedure. Assume a single pile analysis has been performed using a p-y curve approach. Based on this analysis, a single pile load-deflection curve is produced, and at a load of 100 [kips] the expected lateral deflection is 0.38 in (Figure 2.4). The spacing of the piles in the group is three diameters (3D), therefore, a group reduction factor of 0.25 was used following one interpretation of the AASHTO specifications. Using these parameters, the expected horizontal capacity of each pile in the group, at a deflection of 0.38 in is:

$$P_{\text{per pile}} = \left(P_{\text{single pile}}\right)E = (100\text{ kips})(0.25) = 25\text{ kips}$$

Until relatively recently, it was common in Arizona practice to use a group reduction factor of 1.0 for a spacing of at least 3D. With the arrival of the most recent AASHTO specifications, these values were dramatically reduced (Table 2.2). It is important to note that the factors presented in the AASHTO specifications maybe applied in another manner discussed later. An interim policy was adopted based on a brief literature study (Walsh et al., 1998).

Table 2.2: Reduction Factors for Group Reduction Factor Design

			Interim Policy	
Center to Center Shaft Spacing for In-Line-Loading	Historical Practice	Ratio of Lateral Resistance of Shaft in Group to Single Shaft (AASHTO)	Cap above soil	Cap in Contact w/ soil
8B	1.00	1.00	1.00	1.00
6B	1.00	0.70	0.84	0.92
4B	1.00	0.40	0.68	0.84
3B	1.00	0.25	0.60	0.80

2.2.3.1 Advantages and Disadvantages

The advantages of the group reduction factor design method are its simplicity and ease of application to design. The major disadvantage is that the group reduction factor is an average value for the entire group, and thus does not account for variations from row to row. Most recommendations provide only a single set of recommendations. Therefore, these factors are applied to a range of conditions and deflection levels. The reduction factors are based primarily on lab and field measurements, rather than theory. However, a limited number of finite element analyses have produced results generally consistent with field measurements. The procedure reduces the pile stiffness along with the soil stiffness, even though the pile stiffness is not affected by group interaction.

2.2.3.2 Definitions of Group Reduction Factors

Aside from the values presented in Table 2.2, other reduction factor values can be found in the literature. However, one of the most difficult challenges to overcome when one assesses group reduction factors in the literature is the lack of a widely accepted definition. Group reduction factors were found based on the following definitions:

- Computing the ratio of the average ultimate load on the piles in a pile group to the ultimate load of a single pile of the same size as each of the piles in the group.
- Computing the ratio of the load resisted by a pile in a pile group to the load resisted by a single pile of the same size as each of the piles in the group at some level of deflection, this deflection being the same for both the group and the single pile.
- Computing the ratio of the deflection of a single pile to the deflection of a pile group composed of the same size piles and at the same average load per pile.

Thus, there are different schools of thought regarding how the group reduction factor could be defined, and broadly these schools could be classed as either based on ultimate load, based on load, or based on deflection. It is the authors' belief that definitions based on the ultimate load are not generally applicable in the urban systems of central Arizona, as the deflection of the pile cap is typically limited in design to some low value (in the range of 0.5-1.0 in, or around 3 percent of the pile diameter for common drilled shaft sizes). However, such a definition may be implemented in parts of the state where seismic design may be more important. To the extent that linear elastic conditions exist in the soil, which is often assumed at low stress levels, the choice of evaluating the efficiency based on deflection or load should make no difference as the ratio should be more or less equivalent for either definition.

The issue is clouded even more by the structural boundary condition at the top of each pile. The pile head is idealized as either free to rotate at the top (free head pile) or fixed against rotation at the top (fixed head pile). For the most part, in field and model testing the single pile is a free head pile, because of the difficulty involved in fixing the top of a single pile. However, it is very common to assume that pile groups are connected by an effectively rigid pile cap and therefore cannot rotate. In analytical studies, it is no particular problem to enforce a fixed head boundary condition in either a single pile or a pile group, and this is often done in computational solutions. So, one finds rapidly that there are a number of potential definitions of efficiency, based on whether one chooses to consider loads or deflections, and based on how one chooses to fix the top of the pile. The possible definitions are summarized in Table 2.3.

Table 2.3: Definitions of Group Reduction Factors

	Single Pile Free Head, Group Piles Fixed Head	Single Pile Fixed Head, Group Piles Fixed Head	Single Pile Free Head, Group Piles Free Head
Definition Based on Ratio of Deflections	$\left(E_{free/fix}\right)_{\Delta}$	$\left(E_{fix/fix}\right)_{\Delta}$	$\left(E_{free/free}\right)_{\Delta}$
Definition Based on Ratio of Loads	$\left(E_{free/fix}\right)_{P}$	$\left(E_{fix/fix}\right)_{P}$	$\left(E_{free/free}\right)_{P}$

Given that so many bases for defining the group reduction factor exist, the problem that arises is that of knowing which definition to use for a given application. The solution to this problem is to use the definition which is most appropriate to the conditions of the design process. For purposes of design in Arizona, most piles are fixed into rigid caps so that rotation of the top is unlikely. Therefore, design based on a fixed group is probably best. The free/fix condition will probably only be useful when a free-head single pile load test is performed. Even in this case, it would be possible to use trial and error with COM 624 to find p-y curves which produce a match with the measurements from the free head pile load test. Then the fixed head single pile response could be computed and a definition of the reduction factor based on the fix/fix condition could be implemented.

Another issue is whether the definition should be based on load ratios or deflection ratios. In practice this selection would depend on how the designer intended to limit deflection. Based on the survey information to be presented later in this report, the most common approach in Arizona is to design based on a p-y curve developed from curves in the literature and scaled according to the observed soil properties as measured in laboratory tests and/or the experience of the geotechnical designer. This p-y curve is then used to obtain a load-deflection curve for the single pile with a fixed head using one of the available computer programs. Then, the load at the acceptable pile group deflection is picked from the single pile load-deflection curve, and this load is multiplied by the group reduction factor to obtain the allowable lateral load capacity of each pile in the group. Given this procedure, the most interesting reduction factors from the literature would be those which correspond to $(E_{fix/fix})_P$.

So, while in concept the group reduction factor appears to be relatively simple to apply, it has been found that the value one would use could potentially be related to a number of factors. First among these are the issues presented above, which relate directly to the definition of the group reduction factor. Additional dependencies exist, however. The group reduction factor is related as well to the deflection level, the soil type, the pile cap boundary conditions, and the relative stiffness of the pile and the soil. These dependencies will be treated individually in the following sections.

2.2.3.3 Impact of Definition Chosen

Lateral load reduction factors can be found which relate single pile response to group response in many ways, including deflection-based factors and load-based factors, including free-head or fixed-head conditions, including different boundary conditions for the pile group, and including different strain levels. An attempt was made to report group reduction factors solely in terms of the response of the single pile compared to the average response of the pile group, and only at strain levels expected in the prototype foundations for full-height abutment walls, on the order of 3% of the pile diameter. This meant entering the tables or figures of the papers reviewed to calculate group reduction factors, which in many cases were different from those reported by the authors of those papers. A complete tabulation of all group reduction factors found or calculated with the appropriate conditions and a bibliographic citation can be found in Appendix D.

In an effort to observe any differences that might exist, the data were separated by definition and by fixity at the ground surface. Figure 2.8 presents group reduction factors calculated as a ratio of loads at a deflection near 3% of the pile diameter and calculated as a ratio of deflections at a pile load able to create around 3% deflection, versus pile spacing. The data in Figure 2.8were derived primarily from actual pile load tests on single piles and pile groups. Most of the data points come from small-scale model tests and a small number of the data points come from analytical studies. At first glance, one might assume that the data points plotting above one are erroneous, given that group reduction factors should theoretically be one or less. However, these data points correspond to full scale load tests where the pile cap for the group was in contact with the soil and the single pile had no cap. Thus differing boundary conditions explain these data points.

The solid symbols in Figure 2.8 were developed from load-based definitions, the open symbols arise from deflection-based definitions. Furthermore, the tests with the single pile fixed and the group pile fixed (fix/fix) have been separated from the tests with both the single pile and the group free (free/free). While deflection-based reduction factors are generally slightly lower than load-based reduction factors, the differences do not appear to be extremely significant. Note from Figure 2.8 that the open symbols tend to be the lowest points in both graphs at most spacings shown, and the closed symbols tend to be the highest points. However, there are a number of exceptions, and the body of the data in these two figures show very similar trends.

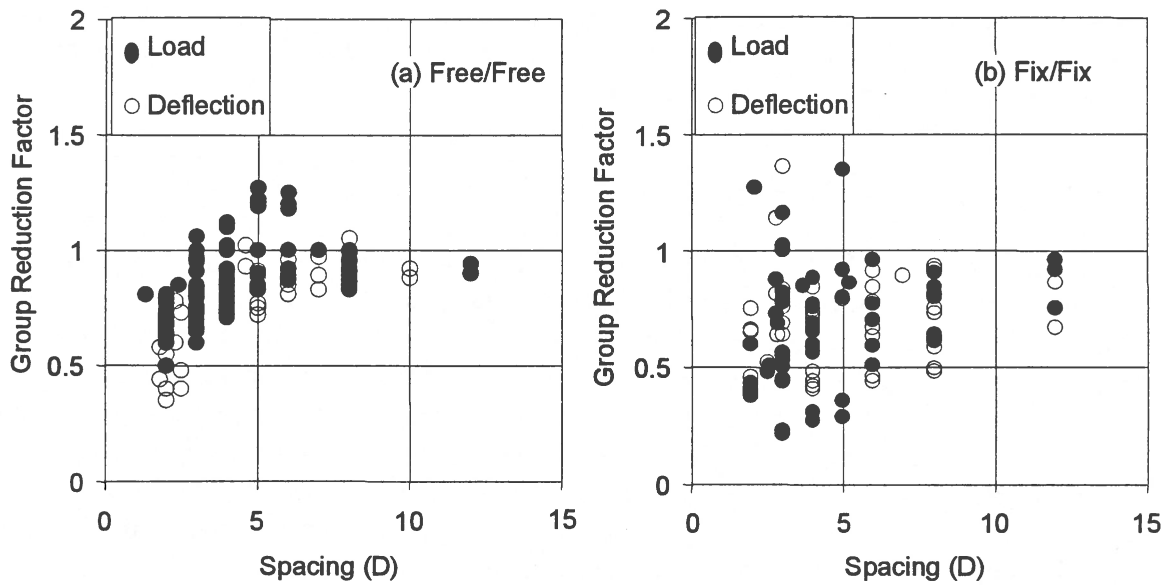


Figure 2.8: Effect of Basis of Definition

It is also apparent from Figure 2.8 that there are only minor differences between the free/free and fixed/fixed reduction factors. At any given spacing, the fixed/fixed results tend to have a slightly lower low-end, and the free/free results tend to have a slightly higher high-end, but the trends are again very similar. This result is not entirely unexpected as, in the free/free and fixed/fixed cases, the same rotational constraints at the pile head exist for the single pile and each pile in the group, so that the effect of a change in rotational constraint would appear in both the numerator and the denominator in the group reduction factor calculation. However, there are very significant changes introduced when the rotational constraint is different for the single pile and the piles in the pile group. Figure 2.9 shows the free/free and fixed/fixed results together with the free/fix load tests – these latter being those cases reported in which the single pile was tested free head, and the pile group was tested in a fixed head condition. Again, solid symbols are used for efficiencies based on a ratio of load, and open symbols for efficiencies based on a ratio of deflection.

The circular symbols in Figure 2.9 represent the free/fixed case, and represent in large part the increase in pile stiffness which arises when the rotational constraint changes from the free-head condition of the single pile to the fixed-head condition of the piles in the group, and hence the group reduction factor reported is very large. In the authors' opinion, this group reduction factor is only useful for the case when a lateral load test on a single pile is actually performed as part of the design process, and in fact all of the points shown result from exactly that sort of process. Given that lateral load testing of single piles is relatively rare in engineering practice, and because it is more appropriate to consider a consistent set of reduction factors, henceforth the test results corresponding to the free/fixed state will be reported only after conversion to a fixed/fixed state. This process requires a conversion factor for the difference in fixity and henceforth the tests shown in the free/fixed state will be reported only after conversion to a fixed/fixed state. This process requires a conversion factor for the difference in fixity of the single pile. In some cases, the appropriate factor was reported in the literature (e.g. Matlock and Foo's (1976) discussion of Kim and Brungraber (1976) contains the appropriate factor for that case). In other cases, a COM624 analysis was performed on the single pile, using the p-y curves given or assumed p-y curves, until the pile head load-deflection curve was matched very well for the free-head condition. This model was then rerun with the single pile changed to the fixed head case, allowing the ratio of fixed-head response to free-head response to be calculated. The exact value of this ratio is a function of the pile and soil properties, but seems to range from about 2 to about 2.4 for typical pile sizes.

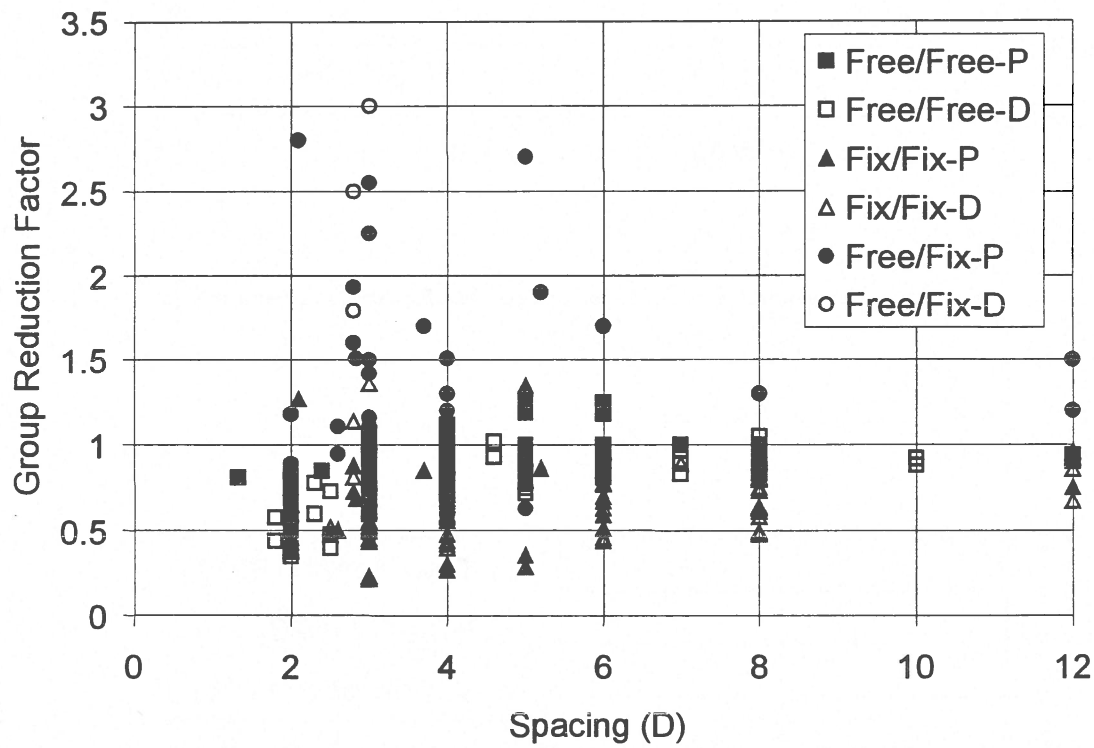


Figure 2.9: Effect of Fixity Condition

2.2.3.4 Deflection Level and Soil Type

As stated previously, the most interesting group reduction factor is that based on a ratio of loads. If a ratio of loads is used then they must be taken at a certain deflection level. The deflection level one chooses to determine group reduction factor has a large impact on the answer one obtains. The results from a few load tests reported in the literature have been analyzed to determine the effect of deflection on group reduction factor. Examples of these results are shown in Figure 2.10, Figure 2.11, and Figure 2.12.

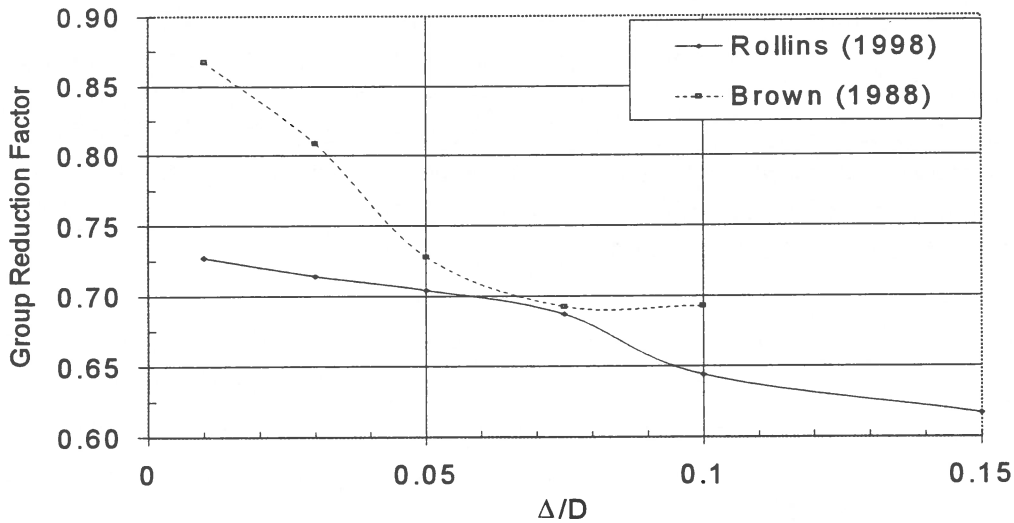


Figure 2.10: Group Reduction Factor versus deflection for Full-Scale Tests in Clay

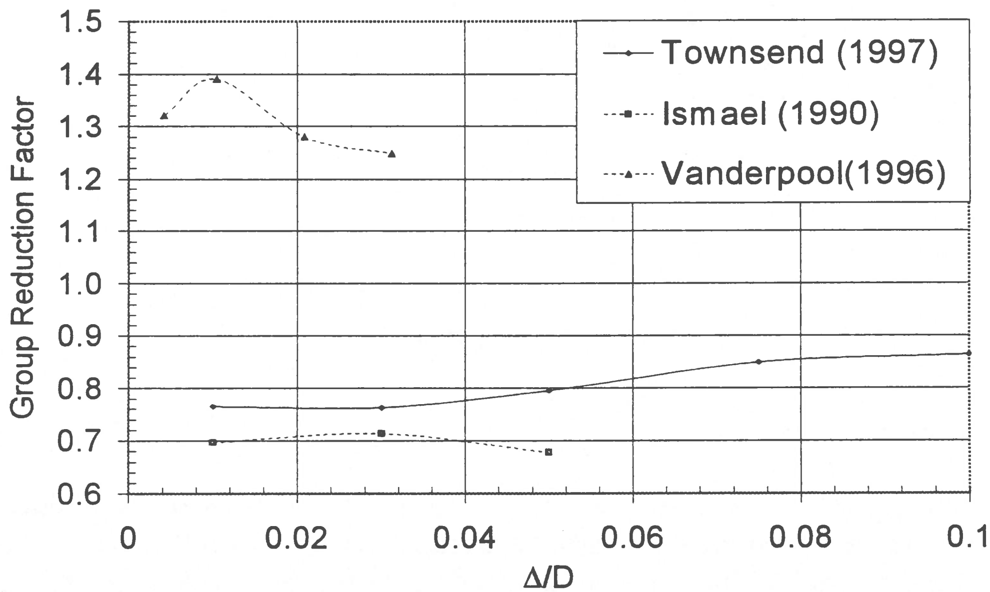


Figure 2.11: Group Reduction Factor versus Deflection for Full-Scale Tests in Sand

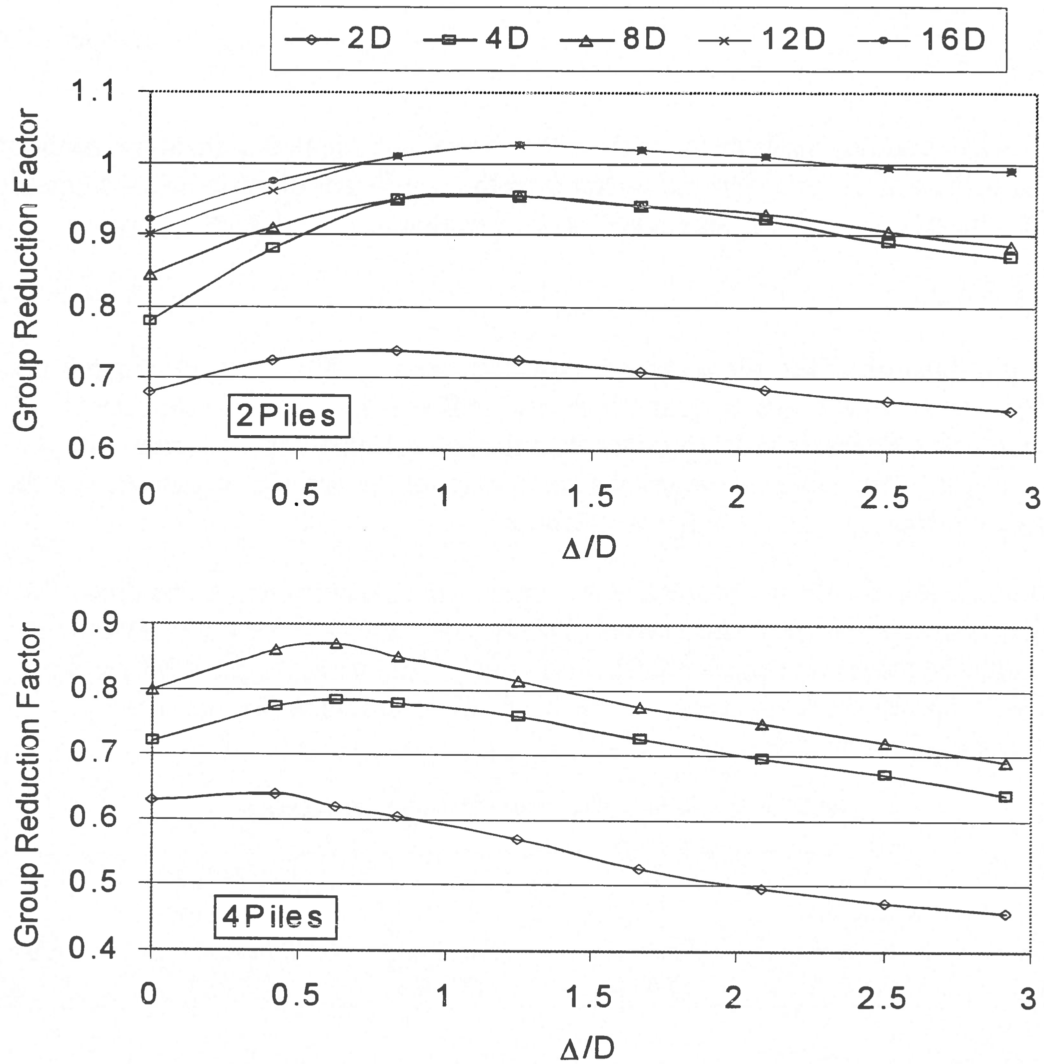


Figure 2.12: Group reduction factor versus Deflection for Model Pile in Sand (From Sarsby, 1985)

For full scale tests it appears that for clay soils (Figure 2.10) the group reduction factor has a tendency to decrease with increasing deflection. For sand (Figure 2.11) the trend is less clear. For small scale tests in sand the results of Sarsby (1985) are presented (Figure 2.12). At first glance it appears that the group reduction factor increases and then decreases at high levels of deflection. However, it is important to note that the model piles used in the study were 6 [mm] in diameter. Therefore, at all deflection levels up to about 50% of the diameter the group reduction factor increased.

2.2.3.5 Relative Stiffness of Pile and Soil

The most commonly used expression for relative stiffness of the pile compared to the soil is T, where:

$$T = \sqrt[5]{\frac{E_p I_p}{n_h}} \qquad \text{Equation 2.18}$$

Here, E is the modulus of elasticity of the pile material and I is the moment of inertia of the cross section of the pile. The parameter n_h is the coefficient of variation of subgrade reaction. The modulus of subgrade reaction, k, is related to n_h and depth, z, by

$$k = n_h z \qquad \text{Equation 2.19}$$

where k is a function of the pile width (as shown on page 5). Note that, although T is a parameter that is widely used to quantify relative stiffness, n_h is actually the rate of increase in soil stiffness with depth rather than the soil stiffness itself. Nonetheless, n_h tends to be higher for stiffer, stronger materials, particularly granular materials, and thus T provides a general indication of relative stiffness.

Prior to discussing the design procedure it is important to have an understanding of what the variables used in the procedure mean. This is very important for the coefficient of lateral subgrade reaction because different researchers and manuals use different notation. Table 2.4 compares the terminology for the modulus of subgrade reaction used in a number of important studies.

Table 2.4: Comparison of Modulus of Subgrade Terminology

Property of Interest	Matlock & Reese (1961)	Prakash (1962)	Davisson (1970)	*Canadian Found. Eng. Manual* (1985)	NAVFAC (1986)
Modulus of Subgrade Reaction	N/A	k	k	K_s	N/A
Coefficient of Variation of Subgrade Reaction*	k	n_h	n_h	n_h	f
Relative Stiffness (T)	$T = \sqrt[5]{\frac{E_p I_p}{k}}$	$T = \sqrt[5]{\frac{E_p I_p}{k}}$	$T = \sqrt[5]{\frac{E_p I_p}{k}}$	$T = \sqrt[5]{\frac{E_p I_p}{k}}$	$T = \sqrt[5]{\frac{E_p I_p}{k}}$

*Many of the researchers used different names when identifying this parameter. For the remainder of this report the names in the first column of the table and the variables as defined by Davisson will be used.

For typical conditions in full-height abutments in Arizona (3 foot diameter concrete shaft, soil properties on the order of ϕ=32°, c =500 [psf], relatively high density), one can compute a T value in the range of 90-100 inches. Model piles commonly reported in the literature typically have T-values which are much lower. It should be pointed out that if EI is low enough, T can be low for almost any soil one might encounter. This is illustrated in Table 2.5 below for a hypothetical 1/2 in diameter model pile made of aluminum.

Table 2.5: Range in *T* Values for a Pile of Very Low *EI*

Soil Type	n_h (pci)	EI (lb in^2)	T (in)
Very soft clay	1	30679	7.9
Very dense granular soil	60	30679	3.5

The values of T shown in Table 2.5 are typical of the values corresponding to almost all of the model tests reported in the literature. For example, a very important set of model tests frequently cited is that of Prakash (1962). For Prakash's tests, the 0.5 in O.D. aluminum tubes used for piles and the dense sand produced a T value of 2.85 in. Table 2.6 shows the range in T values for actual prototype piles that one might encounter in practice. These data show that for the full range of soil types, T ranges from about 36 in to about 350 in.

Table 2.6: Range in *T* Values for Common Prototype Piles

		Pile Types				
f = n_h [tons/ft^3]	Soil Type and Condition	1 ft dia. reinf. conc.	3 ft dia. reinf. conc.	5 ft dia. reinf. conc.	10.75 in dia. Steel, 3/8 in wall	8 ft dia. Steel, 2 in wall
3	very soft cohesive (S_u = 300 [psf])	65	157	236	68	355
5	very loose sand (D_r = 25%)	59	142	213	61	321
12	med. – stiff cohesive (S_u = 1000 [psf])	49	119	179	51	269
22	med. Dense sand (D_r = 50%), stiff - v. stiff cohesive (S_u = 2000 [psf])	44	105	158	45	238
55	Very dense sand (D_r = 93%), v. stiff cohesive (S_u = 4000 [psf])	36	88	132	38	198

Using the relative stiffness factor, T, one can develop an appreciation for the impact of soil-pile stiffness on the response. Consider a hypothetical condition in which a pile group is founded in very soft material of near zero shear strength, as shown in Figure 2.13. A horizontal load applied near the top of this group would create a deflection pattern similar to a cantilever beam, and the deflection would be mostly a function of the pile properties. This is because essentially no loads would be transferred to the upper soil of near zero resistance, each pile would deflect identically and carry the same horizontal load -- and also the same load as a single pile deflected to the same shape. Thus, one would expect the pile to behave in a linear elastic fashion, and the group reduction factor to be very near one whether the group reduction factor is evaluated from load or deflection. The effect of pile-soil interaction would be minimal. In this case, T would be very high, as EI would be a large number compared to a very small n_h, provided n_h for the upper very soft material were used. The important aspect of this example is that lateral load is not transferred to the soil quickly. As long as the lateral load remaining in the pile is high, the pile or group of piles behaves like cantilever beams and the efficiency is high as described above.

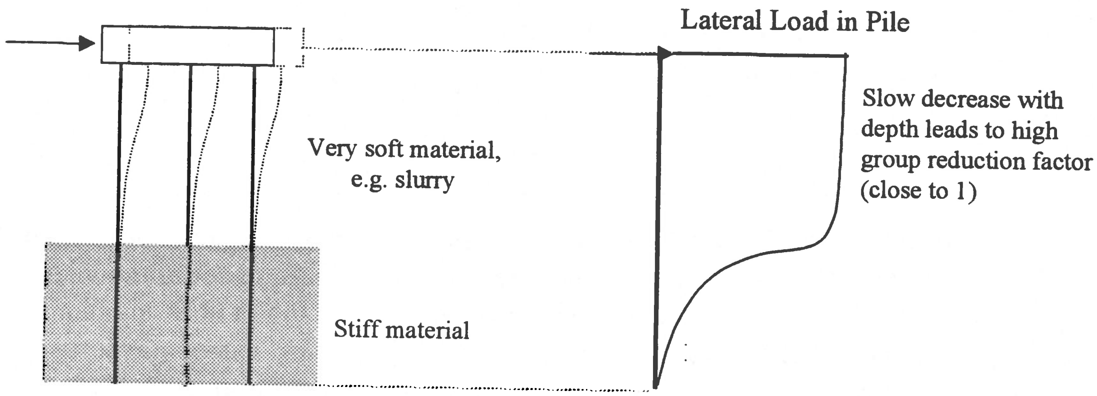


Figure 2.13: Pile Group in Very Soft Material and Lateral Load Remaining in Pile with Depth

At the other extreme, consider a very flexible pile in a very stiff uniform soil. In this case, the deflection would be heavily controlled by the soil behavior, because lateral load is quickly transferred to the soil and cantilever action is relatively less important, Figure 2.14. Overlapping stress fields lead to low group reduction factors. The load-based group reduction factor would be expected to be different from the deflection-based group reduction factor. The deflection would be a function of the stress state in the soil, and the stress in front of any one pile in the group will be increased by the stress distribution from other piles, tending to increase the deflection. In other words, loads would be transferred quickly to the soil due to the low flexural stiffness of the pile, stress superposition would affect all the piles in a group, and group reduction factor could be well below 1. In this case, *T* would be relatively low, as *EI* would be a smaller number relative to n_h.

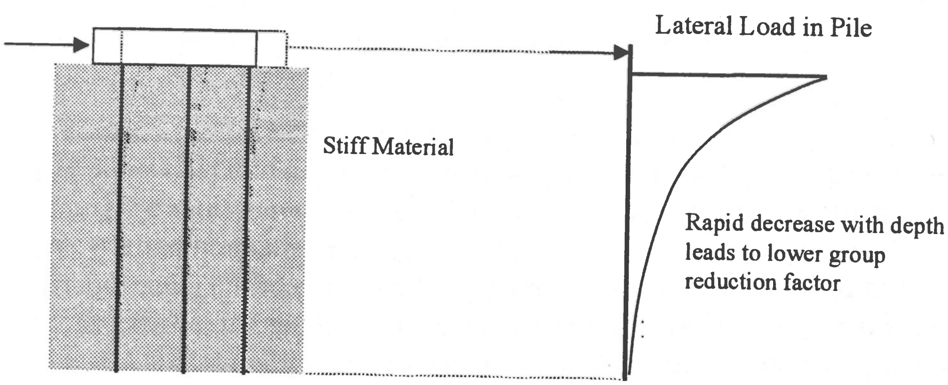


Figure 2.14: Pile Group in Uniform Material and Accompanying Load Remaining in Pile with Depth

In fact, one can observe that there is a difference in the group reduction factors reported in the literature for small scale (model, low-*T*) pile groups and prototype scale (full-scale) pile groups (Figure 2.15).

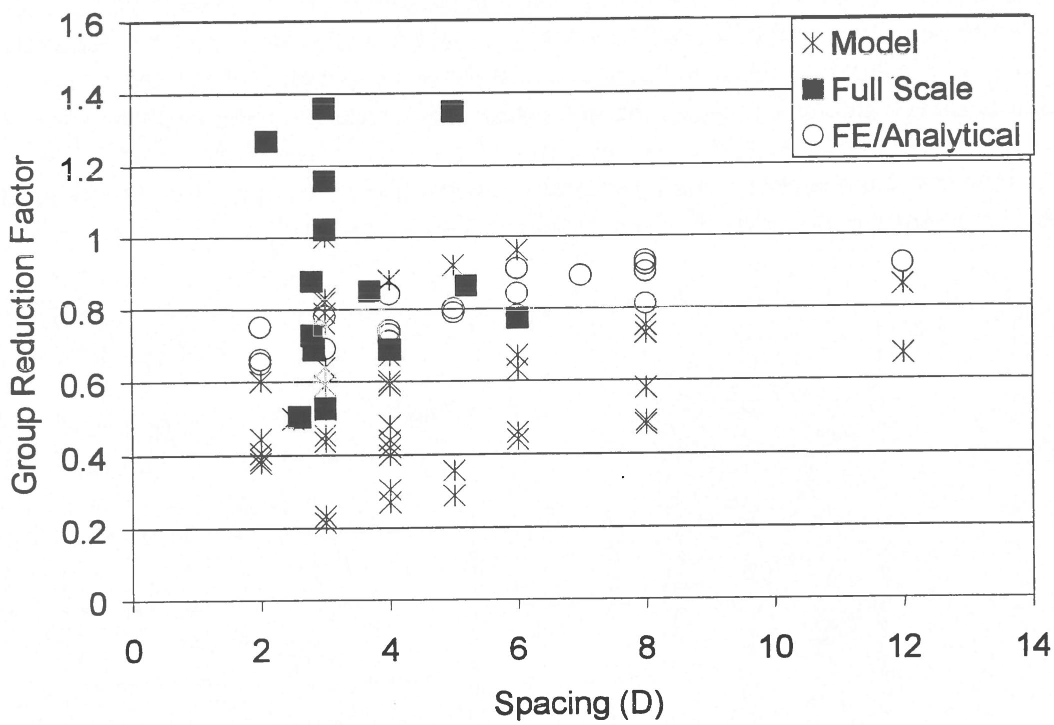


Figure 2.15: Effect of Type of Study for Fixed/Fixed Group reduction factor

The model tests typically have the lowest *T* values, and full scale results correspond to very high *T* values. Finite element studies may be performed with a larger simulated *T* than the model tests, but non-linear constitutive models are not often used, leading to somewhat higher degrees of stress overlap in most cases than might be expected in the field. Clearly, the lowest group reduction factors across the range of *T* values are for the model tests, and the highest are for the full-scale tests, with finite element tests somewhere in between.

The importance of T is made even more clear if one separates those results for which T of the test is in the neighborhood of a realistic prototype pile foundation (to be referred to as "high T," with a threshold of about 35 inches), from those for which T of the test has a value which is unrealistically low given the dimensions and materials used in prototype piles (to be referred to as "low T," values less than 13 inches). Clearly from Figure 2.16 there have been many more tests conducted with very low T than with a T which would be reasonable for prototype pile dimensions, no doubt because of cost.

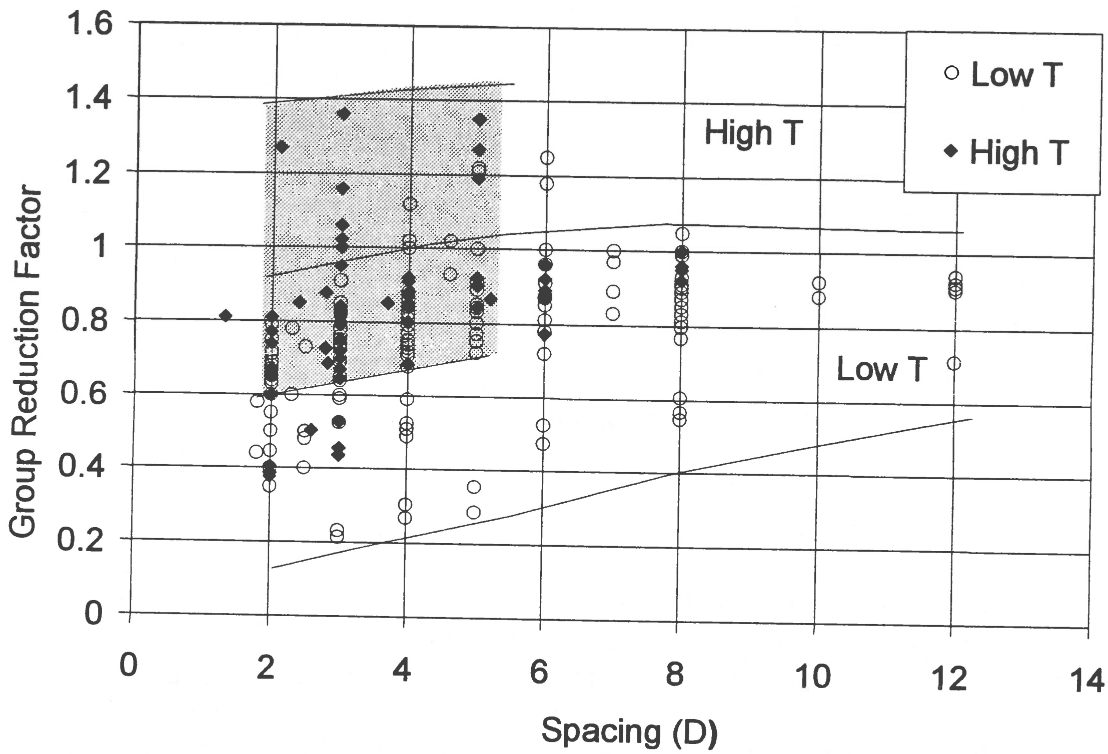


Figure 2.16: Effect of Relative Stiffness of Pile and Soil

The high-T tests form a significantly higher band than the low-T tests. At a spacing of 3D, the middle of the low-T range is approximately 0.65, while the middle of the high-T is approximately 1.0.

There are two important conclusions from these comparisons. First, the values of group reduction factor should be higher for piles of high T than for piles of low T. Second, in order to get reduction factors which are appropriate for full-scale field piles, it is necessary to use tests or analyses in which the T values of the test match the T values of the prototype piles.

2.2.3.6 Pile Cap Boundary Conditions

In the case of a buried pile cap, which is common for a full-height abutment in Arizona with drilled shafts to provide for added lateral resistance, the pile cap would be fairly stiff.

The soil which is in contact with the cap provides substantial resistance to rotation and lateral movement. These boundary conditions are typical for the abutment pile cap, but are not present for the single pile, which is very important because it accounts for the fact that essentially all of the full-scale lateral load tests on pile groups where abutment boundary conditions are matched show reduction factors above, and often well above, 1.0. The solid symbols on Figure 2.17 come from tests with pile cap boundary conditions similar to the prototype piles for full-height abutments, in that the cap was in contact with the soil surface. In the tests represented by the open symbols, the pile cap boundary conditions were unlike the prototype, in that the cap was not in contact with the soil surface. Clearly, this factor is extremely important; when the prototype boundary conditions are matched, a group reduction factor above one often results, even at a spacing of 3*D*.

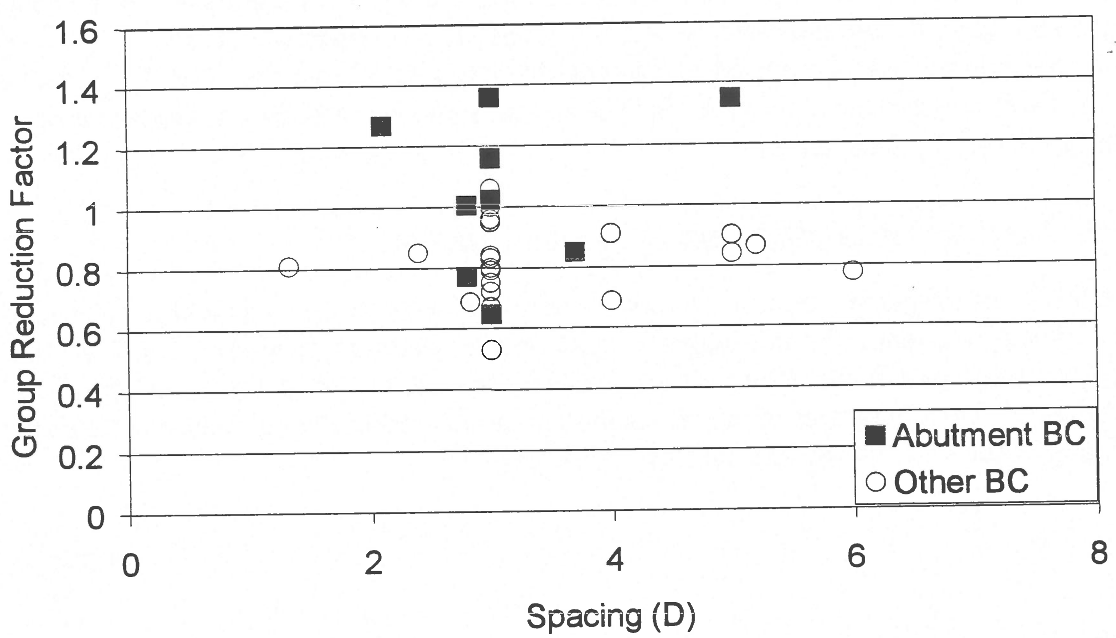


Figure 2.17: Prototype Scale Results

2.2.3.7 Use of AASHTO Guidelines

During the technical literature review the procedure outlined in the *AASHTO 1996 Standard Specifications for Highway Bridges* was reviewed. As ADOT and others have interpreted AASHTO, the specification for groups of laterally loaded drilled shafts (Sec. 4.6.5.6.1.4) states that the "effects of group action for in-line CTC (center-to-center) < 8B may be considered using ....the ratio of lateral resistance of shaft in group to single shaft." There is no other guidance on how to apply these ratios to the group action problem. Many have interpreted the word "resistance" in its common usage elsewhere in AASHTO as implying force, and accordingly developed an understanding that the values given for lateral load resistance are ratios of forces. The source in AASHTO is the Canadian

Geotechnical Society (1985). This was interpreted to mean the Canadian Foundation Engineering Manual (CFEM) which is published by the Canadian Geotechnical Society. This reference recommends applying the reduction factors to the coefficient of lateral subgrade reaction rather than reducing the lateral force capacity of the single pile. This method was further substantiated when examining the origin of the CFEM recommendations, in Davisson (1970). Davisson stated that the effective value of k (k_{eff}) for a pile group is less than that for a single pile. He provided the following relationships between spacing and k_{eff}:

$$
\begin{aligned}
@3D, k_{eff} &= 25\% k \\
@8D, k_{eff} &= 100\% k
\end{aligned}
\qquad \text{Equation 2.20}
$$

These relationships clearly show that Davisson intended the reduction factors to be applied to the modulus of subgrade reaction and not the single pile capacity. These recommendations have been used in *Foundations and Earth Structures* (1986). It is clear that the source referenced in the AASHTO specifications has a different interpretation than that in use by most states.

2.2.4 Coefficient of Lateral Subgrade Reaction Reduction

The theory of subgrade reaction is another solution of a foundation element under lateral load. The coefficient of lateral subgrade reaction k is related to E_s in Equation 2.21. It models the soil as a Winkler foundation, which is then used to solve the beam on elastic foundation. The coefficient of lateral subgrade reaction reduction procedure reduces the stiffness of the soil, but the pile stiffness is unchanged

$$k_0 = \frac{E_s}{D} \qquad \text{Equation 2.21}$$

where:

D = the pile diameter

The following procedure is used for calculating the lateral load capacity as stated in *Foundations and Earth Structures* (1986) also referred to as DM 7.2. The design procedure includes design for a single pile and reduction factors for pile groups. The procedure defined in DM 7.2 assumes that the lateral load does not exceed about one-third of the ultimate lateral capacity. The lateral load analysis is dependent on two criteria: the soil conditions and the loading conditions.

2.2.4.1 Soil Conditions

The soil conditions in-situ are modeled by the coefficient of lateral subgrade reaction, k. The value of k is a function of the soil type. DM 7.2 classifies soils into two categories: 1) granular soil and normally to slightly overconsolidated cohesive soil and 2) heavily overconsolidated cohesive soils

2.2.4.1.1 Granular Soil and Normally to Slightly Overconsolidated Cohesive Soil

Soils that fit into this category have in-situ k_0 values that increase linearly with depth. The formula used to define k_0 is:

$$k_0 = \frac{f \times z}{D} \qquad \text{Equation 2.22}$$

where:

k_0 = coefficient of lateral subgrade reaction [tons/ft^3]
f = coefficient of variation of lateral subgrade reaction [tons/ft^3]
z = depth ft
D = width/diameter of loaded area ft

Selection of f is dependent on whether the soil is fine-grained or coarse-grained. The value of f can be obtained from Figure E1 (Appendix E) using the unconfined compressive strength, Q_u, or the relative density of the soil, D_r.

2.2.4.1.2 Heavily Overconsolidated Cohesive Soils

When heavily overconsolidated cohesive soils are encountered, it is common to assume that the coefficient of lateral subgrade reaction is constant with depth, and defined within the limits presented below:

$$35 \times c < k_0 < 70 \times c \qquad \text{Equation 2.23}$$

where:

k_0 = coefficient of lateral subgrade reaction [tons/ft^3]
c = undrained shear strength

Soils fitting this description are analyzed using the analysis of beams on elastic foundation directly.

2.2.4.2 Boundary Conditions

There are three principal boundary conditions that were considered in the DM 7.2 procedure. The boundary conditions are: 1) pile with a flexible cap or hinged end condition (free); 2) pile with a rigid cap fixed against rotation at the ground surface; 3) pile with rigid cap above the ground surface. These principal loading conditions are illustrated with design procedures in Figure E2. The procedure for a fixed head pile subject to a horizontal load is presented because it most resembles Arizona conditions. The remaining procedures with accompanying figures are presented in Appendix E. The equation for the lateral deflection for a fixed head pile subjected to lateral load is presented for later use (Equation 2.24). The variables are defined in Figure 2.18.

$$\delta_p = F_\delta\left(\frac{PT^3}{EI}\right)$$ Equation 2.24

The design of pile groups must take into account the interaction of one pile with the other piles in the group. The recommendation in DM 7.2 is that group action should be considered when the spacing between piles is less than 8 pile diameters in the direction of loading. Group action is evaluated by applying a reduction factor, R, to the coefficient of lateral subgrade reaction, k, in the direction of loading (Table 2.7).

Table 2.7: Reduction Factor, R vs. Pile Spacing

Pile spacing in direction of loading D = pile diameter	Subgrade reaction reduction factor [R]
8D	1.00
6D	0.70
4D	0.40
3D	0.25

2.2.4.3 Application of R in Design

The application of the reduction factor is not entirely straightforward. The definition of k_0 was presented in Equation 2.22. It is clear that for a given pile/shaft geometry at a given depth, both D and z remain constant. Therefore, a reduction in k is equivalent to a reduction in f. One then multiplies f for the soil at the site by R to obtain a reduced value of f for the group, f_{group} (Equation 2.25).

$$f_{group} = f_{\sin gle} \times R$$ Equation 2.25

To obtain the lateral capacity of a pile in the group, T_{group} must be calculated using f_{group} in Equation 2.18, and then Equation 2.24 is solved for P using the data in Table 2.1 and Figure 2.18 and T_{group}. The capacity of the group is equal to the capacity obtained for a pile in the group with reduced k_0, multiplied by the number of piles in the group, N (Equation 2.26).

$$P_{\text{group}} = N \times P_{\text{reduced single}}$$ Equation 2.26

2.2.4.4 Advantages and Disadvantages

The DM-7.2 procedure, like the p-multiplier procedure, accounts for group effects by reducing only the soil stiffness, in this case through the modulus of subgrade reaction. A minor disadvantage of this method is that it is only applicable up to one-third of ultimate lateral capacity because the method assumes a linear relationship between load and deflection. The limitation on lateral capacity would be a problem when large loads associated with extreme events such as earthquakes, floods, or ship impact need to be considered.

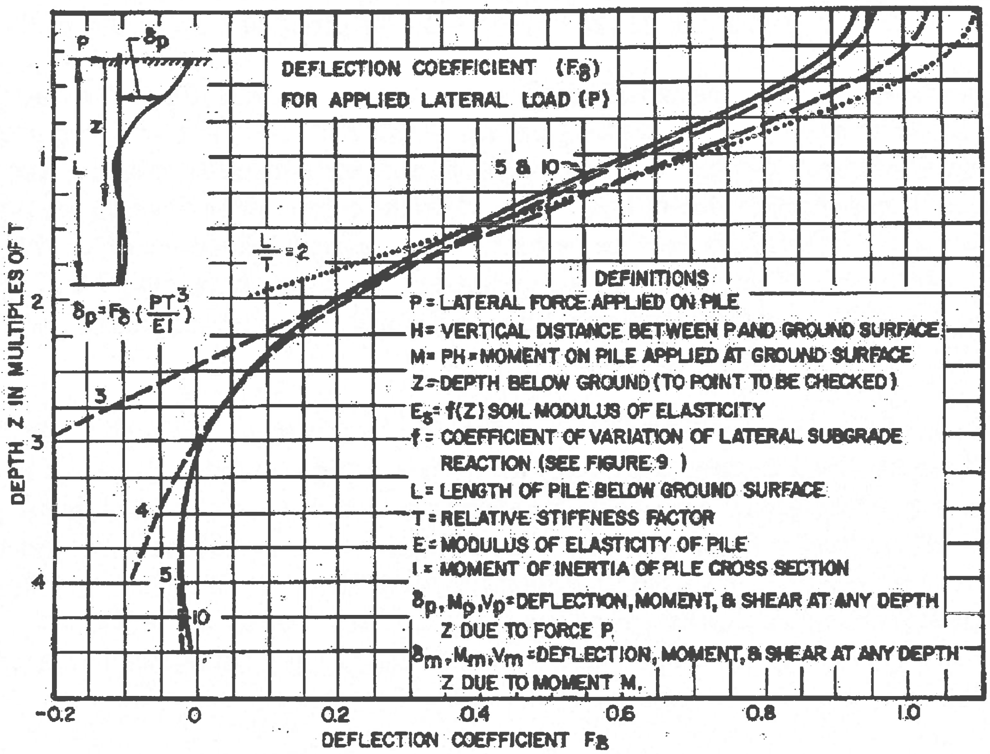


Figure 2.18: Influence values for modulus of subgrade reaction reduction procedure for pile fixed against rotation (from DM7.2 1986)

An example of the modulus of subgrade reaction multiplier method is presented using the soil and pile properties from Table 2.1. From the applied load, P = 100 [kips], Figure 2.18 gives $F_\delta = 1.05$. Solving Equation 2.24 using these properties results in $(\delta_p)_{single} = 0.42$ in.

$$\delta_p = F_\delta\left(\frac{PT^3}{E_pI_p}\right) = 1.05\left(\frac{(100\text{ kips})(134\text{ in})^3}{(6.22x10^{11}\text{ lb}-\text{in}^2)}\right) = 0.42\text{ in}$$

If the piles are spaced at 3D, the reduction factor R = 0.25 (Table 2.7) is applied to f_{single}.

$$f_{group} = f_{single} \times R = (13.9\text{ pci}) \times 0.25 = 3.5\text{ pci}$$

Then, T_{group} is calculated using f_{group}:

$$T = \sqrt[5]{\frac{E_pI_p}{n_h}} = \sqrt[5]{\frac{(6.22x10^{11}\text{ lb}-\text{in}^2)}{3.5\text{ pci}}} = 177\text{ in}$$

Using T_{group} results in a new value of $F_\delta = 1.10$ from Figure 2.18. Equation 2.24 is used to solve for P at $\delta_{group} = \delta_{single}$ below:

$$P = \frac{\delta_p}{F_\delta}\left(\frac{E_p I_p}{T^3}\right) = \frac{0.42 \text{ in}}{1.1}\left(\frac{\left(6.22x10^{11} \text{ lb} - \text{in}^2\right)}{\left(177 \text{ in}\right)^3}\right) = 42 \text{ kips}$$

The result is a lateral capacity of 42 [kips] per pile in the group at a deflection of 0.42 in.

An alternative approach for using the coefficient of lateral subgrade reaction reduction could be performed using the computer program COM624P. This alternative approach reduces the coefficient of lateral soil reaction, k, by the recommended reduction factor as in DM-7.2. The modified value of k is then used by the computer to develop the p-y curves internally. COM624P can than be run to obtain a load deflection curve with the reduced k p-y curves. This procedure is simplified compared with the DM-7.2 procedure. This procedure is also easier to implement due to the fact that many engineers are already using the COM624P or similar program.

There are significant drawbacks to this procedure. First, this method is only applicable to sands. The internally generated p-y curves for other soil types are not a factor of k. Therefore, a modification of the k value for a clay material doesn't effect the development of p-y curves resulting in no effect on the load-displacement curve of the shaft. Secondly, the k values for sands are only used to predict the initial slope of the p-y curve. This means that only the initial portion of the p-y curve is affected by the reduction in k. After relatively small deflections, the single pile and the group pile p-y curves are the same (Figure 2.19).

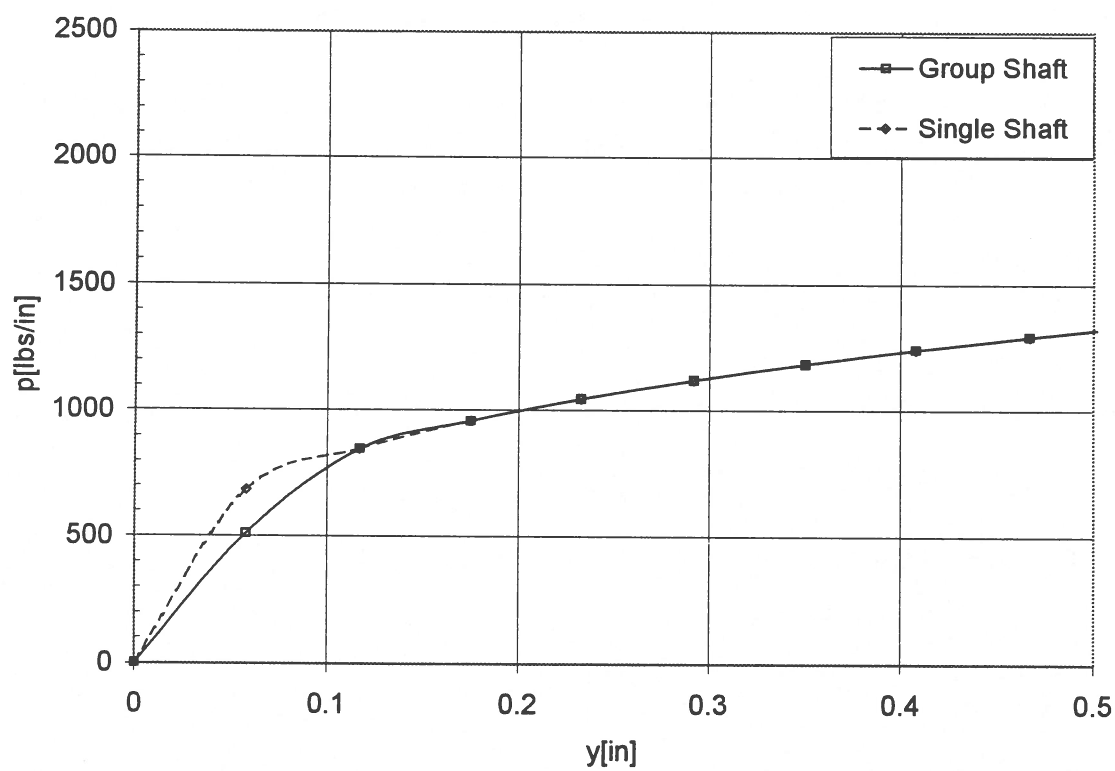


Figure 2.19: Comparison of p-y curves for a Single Shaft and a Shaft in a Group at a Depth of 5 ft.

2.2.5 P-multiplier Design

Another method used for the design of pile groups is the p-multiplier method, which is commonly attributed to Brown (1988). The p-multiplier method is similar to the modulus of subgrade reaction procedure except that the p-axis of the p-y curve is reduced instead of the modulus of subgrade reaction, k,. The p-y curve approach can be related to the modulus of subgrade reaction using the soil modulus, E_s. The equation relating the p-y curve to E_s is as follows:

$$E_s = \frac{p}{y} \qquad \text{Equation 2.27}$$

From the Equation 2.21 and Equation 2.27 it is clear that a reduction in p of the p-y curve is analogous to a reduction in k. The main difference between the two procedures is that E_s is independent of deflection for the modulus of subgrade reaction procedure while the p-multiplier method allow E_s to vary with deflection.

The following procedure is used for calculating the lateral load capacity as stated in *Design and Construction of Driven Pile Foundations*. GROUP and FLPIER are widely available and commonly used computer programs that employ the p-multiplier method to analyze a group of laterally loaded piles. Instead of modifying the load response of the single pile, the p-multiplier method involves modifying the p-y curve obtained for a single isolated pile by multiplying the p-values in the p-y curve by the p-multiplier, denoted as p_m. An example is presented in Figure 2.21. It is common to apply different p-multipliers to each row of piles/shafts in the group as a function of its position within the group, with respect to the loading conditions (Figure 2.22). There are a number of results and recommendations available for values of p_m (Table 2.8). It is, of course, advised that the engineer use p-multipliers from the case most similar to the design under consideration and in this context the lack of drilled shaft testing is significant for Arizona usage. The p-multiplier design approach can be summarized by the following steps (FHWA 1996):

Step1) Develop p-y curves for a single isolated pile

There are three ways a p-y curve can be developed: 1) an instrumented lateral pile load test; 2) based upon published correlations in the literature with soil properties; and 3) based on in-situ test data. LPILE and COM624P combine options 2 and 3 to develop p-y curves internally.

Step 2) Develop load-deflection and load-moment data

For each row in the pile group, generate a separate load-deflection curve using the p-y curve from Step 1, with p at each y multiplied by the appropriate value of p_m for that row.

Step 3) Develop load deflection curve for pile group

Enter the load-deflection curve for each row at a given deflection and obtain the lateral resistance for that row at that deflection. Sum the lateral resistance for all the rows together to obtain the lateral resistance of the group, and plot against the deflection assumed. (This procedure is only applicable if all rows in the group have the same number of piles, otherwise the procedure should be done on a pile by pile basis.) Repeat the procedure for several deflections to obtain a load-deflection curve for the pile group.

Step 4) Group lateral capacity

To obtain the lateral capacity of the pile group enter the group load-deflection curve at an acceptable level of deflection for the design.

Step 5) Evaluate pile structural acceptability.

Once the deflection and loading criteria have been established, it is necessary to evaluate the structural adequacy of the piles. First, plot the maximum bending moment, from the computer analysis, versus deflection for each row of piles. Determine the maximum bending moment and the resulting stress for a single pile in each row. Check if the maximum pile stress is less than the pile yield stress, if not, the design must be modified.

Step 6) Refine evaluation

Refine the pile group response evaluation taking into account the superstructure-substructure interaction.

Programs that solve only the single pile problem, such as LPILE or COM624P will be used for Steps 1 through 3 only, and the balance will be completed by hand. Computer programs such as GROUP and FLPIER complete Steps 1 through 3 internally, with the remaining steps done by the engineer. These programs have default p-multipliers that can be modified by the engineer to site specific conditions using engineering judgement.

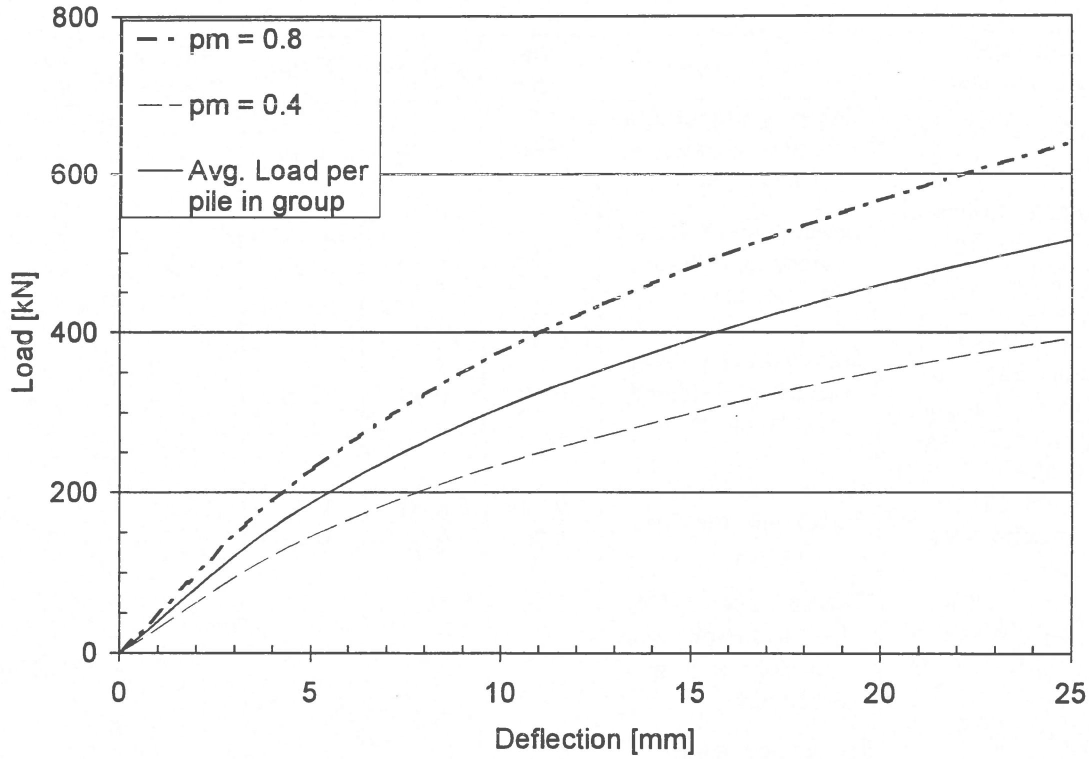


Figure 2.20: Load-deflection curve for p-multiplier example

An example to illustrate the procedure is as follows. The single pile is the same as used previously for the group reduction method. At a load of 100 [kips] the expected lateral deflection is 0.38 in. The pile group is composed of ten piles in two rows (Figure 2.7). Figure 2.20 shows the load-deflection curves for the pile group, a single pile with a p-multiplier equal to 0.8, and a single pile with a p-multiplier equal to 0.4. These p-multipliers are used for the first and second row of the pile group using the FHWA (1996) recommendations. The average load-deflection curve for each pile in the group is obtained by adding five times the first row plus 5 times the second row (there are five piles in each row) then dividing by ten. From Figure 2.20 the average load capacity for each pile in the group is equal to 67.5 [kips) at a deflection of 0.38 in (9.65 [mm]).

Table 2.8: P-multipliers vs. Row Position

Reference: Soil Properties	Pile Properties	Lead Row	2^{nd} Row	3^{rd} Row	Trailing Rows	Last Row
Rollins (1998) Clayey silt (CL-ML, ML);	Driven steel pipe pile filled with concrete. D=0.324 [m]	0.60	0.40	0.40	——	——
Ruestra and Townsend (1997) Loose fine sand (SP) Dr ≈ 20-40%	Jetted/driven 0.76 [m] square prestressed concrete pile	0.80	0.70	0.30	0.30	——
Brown et al. (1988) Clean medium sand (SP) Dr ≈ 50% sand placed after driving	Driven 0.272 [m] OD steel pipe pile filled with grout	0.80	0.40	0.30	——	——
Brown et al. (1987) Stiff clay (CL to CH); over-consolidated by dessication	Driven 0.272 [m] OD steel pipe pile filled with grout	0.70 0.70	0.60 0.50	0.50 0.40	——	——
Meimon et al. (1986) Silty clay (CL); Su = 500 [psf]; ϕ' =38-42°; c' = 0	Driven 0.284 x 0.270 [m] steel H-pile with side plates to form a box section	0.90	0.50	——	——	——
McVay et al. (1995) medium dense sand Dr ≈ 55%, Centrifuge	Driven open-ended pipe pile 0.43 [m]	0.80	0.40	0.30	——	——
McVay et al. (1995) loose medium sand Dr ≈ 33%, Centrifuge	Driven open-ended pipe pile 0.43 [m]	0.65	0.45	0.35	——	——
Pinto et al. (1997) sand Dr ≈ 55%, Centrifuge	Driven open –ended aluminum pipe pile; D = 0.43 [m]; L = 13.3 [m]	0.80	0.45	0.30	——	——
Pinto et al. (1997) sand Dr ≈ 33%, Centrifuge	Driven open –ended aluminum pipe pile; D = 0.43 [m]; L = 13.3 [m]	0.65	0.45	0.35	——	——
Zhang (1999) uniform low to medium dense sandy silt, ϕ' =35	Bored concrete shaft D = 1.5 [m]	0.50	0.40	0.30	——	——
Zhang (1999) uniform low to medium dense sandy silt, ϕ' =35	Driven 0.8 [m] square prestressed concrete piles	0.90	0.70	0.50	0.40	——
GROUP (1996)	Recommendation	*			**	——
FLPIER (1996)	Recommendation	0.80	0.40	0.30	0.20	0.30
FHWA (1996)	Recommendation	0.80	0.40	0.30	0.30	——

*Leading Row $p_m = 0.7309\left(\frac{s}{D}\right)^{0.2579}$ **Trailing Rows $p_m = 0.5791\left(\frac{s}{D}\right)^{0.3251}$

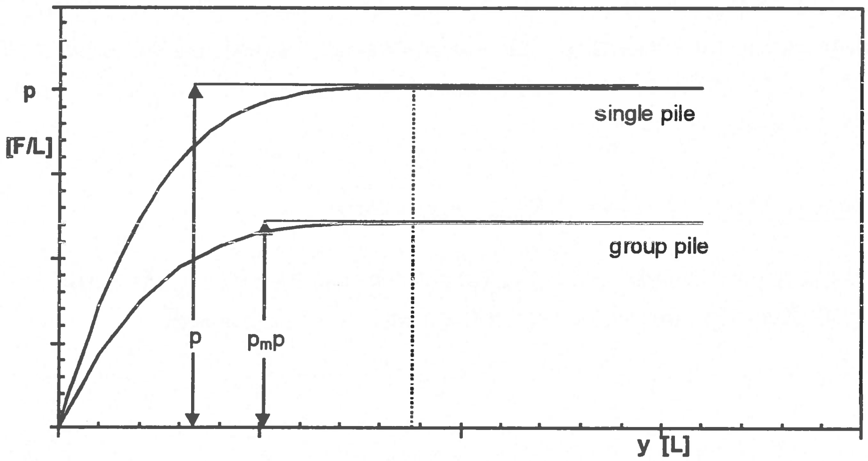


Figure 2.21: Example of p-multiplier

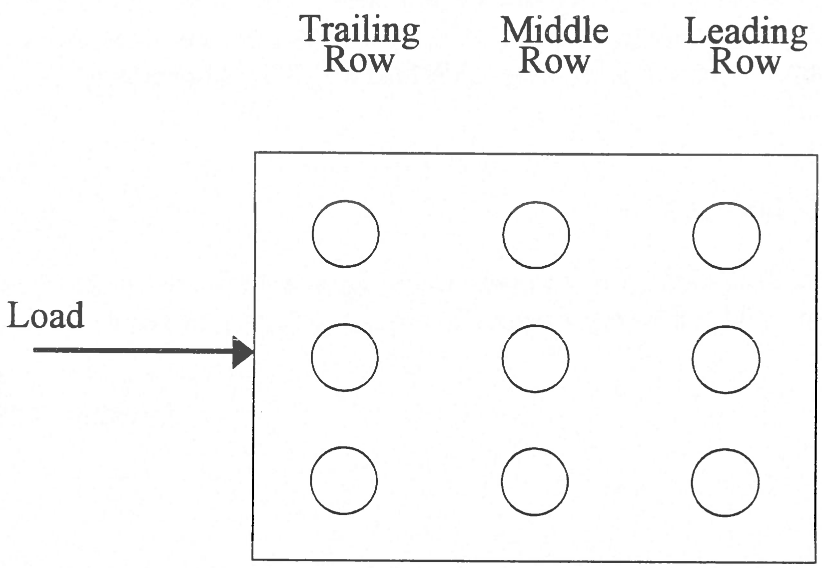


Figure 2.22: Identification of Rows with respect to Applied Load

2.2.5.1 Advantages and Disadvantages

In the p-multiplier method, the soil stiffness is reduced by the p-multiplier; the pile stiffness is unchanged. The p-multiplier accounts for the fact that not all the piles resist the same amount of load for a group in which all piles deflect equally, so one can choose to design for the worst case pile. However, as pointed out earlier, not all structural designers will take advantage of this result. The method allows the soil modulus, E_s, to vary independently with depth. The major disadvantage of the p-multiplier method is that the same p-multiplier is used for the entire length of the pile. This is a disadvantage because the stress overlap, and thus the degree of influence of surrounding piles, is a function of pile deflection, and the pile deflection varies along the length of the pile (Norris, 1994). Furthermore, nearly all p-multiplier recommendations have been developed from tests on driven piles (Table 2.8), and there is some evidence that the behavior of groups of drilled shafts is somewhat different (Zhang, 1999).

2.2.6 Load-and-Resistance Factor Design (LRFD) Procedure

LRFD is a relatively new design procedure with regards to geotechnical engineering. Therefore a complete and thorough background on the method is presented.

2.2.6.1 History

Commonly design in the U.S. has been performed using the allowable stress design (ASD) method, in which a factor of safety is used to take into account all uncertainty in loads and material resistance. At the end of the 1980s, a new specification was under development in which the uncertainty in load(s) and material resistance(s) were to be represented by factors. Uncertainties in load would be represented by load factors, which generally would have a value greater than one. The uncertainties in material resistance would be represented by resistance factors, which generally have a value less than one. A relevant procedure based on this approach was approved by ASSHTO in 1994. Adoption is scheduled in the near future.

2.2.6.1.1 Allowable Stress Design (ASD)

As previously mentioned the ASD method is the common design method in the U.S.. The general design procedure for ASD can be represented by Equation 2.28 as follows:

$$\frac{R_n}{FS} \geq \Sigma Q_{applied} \qquad \text{Equation 2.28}$$

where:

R_n = Nominal (ultimate) resistance
FS = Factor of safety
$\Sigma Q_{applied}$ = Summation of all applied forces

In general the factor of safety is qualitative and not directly related to the uncertainties associated with the load or resistance. The engineer determines the factor of safety based upon his/her experience and the current state of practice.

2.2.6.1.2 Load and Resistance Factor Design (LRFD)

The LRFD method is a quantitative procedure. The procedure attempts to quantify the risks and uncertainties associated with the safety of a system in mathematical terms, in a reliability-based design method. The basic LRFD equation is as follows:

$$\Sigma \gamma_i Q_i \leq \phi R_n \qquad \text{Equation 2.29}$$

where:

γ_I = Statistically-based load factor, generally greater than one
Q_I = Load
R_n = Nominal (ultimate) resistance
ϕ = Statistically-based resistance factor, generally less than one.

2.2.6.2 Limit States

In the LRFD method the safety of a structure is evaluated at various limit states. A limit state is defined as a condition beyond which a structural component, such as a foundation or other bridge component, ceases to fulfill the function for which it was designed. The limit states which must be evaluated in the AASHTO LRFD specification (AASHTO, 1997a) method are:

- Service Limit State
- Strength Limit State
- Extreme Limit State
- Fatigue Limit State

Each limit state is explained briefly below. It is important to note that several load combinations define each limit state. Therefore, several load cases must be checked at each limit state.

2.2.6.2.1 Service Limit State

Service limit state refers to the structural performance of the structure under service load conditions. Evaluation of a structure at this state is performed to determine if the bridge components under regular service conditions exceed restrictions on stress, deflection, and crack widths.

2.2.6.2.2 Strength Limit State

The strength limit state refers to providing sufficient strength or resistance to counteract the applied loads for the statistically significant load combinations that a bridge is expected to experience in its design life. The strength limit state is reached when either partial or total collapse of the structure occurs.

2.2.6.2.3 Extreme Limit State

The extreme limit state refers to the structural survival of a bridge during extreme events. An extreme event doesn't always occur during the life of the structure. There is some probability associated with it happening during the life of the structure. The events that are considered extreme events include earthquakes, floods, or collision by vessel, vehicle, or ice flow. Due to the classification of these events as extreme, the probability of these events occurring simultaneously is extremely low, therefore, these events are analyzed separately. However, there is an exception to this rule. Flooding, due to the possibility that it can occur in combination or as a result of other extreme events, is often considered in conjunction with other extreme events.

2.2.6.2.4 Fatigue Limit State

The fatigue limit state is an analysis of the structure due to repeated loads. A set of restrictions is placed on the stress range that would be caused by repeated loading of a design truck.

2.2.6.3 Load and Resistance Factors

The LRFD method uses both load and resistance factors in the design process. The actual values of these factors vary depending on the limit state under consideration. However, even though the values are different, the criteria that are considered in determining their values do not change.

2.2.6.3.1 Resistance Factors

The resistance factor, ϕ, for a particular limit state must account for uncertainties in (FHWA, 1998):

- Material properties (variability)
- Reliability of the equations used to predict resistance, i.e. amount of scatter in the data.
- Quality of construction workmanship. Normally a structure is designed assuming that good construction practices will be used.
- Extent of soil exploration, i.e. amount of knowledge the designer has about the site. The more detailed the soil exploration the better the designer is able to account for the heterogeneity of the site.
- Consequence of failure. As the importance of a structure increases then the consequence of failure also increases.

2.2.6.3.2 Load Factors

The load factor, γ_i, chosen for a particular load type must consider uncertainties in (FHWA, 1998):

- Magnitude of loads. As the certainty of the designer about the estimated loads on the structure decreases the load factor increases.
- Arrangement (positions) of loads. The arrangement of loads is very important. There are some cases where an increase in the load will help the stability of the structure. In this case a load factor close to 1 will be chosen.
- Possible combinations of loads. The combination of loads is taken into account as to the probability of certain loads acting in combination with one another or separately.

2.2.6.4 General LRFD Equation

The general LRFD equation is composed of three components: load modifiers, load factors, and resistance factors. The previously presented equation for LRFD (Equation 2.29) assumes that the load modifier, η_i, was equal to 1 and was not included. The complete equation is presented in Equation 2.30.

$$\Sigma \eta_i \gamma_i Q_i \leq \phi R_n \qquad \text{Equation 2.30}$$

2.2.6.4.1 Load Modifiers

In the AASHTO LRFD specification, each factored load is adjusted by a load modifier, η_i, to account for the combined effects of ductility, η_D, redundancy, η_R, and operational importance, η_I. There are two limiting conditions for η_i that must be satisfied.

Loads for which a maximum value of γ_i is appropriate:

$$\eta_i = \eta_D \times \eta_R \times \eta_I \geq 0.95 \qquad \text{Equation 2.31}$$

For loads for which a minimum value of γ_i is appropriate:

$$\eta_i = \frac{1}{\eta_D \times \eta_R \times \eta_I} \leq 1.00 \qquad \text{Equation 2.32}$$

There are primarily only two limit states that consider the load modifier: strength and service. For design the following guidelines are presented for selection of the load modifier. The design guidelines vary depending on the limit state under consideration.

Designing at the strength limit state, values of η_i range as follows (FHWA, 1998):

- Ductility - η_D
  - $\eta_D \leq 1.05$ for non-ductile components and connections
  - $\eta_D = 1.00$ for conventional designs and details

    - $\eta_D \geq 0.95$ for components and connections for which additional ductility enhancing measures are specified

- Redundancy - η_R
    - $\eta_R \leq 1.05$ for non-redundant components and connections
    - $\eta_R = 1.00$ for conventional levels of redundancy
    - $\eta_R \geq 0.95$ for exceptional levels of redundancy

- Operational Importance - η_I
    - $\eta_I \leq 1.05$ for important structures
    - $\eta_I = 1.00$ for typical structures
    - $\eta_I \geq 0.95$ for relatively less important structures

The criteria used to determine the operational importance of a structure should be based on social, survival and/or security or defense requirements.

For design at the service limit state, $\eta_D = \eta_R = \eta_I = 1.0$.

2.2.6.4.2 Load Factors

The primary loads of concern to a geotechnical engineer are earth loads, surcharge, and downdrag. These loads can be classified into two main categories: permanent and transient.

2.2.6.4.2.1 Permanent Loads

Permanent loads are defined as those loads that are considered to be acting on the structure at all times. The following is a brief list of permanent loads (FHWA, 1998):

- Dead
    - structural components
    - utilities
    - vertical pressure from earth
- Downdrag
- Lateral earth pressure
- Earth surcharge loading

2.2.6.4.2.2 Transient Loads

Transient loads are loads experienced by the structure only some of the time. The following is a brief list of transient loads that are or may be considered during design (FHWA, 1998):

- Loads associated with vehicles
- Pedestrian live load

- Live load surcharge due to traffic loads on backfill
- Water load and stream pressure force
- Wind load
- Friction force
- Force effects due to superimposed deformations, i.e. temperature change, settlement, and creep or shrinkage
- Earthquake
- Collision due to ice, vehicle , or vessel

2.2.6.4.2.3 Load Factors and Load Combinations

Most substructure designs require the evaluation of foundation and structure performance at the Strength I and Service I limit states. These limit states are chosen because they are analogous to evaluations of ultimate capacity and deformation behavior in ASD, respectively. A table of load factors can be found in *Load and Resistance Factor Design (LRFD) for Highway Bridge Substructures* (FHWA, 1998).

2.2.6.4.3 Resistance Factors

The resistance factors recommended by the *AASHTO LRFD* specifications are not related in a precise manner to variables such as the number of borings, exploration depth, or boring spacing, but instead reflect the standard of care representative of each data set considered. Guidelines for exploration and testing programs are based on three criteria: FHWA (1998) recommendations, the availability of information from previous explorations in the vicinity of the site; and engineering experience and judgement.

A simple quantitative measure of the variability, or dispersion, of data or an engineering property is the coefficient of variation, COV, which is the ratio of the standard deviation to the mean of the data set. The greater the value of COV the less reliable the data and the lower the COV the greater the resistance factor. For any given COV the site characteristics are obviously quantified with greater certainty by obtaining more samples and doing more testing..

It is important to remember that while optimizing the exploration program one should also consider the reliability of the different methods available for engineering property assessment of soil and rock. There are three primary sources of error which contribute to the uncertainty of material resistance estimates (FHWA, 1998):

- Inherent spatial variability represented by the uncertainty in using point measurements compared to measurements reflecting a larger volumetric extent
- Measurement error due to equipment and testing procedures
- Model error reflected by the uncertainty of the predictive method

The LRFD manual (1994) provides many tables for this endeavor. Table 2.9 is presented as an example to the reader.

Table 2.9: Summary of Inherent Soil Variability and Measurement Variability Index (after Phoon et al. 1995)

Various Soil Properties	Soil Type	Inherent Soil Variability Mean COV	Measurement Variability Mean COV	ASTM Precision Estimate COV
W_n	Fine-grained	0.18	0.08	----
W_l	Fine-grained	0.18	0.07	0.05
W_p	Fine-grained	0.16	0.1	0.17
PI	Fine-grained	0.29	0.24	----
LL	Clay, Silt	0.74	----	----
γ	Fine-grained	0.09	0.01	----
γ_d	Fine-grained	0.07	----	----
D_r	Sand	0.19	----	----
	Sand	0.61	----	----

The resistance factors presented in the LRFD manual are not absolute. The engineer can modify resistance factors presented in LRFD when unusual or highly variable soil and rock conditions are encountered, or when very uniform or well defined soil and rock conditions exist. Any modification of resistance factors would be similar to modifying the factor of safety in ASD when critical situations or geologic conditions are present.

2.2.6.5 Design of Laterally Loaded Pile

During the design of a laterally loaded drilled shaft group using the LRFD method, two failure criteria are checked: structural failure of the shaft and excessive deflection of the shaft at ground line. Passive failure is another potential failure mode, but is not considered because failure of this type generally occurs at relatively excessive deflection, which exceeds tolerable movements.

Structural failure of the shaft is checked after determining that there are not excessive deflections at the ground line due to the applied load. The structural integrity of the shaft is checked based on the design of reinforced concrete. This is beyond the scope of this research project and is not presented here.

Section 10.8.2.4 in LRFD provides the criteria for designing and checking deflection for a laterally loaded drilled shaft. This section states that the lateral displacement of a single pile and a pile group can be calculated using a p-y analysis.

There is no mention of group capacity for laterally loaded drilled shafts in LRFD. However, a procedure is outlined in the driven pile section (10.7.3.11). In this section an equation is presented to calculate the pile group factored resistance as follows:

$$Q_R = \varphi Q_n = \eta \varphi_L \Sigma Q_L \qquad \text{Equation 2.33}$$

where:

Q_R = factored resistance
φ = resistance factor
Q_n = nominal resistance
Q_L = nominal lateral resistance of a single pile
φ_L = pile group resistance factor specified in LRFD Table 10.5.4-2.
η = group efficiency factor
= 0.75 for cohesionless soil
= 0.85 for cohesive soil

An important thing to note is that the group efficiency factor presented in LRFD section 10.7.3.11 is not a function of pile spacing. Furthermore, the pile group resistance factor can't be located in the LRFD manual. Without this information it is difficult to determine the actual group reduction factor for a drilled shaft group.

In an effort to develop a greater understanding of the LRFD procedure Mr. Kamel Alqalam with the FHWA was contacted. Mr. Alqalam put the researchers in touch with Mr. Tony Allen of the Washington DOT (WashDOT). A copy of the WashDOT procedure was obtained by the researchers, and after review of the procedure Mr. Allen was contacted with questions about the procedure. From this conversation it was clear that WashDOT uses the p-y curve analysis to analyze a single pile and a pile group. To analyze a pile group WashDOT uses the procedure that they have used in the past. Group effects are accounted for by multiplying the modulus of subgrade reaction and the soil strength parameters (c and ϕ) by a group reduction factor.

2.2.7 Group Amplification Procedure

The group amplification procedure can also be classified as a hybrid model. However, due to its use in the National Cooperative Highway Research Program (NCHRP) Report 343 and the use of the moment amplification factor in LRFD as a group reduction factor, it is presented separately. The procedure involves the development of the single pile response using the characteristic load method (CLM) as described by Duncan et al. (1994). The results of the single pile are then multiplied by amplification factors to determine the deflections and moments of the pile group.

2.2.7.1 Single Pile

The characteristic load method is used to determine the deflection at the ground line and the maximum moments for a single pile. The method was originally developed by Evans and Duncan (1982). This method approximates the results of a single pile using nonlinear p-y analyses. The results of the analysis are represented in dimensionless parameters that make it possible to represent a wide variety of real situations with a single relationship. This relationship is created by dividing the actual load by a characteristic load, P_c; the moments are divided by a characteristic moment, M_c; and deflections are divided by the diameter, D. The expressions for the characteristic load and moment are presented as follows:

For clay

$$P_c = 7.34D^2(E_P R_I)\left(\frac{S_u}{E_P R_I}\right)^{0.68}$$ Equation 2.34

For sand

$$P_c = 1.57D^2(E_P R_I)\left(\frac{\gamma' D \phi' K_P}{E_P R_I}\right)^{0.57}$$ Equation 2.35

For clay

$$M_c = 3.86D^3(E_P R_I)\left(\frac{S_u}{E_P R_I}\right)^{0.46}$$ Equation 2.36

For sand

$$M_c = 1.33D^3(E_P R_I)\left(\frac{\gamma' D \phi' K_P}{E_P R_I}\right)^{0.40}$$ Equation 2.37

where:

P_c = characteristic load (F)
M_c = characteristic moment (F-L)
D = pile or drilled shaft width or diameter (L)
E_P = modulus of elasticity of pile or drilled shaft (F/L^2)
R_I = moment of inertia ratio = ratio of moment of inertia of the pile or drilled shaft to the moment of inertia of a solid circular cross section (dimensionless) $I_p/I_{circular}$

$$I_{circular} = \frac{\pi D^4}{64}$$

S_u = undrained shear strength of clay (F/L^2)
γ' = effective unit weight of sand, which is equal to the total unit weight above the water table = buoyant unit weight below the water table (F/L^3)
ϕ' = effective friction angle for sand (degrees)
K_P = Rankine coefficient of passive earth pressure = $\tan^2(45 + \phi'/2)$ dimensionless)

The soil near the top of the pile or drilled shaft is the most important in terms of resisting lateral loads. For the purpose of evaluating the characteristic load and moment the value of S_u or ϕ' should be averaged over the depth equal to 8D below the ground surface.

The flexural stiffness, E_pI_p, of a concrete pile or drilled shaft pile should be reduced when the tensile stresses are large enough to cause cracking. Normally the value of E_pI_p is reduced by 40 – 50% of the un-cracked section.

2.2.7.1.1 Deflections

Deflection of the single pile can be analyzed for three cases: loads applied at the ground line, moments applied at the ground line, and loads applied above the ground line.

2.2.7.1.1.1 Loads Applied at Ground Line

To determine the deflection of the pile top due to loads applied at the ground line, one can use the dimensionless relationships between load and deflection shown in Figure 2.23.

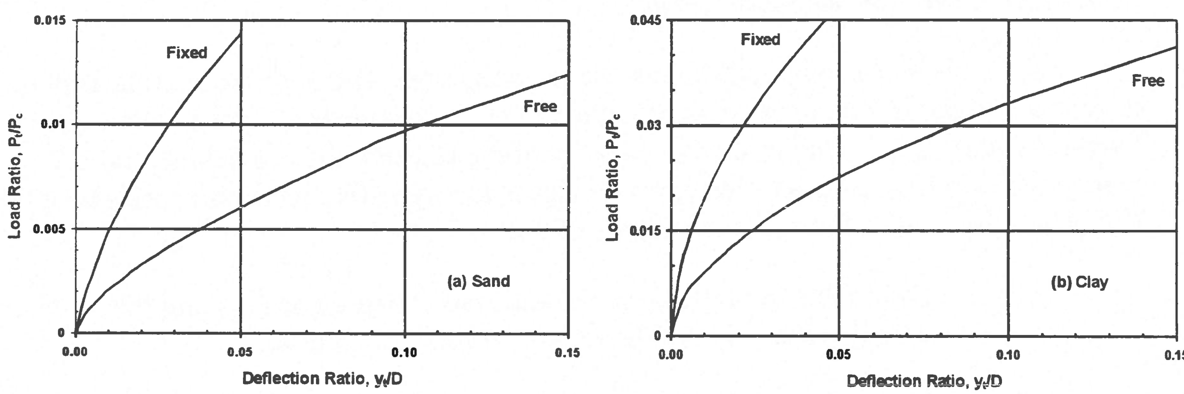


Figure 2.23: Load-Deflection Curves (a) Sand; (b) Clay (After Duncan et al. 1994)

2.2.7.1.1.2 *Moments Applied at Ground Line*

The procedure used to determine the deflections at ground-line due to moments applied at the ground-line is similar to that for loads applied at the ground-line. However, Figure 2.24 should be used for moments.

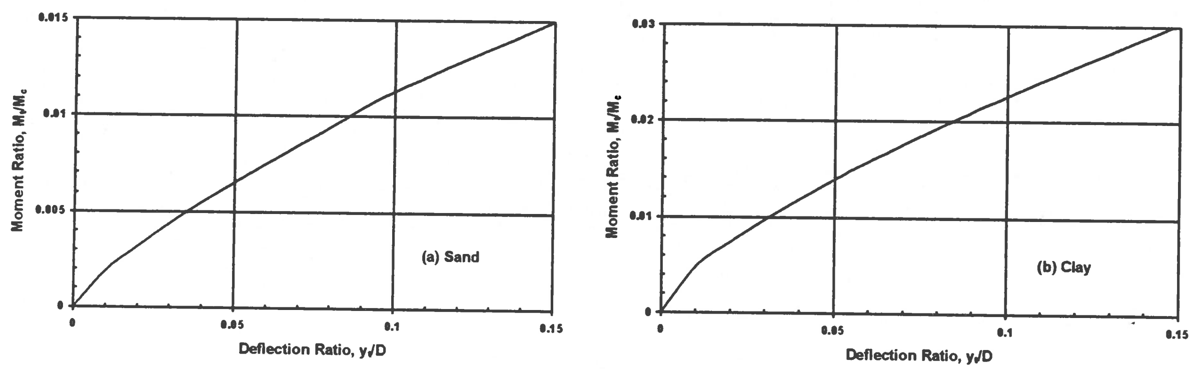


Figure 2.24: Moment Deflection Curves: (a) Sand; (b) Clay (After Duncan et al. 1994)

2.2.7.1.1.3 *Loads Applied Above Ground Line*

This load case is more complicated than the previous one. The application of lateral loads above the ground line induces a horizontal load and a moment at the ground line, as shown in Figure 2.25. Due to the nonlinearity of the solution the normal method of superposition cannot be used. Instead a nonlinear superposition procedure must be used. The procedure is as follows (Duncan et al., 1994):

1 Determine deflections due to the load acting alone (y_{tp}) and the moment acting alone (y_{tm}), as shown in Figure 2.25(a and b).

2 Determine the load that would cause the same deflection as the moment (P_m), and a value of moment to cause the same deflection as the load (M_p). These are determined as shown schematically in Figure 2.25(c and d).

3 Determine the ground line deflections due to the real load plus the equivalent load ($P_t + P_m$), and the real and equivalent moment ($M_t + M_p$), as shown in Figure 2.25(e and f).

The estimated value of deflection due to both load and moment is the average of the two values, y_{tpm} and y_{tmp}. It can be calculated using the following equation:

$$y_{combined} = 0.5\left(y_{tpm} + y_{tmp}\right) \qquad \text{Equation 2.38}$$

where:

$y_{combined}$ = estimated ground-line deflection (L)
y_{tpm} = ground-line deflection due to the real plus equivalent load (L)
y_{tmp} = ground-line deflection due to the real plus equivalent moment (L)

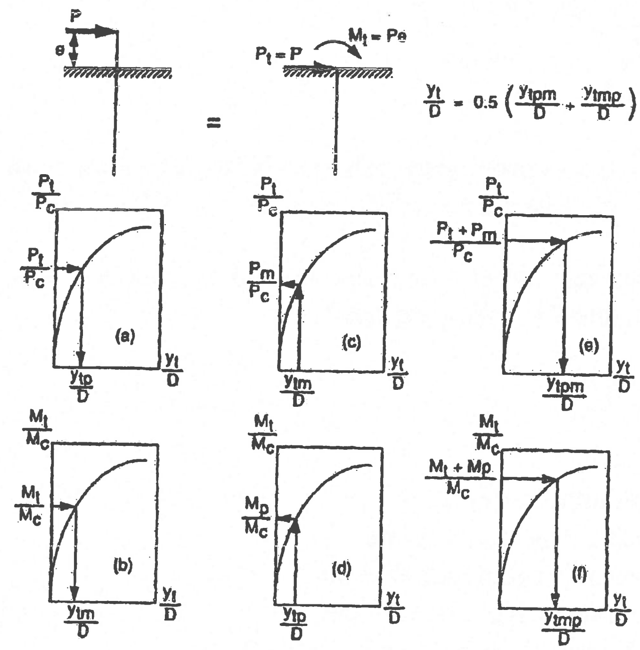


Figure 2.25: Nonlinear Superposition of Deflection due to Load and Moment: (a) Step 1; (b) Step 2; (c) Step 3; (d) Step 4; (e) Step 5; (f) Step 6 (From Duncan et al , 1994)

2.2.7.1.2 Moments

Dimensionless relationship between moment and load have been developed in Figure 2.26. Using the appropriate curve in Figure 2.26, the maximum bending moment for a fixed head pile can be calculated directly. The location of maximum bending moment is the top of the pile.

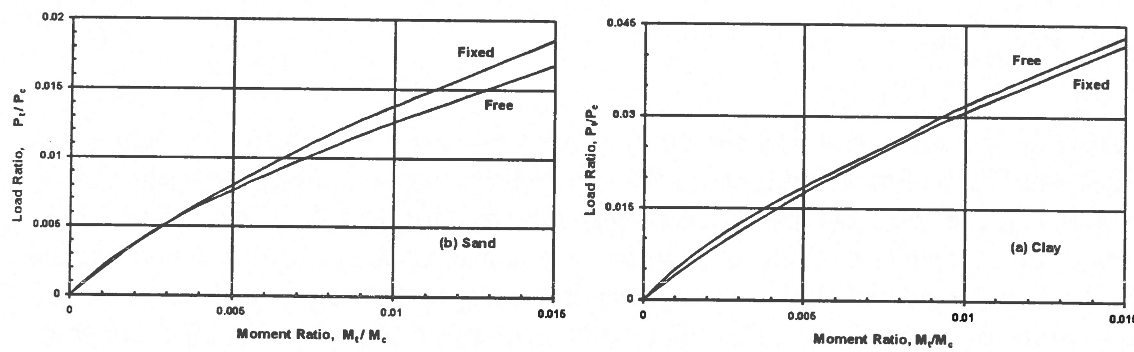


Figure 2.26: Load-Moment Curves: (a) Clay; (b) Sand

To determine the value and location of the maximum bending moment in a free head pile is more complicated. The first step is to calculate the "characteristic length" (T) for the pile and soil conditions being analyzed. The following equation must be solved for T:

$$y_{combined} = \frac{2.43P_t}{E_pI_p}T^3 + \frac{1.62M_t}{E_pI_p}T^2 \qquad \text{Equation 2.39}$$

where:

all the variables are as previously defined. Note that this T is the same as that presented earlier as the relative stiffness factor.

When the value of T has been determined, the bending moments in the upper part of the pile can be calculated using the following equation:

$$M_z = A_mP_tT + B_mM_t \qquad \text{Equation 2.40}$$

where:

M_z = moment at depth z (F-L)
z = depth below ground line
A_m = dimensionless moment coefficient
B_m = dimensionless moment coefficient
The values of A_m and B_m are given in Table 2.10

Table 2.10: Moment Coefficients A_m and B_m (after Matlock and Reese 1961)

z/T	A_m	B_m
0	0.00	1.00
0.5	0.46	0.98
1.0	0.73	0.85
1.3	0.77	0.73
1.5	0.76	0.64
2.0	0.63	0.40

2.2.7.2 Pile Group

The analysis of the behavior of a pile group using the group amplification procedure is a simplification of the Focht and Koch (1973) procedure. As explained previously, the deflection is made up of two parts: a nonlinear soil behavior near the pile and linear soil behavior due to the pile-soil-pile interaction. The nonlinear portion of the deflection due to the load on the individual pile can be computed using nonlinear p-y analysis. For this portion of the problem Ooi and Duncan (1994) recommend the use of the characteristic load method. The interaction amongst piles in the pile group, through pile-soil-pile interaction, is estimated using Poulos' (1971) elastic interaction coefficients.

To simplify the procedure involved in determining the pile-soil-pile interaction effect Ooi and Duncan (1994) developed the deflection amplification factor, which when multiplied by the deflection of a single pile yields the deflection of the group. This is expressed in equation form below:

$$y_g = C_y y_s \qquad \text{Equation 2.41}$$

where

y_g = group deflection (L)
C_y = deflection amplification factor (dimensionless)
y_s = single pile deflection under the same load (L)

A similar amplification factor used to determine the maximum bending moment in the pile group when multiplied by the maximum bending moment in the single pile was developed.

$$M_g = C_m M_s \qquad \text{Equation 2.42}$$

where:

M_g = maximum moment in a pile group (F-L)
C_m = moment amplification factor (dimensionless)
M_s = maximum moment in a single pile under the same load (f-L)

The equation for the deflection amplification factor (C_y) was developed through a parametric study based on the CLM and Focht and Koch (1973). The following equation was developed from the study:

$$C_y = \frac{A + N_{pile}}{B\left(\frac{S}{D} + \frac{P_s}{CP_N}\right)^{0.5}} \qquad \text{Equation 2.43}$$

where:

A = constant (dimensionless)
= 16 for clay
= 9 for sand
B = constant (dimensionless)
= 5.5 for clay
= 3.0 for sand
C = constant (dimensionless)
= 3 for clay
= 16 for sand
N_{pile} = number of piles in the group
S = average spacing of piles (L)
D = diameter of single pile (L)
P_s = P_g / N_{pile} = average lateral load on pile (F)

P_g = total lateral load on pile group (F)

P_N = S_uD^2 for clay (F), and $K_p\gamma D^3$ for sand (F)

where:

S_u, D, K_p, γ, are defined as previously.

The procedure that involves the determination of the maximum bending moment was developed by modifying the theory described in Focht and Koch (1973). A parametric study was conducted and the equation representing the moment amplification factor is described below:

$$C_M = \left(C_y\right)^n \qquad \text{Equation 2.44}$$

where:

C_M = moment amplification factor (dimensionless)

C_y = deflection amplification factor (dimensionless)

$n = \dfrac{P_s}{150P_N} + 0.25$ for clay (dimensionless)

$n = \dfrac{P_s}{300P_N} + 0.30$ for sand (dimensionless)

Using this amplification factor it is possible to estimate the maximum bending moment in the most severely loaded pile within the group.

2.2.7.3 Advantages and Disadvantages

The group amplification procedure is based upon the hybrid method of Focht and Koch. The main advantage is that it reduces the amount of work necessary to design a pile group compared to the hybrid method. Instead of needing to determine an influence factor for each pile in the group, an empirical equation is used that requires only one factor to be read from a chart.

The principle limitation of the characteristic load method is that it is only applicable to piles and drilled shafts that are long enough that their behavior is unaffected by their length. Minimum lengths necessary to satisfy the criterion depend on the relative stiffness of the pile or shaft in relation to the stiffness of the soil. Minimum lengths for a number of criterion is presented in Table 2.11 (Duncan et al., 1994). If the length of a pile or drilled shaft is less than those presented in Table 2.11 then the actual deflections will be greater than those calculated and the maximum bending moments in the pile or shaft will be smaller than calculated.

Table 2.11: Minimum Length Criterion (after Duncan et al., 1994)

Soil Type	Criterion	Minimum Length
Clay	$E_pR_I/S_u = 100{,}000$	6 diameters
Clay	$E_pR_I/S_u = 300{,}000$	10 diameters
Clay	$E_pR_I/S_u = 1{,}000{,}000$	14 diameters
Clay	$E_pR_I/S_u = 3{,}000{,}000$	18 diameters
Sand	$E_pR_I/(\gamma'D\phi'K_p) = 10{,}000$	8 diameters
Sand	$E_pR_I/(\gamma'D\phi'K_p) = 40{,}000$	11 diameters
Sand	$E_pR_I/(\gamma'D\phi'K_p) = 200{,}000$	14 diameters

Other limitations to the group amplification procedure are as follows (Ooi & Duncan, 1994):

- Applicable only to vertical pile groups
- Load distribution among the piles in the group cannot be predicted
- The values of deflection and moment for groups with the same number of piles are the same regardless of their arrangement.
- Does not include effects of the pile cap
- Only able to predict the response for small deflections. For a fixed pile the allowable lateral deflection was approximately 5% of the diameter of the pile
- Difficulty in applying terms "sand" or "clay" to soils commonly encountered in the desert southwest

2.2.8 Strain Wedge (SW) Method

The strain wedge (SW) method (Norris, 1986) predicts the response of a pile subjected to lateral loads by analyzing a three-dimensional wedge of soil that develops in front of the pile. Figure 2.27 gives a graphical representation. The pile is broken into a number of elements for the analysis. The SW method relates stress-strain-strength behavior to the one-dimensional beam on elastic foundation. The traditional one-dimensional beam on elastic foundation pile response parameters are characterized by SW method on the envisioned soil-pile interaction and its dependencies on both soil and pile properties. The variables in Figure 2.27 are defined below:

D = width of pile cross-section
Θ_m = base angles
β_m = base angles
where:

$$\Theta_m = 45 - \phi_m/2$$
$$\beta_m = 45 + \phi_m/2$$

ϕ_m = spread of the wedge fan angle (mobilized friction angle)
$\Delta\sigma_h$ = horizontal stress change at the passive wedge face
τ = side shear

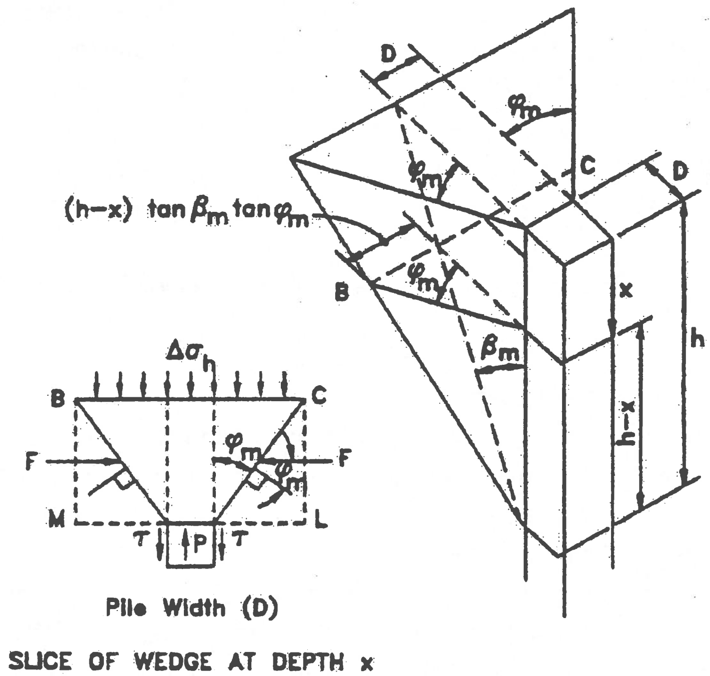


Figure 2.27: Strain Wedge (From Ashour et al., 1998)

2.2.8.1 Advantages and Disadvantages

The strain wedge method uses data that can be obtained directly from the test results of a triaxial specimen. In contrast to the p-multiplier method, interaction effects change along the length of the pile with changing deflection. The main disadvantage of this method is that it cannot handle piles with diameter greater than 2 ft. For piles that are larger than this, the program has difficulty in calculating the side shear associated due to the pile movement. Another disadvantage of the method is that it is not a well known or widely used method, and so it hasn't been rigorously tested by other researchers and practitioners across the country.

2.2.9 Finite Element Modeling (FEM)

The remaining analytical method used for the design of laterally loaded pile groups is three-dimensional Finite Element Modeling (FEM). This method requires the use of a finite element computer program that can handle a 3-D problem. Finite element involves four basic components: mesh generation, failure criterion , calibration, and boundary conditions.

2.2.9.1 Mesh Generation

To obtain a valid solution to a problem using the finite element method it is necessary to develop a proper mesh. A mesh is defined as the discretizing of the soil and pile into elements. The selection of the size and number of elements is important because they define where stresses and strains will be calculated. The exact location within the element where the stresses and strains are calculated is dependent on the computer program that is used.

2.2.9.2 Failure Criterion

Most finite element codes allow the user to select from a number of failure criteria. The results obtained from the FE analysis can be highly dependent on the failure criteria that are used. It is up to the user to specify the criterion that he/she thinks most adequately represents the conditions expected for the specific problem in the field.

2.2.9.3 Calibration

Once the mesh generation and failure criterion have been established it is necessary to calibrate the FE model. The FE model is calibrated with a constitutive model. The ideal constitutive model would apply to the conditions of interest. The parameters defining the constitutive model would then be modified until close agreement could be obtained between the calculated output and some measured or known result. Once the FE model is calibrated it could be used to solve for additional conditions as desired.

2.2.9.4 Boundary Conditions

It is very important that the proper boundary conditions for the problem under consideration be defined. Boundary conditions are usually classed as stress or displacement.

2.2.9.5 Advantages and Disadvantages

The advantage of the finite element method is that it performs analyses based on the stress and strain characteristics of the materials used in the design. The disadvantages of this method are that it is very complicated and the analysis time is very large. Another disadvantage of finite element is the difficulty in modeling soil properties with great precision. Due to the complexity of FE this is more of a research tool more than a tool used in design practice. There will be much more detail on FE in Section 2.4.

2.3 Comparison of Analytical Approaches

Previously discussed were nine design methods with which to determine the lateral load capacity of a pile group. These nine design methods can be divided into two groups based upon use in design today: frequent and infrequent. The determination of infrequent or frequent use was based upon the interviews conducted with state departments of transportation around the country and with Maricopa County area consultants.

The design methods that are common in design today are as follows:

1. Group reduction factor design
2. Coefficient of lateral subgrade reaction reduction
3. P-multiplier design

The design methods that are not common in practice today are as follows:

1. Elastic Analysis
2. Hybrid Analysis
3. Load-and-Resistance Factor Design philosophy (LRFD)
4. Group Amplification Procedure
5. Strain Wedge
6. Finite Element Modeling (FEM)

Of these methods both the elastic and hybrid analysis are not used in current design due to the belief that the more recent methods better approximate field behavior. LRFD is an emerging procedure that may find wide spread use in the future, but currently is not used much. The group amplification procedure is similar to the hybrid analysis and has enjoyed only limited use to date, perhaps because it is perceived to be somewhat complex.

The strain wedge method is a developing procedure and is currently under consideration by Caltrans. Finite element modeling is a very useful tool to solve the laterally loaded pile group problem, however, it is not used very often. This may be due to the amount of time involved in performing the analysis or the prevailing sentiment that design problems do not require such a detailed analysis.

2.3.1 Definitions

The methods commonly used in design have different reduction factors for group effects and apply them at different stages of the design process to determine the lateral load capacity of the pile group. In an effort to eliminate any confusion as to which method is being discussed a list of definitions is presented. Furthermore, a new term, efficiency, is defined in order to perform a meaningful comparison of the methods on a common basis or criterion.

- Group Reduction Factor- Term used to describe the reduction factor that is applied during the group reduction factor method. The reduction factor is applied directly to the single pile capacity to obtain the load/pile in the group.

- Reduction Factor- Term used for the reduction factor in the coefficient of lateral subgrade reaction reduction method. The reduction factor reduces the value of the coefficient of lateral subgrade reaction to account for group effects.

- P-multiplier- A multiplier that is multiplied to the p-axis of the p-y curve. The multiplier reduces the p-y curve of the soil to account for group effects.

- Efficiency (E)- The criterion chosen to compare the results of the procedures on the same basis. Efficiency is a ratio of loads used to compare the load in the pile group divided by the load in a single pile times the number of piles in the group, at the same deflection (Equation 2.45).

$$E = \left(\frac{P_{group}}{P_{single} \times N} \right)_{\Delta} = \left(\frac{P_{group/pile}}{P_{single}} \right)_{\Delta} \qquad \text{Equation 2.45}$$

where:

E = efficiency
P_{group} = the load carried by the pile group
P_{single} = the load carried by the single pile
$P_{group/pile}$ = the average load carried by each pile in the group
N = number of piles in the group
Δ = the deflection at which the loads were compared

2.3.2 Boundary Conditions

Prior to comparing the various methods of analysis for laterally loaded piles it is necessary to have a complete understanding of the boundary conditions that can be imposed. This is important to insure that all methods are compared on the same basis. The two primary boundary conditions that need to be explored when examining the group reduction factor of a pile group are slenderness of the pile and fixity of the pile cap.

2.3.2.1 Pile Slenderness

The term pile slenderness refers to the ratio of the length of the pile, L, to the relative stiffness factor, T (Equation 2.18). There are three basic categories of pile slenderness: flexible or long, intermediate, and rigid or short.

A rigid pile is defined as having a ratio of $L/T \leq 2$. A pile defined as short obtains its lateral resistance due to the lateral resistance of the soil. The pile deflects at the surface in the direction of the applied lateral load and the tip of the pile deflects in the opposite direction. In other words the pile tends to "kick out" stressing the soil behind the pile.

A second classification of a pile based upon the slenderness ratio is long. A pile is defined as flexible when $L/T \geq 4$. A pile defined as flexible means that at some depth along the length of the pile it is essentially fixed. The pile top moves in the direction of the applied load causing the pile to deflect. At some point, approximately $Z/T = 2$, the pile does not deflect from its original position. Directly beneath this point the pile deflects in the direction opposite of the applied lateral load, but not substantially. The pile then quickly returns to its original configuration with depth.

When the L/T ratio is neither greater than four nor less than two, then the pile is classified as an intermediate pile. The exact behavior of a pile that falls into this category is somewhere between a rigid and a flexible pile.

2.3.2.2 Pile Cap Fixity

The load-deflection response and the magnitude and location of maximum moment of a laterally loaded shaft or group depends on the fixity of the butt into the cap. Unless a detailed structural analysis is performed, the fixity of the cap connection is usually assumed to vary between fully fixed and 50 percent partially fixed. The key factors that must be considered in determining or estimating fixity at the cap connection include (FHWA, 1998):

- Depth of shaft embedment into the cap
- Magnitude of bending moment at shaft-cap connection
- Shaft type and geometry
- Shaft-to-cap connection detail

Fixity of a pile refers to the movement of the pile head when subjected to lateral load. There are three possibilities for the pile cap fixity: free, fixed, and partially fixed.

A pile classified as free has a pile head that is completely free to rotate. This occurs when no cap is present, a flexible cap is present, or when the pile is connected with a hinge. The absence of a cap or the presence of a flexible cap or a hinge forces the moment at the pile head to be zero. The failure mode of free head piles is dependent on the pile slenderness ratio. The failure modes for free head piles are shown in Figure 2.28.

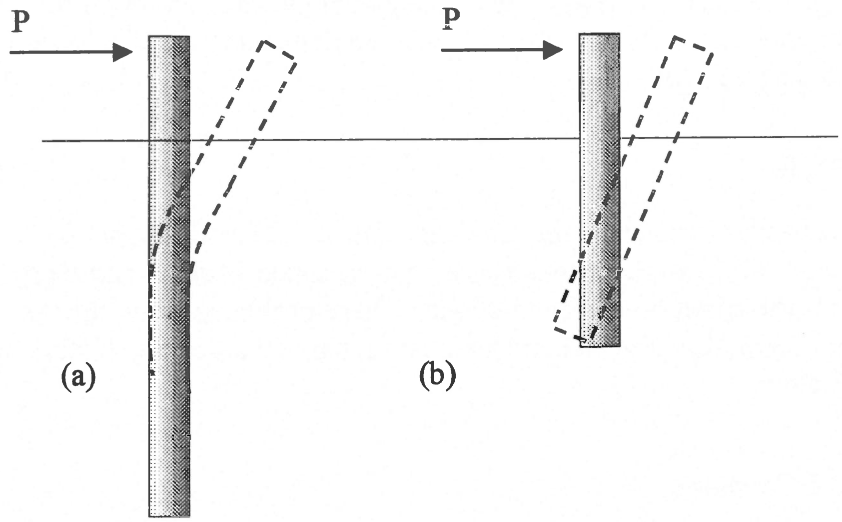


Figure 2.28: Failure Modes for Free, Laterally Loaded Piles: (a) Flexible (b) Rigid (After Broms, 1965)

The second possibility is that the pile head is completely fixed against rotation, commonly referred to as fixed. The pile head deflects due to applied horizontal loads and moments through translation. This boundary condition can be met by the use of a rigid cap that prevents rotation. When rotation is prevented the maximum moment in the pile develops at the pile head. Based on the results of full-scale load tests (Shahawy and Issa, 1992), a depth of embedment of the pile into the cap of 2D to 3D provides full fixity for most service load conditions. The degree of shaft-cap fixity is usually established empirically. The failure modes for fixed head piles are shown in Figure 2.29.

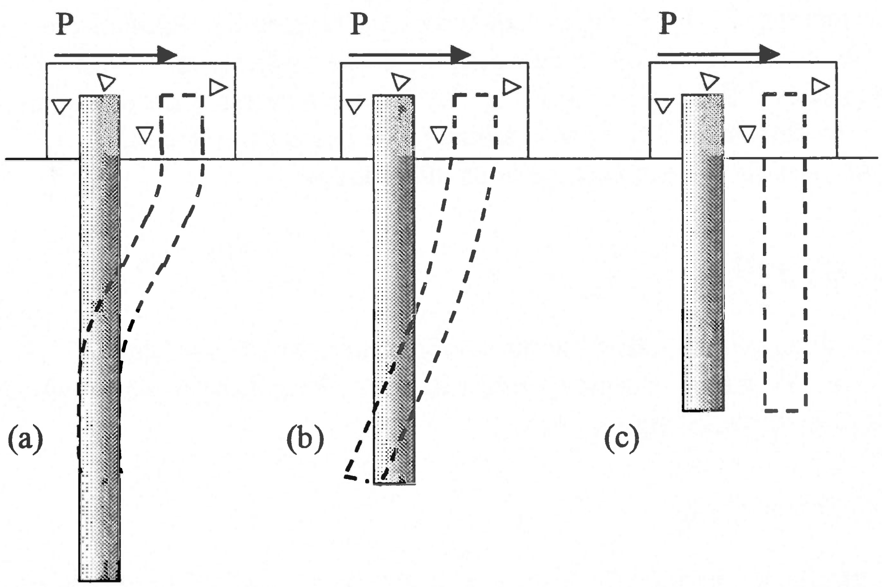


Figure 2.29: Failure Modes for Fixed Laterally Loaded Piles: (a) Flexible; (b) Intermediate; (c) Rigid (After Broms, 1965)

The remaining fixity condition at the pile top is partial fixity. In other words the pile is in-between the free and fixed condition. Partial fixity of the pile in the cap is difficult to model so in practice the pile is designed as either free or fixed depending on whichever more closely resembles the actual case.

2.3.3 Problems for Analysis

Three problems were analyzed to determine the efficiency that would result from use of the most common methods. These three problems are first explained in detail and then analyses are performed for the common design methods. These problems allow for the comparison of the various methods using an equivalent basis, namely efficiency factors as defined in Equation 2.45 above.

2.3.3.1 Problem 1: Initial Condition

The initial condition as defined in this report refers to the conditions that exist at a site where a group of drilled shafts are to be built. The parameters were mimicked from a laterally loaded pile group at the intersection of Warner Road and the Price Freeway. The layout of the pile group is detailed in Figure 2.7.

2.3.3.2 Problem 2: Effect of Slenderness Ratio (L/T)

The initial slenderness ratio of the shaft was 2.5 classifying it as an intermediate pile. To determine the effect of the slenderness ratio on efficiency the slenderness ratio was increased to make the pile behave as a long pile. The two ways to accomplish this are: reduce the value of T or increase L. First T was reduced by decreasing the value of the diameter from 42in to 12in which resulted in a decrease in I from $1.53\text{x}10^5$ to $1.02\text{x}10^3$ [in^4]. Secondly, L was increased from 30 ft to 60 ft. The two parameters were changed independent of the other to determine if there was a difference based on individual parameter or if the slenderness ratio itself was the controlling factor.

2.3.3.3 Problem 3: Pile Head Fixity

The third problem was to determine whether the pile fixity conditions affected the efficiency of the pile group. To gain a complete understanding of cap fixity it was deemed necessary to examine both flexible and rigid piles.

2.3.4 Group Reduction Factor Design

The group reduction factor design method is directly related to the group reduction factor of the pile group. The group reduction factor is applied to the single pile capacity. Therefore, a reduction factor of 0.8 applied to the single pile results in an efficiency of 0.8. This relationship is true regardless of the reduction factor applied or the problem under consideration, i.e. pile characteristics, soil characteristics, and boundary conditions. However, recommendations for different group reduction factors could be made depending on soil and pile type if desired. The efficiencies and reduction factors are tabulated in Table 2.12 for completeness, even though they are numerically equal.

Table 2.12: Efficiency versus Group Reduction Factor Design

Group Reduction Factor	Efficiency
1.0	1.0
0.7	0.7
0.4	0.4
0.25	0.25

2.3.5 Coefficient of Lateral Subgrade Reaction Reduction

The procedure for the use of the coefficient of lateral subgrade reaction reduction has been fully explained earlier. This section is concerned with how the application of the coefficient of variation of subgrade reduction factor affects the predicted efficiency of the pile group. An analysis was performed using the following procedure. The formula used in DM 7.2 to develop a load-displacement curve is as follows:

$$\delta_p = F_\delta \times \left(PT^3 / EI\right) \qquad \text{Equation 2.46}$$

where:

δ_p = deflection at any depth z due to force P
P = applied lateral load
EI = stiffness of the pile
  E = modulus of elasticity of the pile
  I = moment of inertia of the pile
T = relative stiffness factor, a function of (EI, n_h)
  EI = stiffness of the pile
  n_h = coefficient of variation of lateral subgrade reaction
F_δ = Deflection coefficient, is a function of (L, T, z)
  L = length of the pile
  z = depth below ground (i.e. point of interest)
  T = relative stiffness factor

For design purposes it is necessary to develop a load-deflection curve for a specific site and specific pile geometry. This would involve solving Equation 2.46 at several load increments to develop the complete curve. In this case the pile geometry, the soil, and the point of interest are constant. Therefore, the variables *L*, *EI*, n_h, and *z* are constants for a particular problem. If these variables are constant then both F_δ and *T* are constants. Then Equation 2.46 can be simplified into the following form:

$$\delta_p = C \times P \qquad \text{Equation 2.47}$$

where:

$$C = \text{constant} = F_\delta \times \left(T^3/EI\right)$$

Equation 2.47 shows that there is a linear relationship between load and displacement using the procedure outlined in DM 7.2. Using this information it was only necessary to analyze the results from one deflection for this portion of the study. The analysis is broken into the individual problems.

2.3.5.1 Problem 1: Initial Conditions

The analysis for the pile group was performed at a variety of spacings. The following relationship was used between spacing and reduction factor as provided in DM 7.2

The results of the analysis of Problem 1 for a number of reduction factors are presented in Table 2.13 and are plotted in

Figure 2.30. An example calculation is presented below.

Step 1) Perform analysis for the single pile. The result was a load on the single pile, P_{single}, of 120 [kips] at a deflection, Δ, of 0.5 in.

Step 2) Apply reduction factor to the coefficient of lateral subgrade reaction and then perform an analysis on a typical pile in the group. The result was a load on a pile in the group, $P_{group/pile}$, was 49.8 [kips] at a deflection, Δ, of 0.5 in.

Step 3) Calculate efficiency, E, using Equation 2.45.

$$E = \left[\frac{P_{\text{group/pile}}}{P_{\text{single}}}\right]_\Delta = \frac{49.8\ [\text{kips}]}{120\ [\text{kips}]} \approx 0.42$$

Table 2.13: Results of Problem 1 using DM-7.2

Reduction Factor	Efficiency
0.25	0.42
0.4	0.56
0.7	0.80

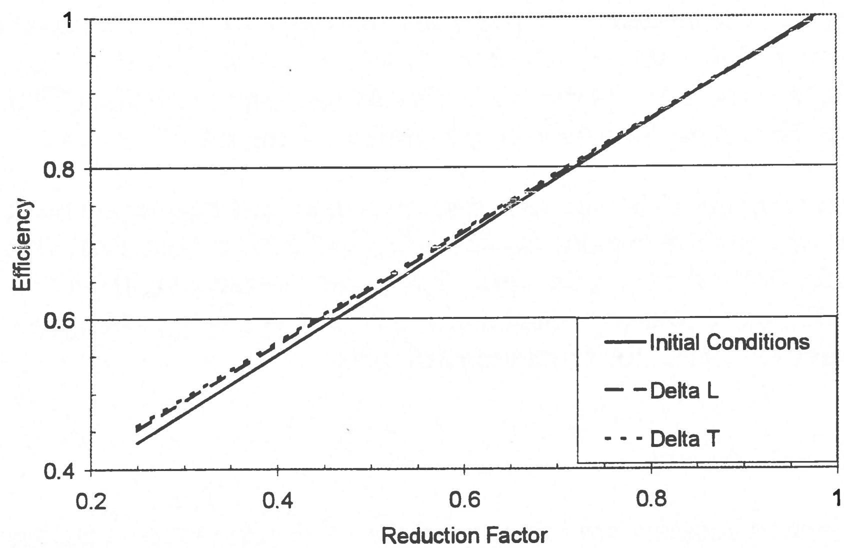


Figure 2.30: Relationship between Efficiency and Reduction Factor using DM-7.2 (Problems 1 and 2)

The solid dark line labeled initial conditions represents the results from Problem 1. It is clear that a linear line fits the data. The equation representing the line is y = 0.7758x + 0.2412 with an R-squared value of .9959. The source of the other two lines is described in the next section.

2.3.5.2 Problem 2: Effect of Pile Slenderness (*L/T*)

To determine the effect of the pile slenderness on the efficiency of the pile the value of *T* and the value of *L* were changed. Both values were changed to determine if the individual parameters had an effect, or if *L/T* was the controlling parameter.

The analysis for the pile group was performed for a variety of spacings. A relationship between spacing and reduction factor was presented in Table 2.7 and will be used here. The results due to the modification of the relative stiffness factor are presented in Table 2.14. Where ΔL and ΔT represent changes in length and relative stiffness respectively, to make the pile behave as a long pile.

Table 2.14: Effect of Slenderness Ratio using DM-7.2 (Problem 2)

	Efficiency	
Reduction Factor	Δ T	Δ L
0.25	0.44	0.44
0.4	0.58	0.57
0.7	0.81	0.81

These results were plotted on Figure 2.30 with the initial conditions for two reasons. The first reason was to determine if the pile slenderness ratio was the variable that effects efficiency or was it one of the parameters that make up the slenderness ratio. The data was plotted on Figure 2.30 and labeled as Delta L (ΔL) and Delta T (ΔT). The equation describing the line through these data points was $y = 0.748x + 0.2665$ with an R-squared value of 0.9957 for Delta L and $y = 0.7446x + 0.2709$ with an R-squared value of 0.9952 for Delta T. These two lines show little difference between ΔL and ΔT.

The second reason was to examine the difference between an intermediate and a flexible pile for the given boundary and site conditions. From Figure 2.30 it is clear that there is little difference in efficiency for the two pile types. The equation describing the initial conditions, i.e. intermediate pile, was presented in the results to Problem 1. However, this conclusion is not necessarily correct for all intermediate piles.

2.3.5.3 Problem 3: Pile Head Fixity

Further analysis was done to compare head fixity with pile slenderness ratio. The analysis of the pile group was performed for a variety of spacings. The results of these analyses are presented in Table 2.15 and Table 2.16 and were plotted in Figure 2.31.

Table 2.15: Free Head using DM-7.2 (Problem 3)

Reduction Factor	Efficiency	
	L/T < 4	L/T >4
0.25	0.28	0.44
0.4	0.45	0.58
0.7	0.73	0.81

Table 2.16: Fixed-Head using DM-7.2 (Problem 3)

Reduction Factor	Efficiency	
	L/T < 4	L/T >4
0.25	0.42	0.44
0.4	0.56	0.58
0.7	0.80	0.81

From Table 2.15, Table 2.16, and Figure 2.31 it is possible to state that the head fixity condition does not have an effect if the pile is flexible. The data points and regression lines plotted on top of each other. The regression line is represented by the following equation $y = 0.7446x + 0.2709$ with an R-squared value of 0.9952.

For intermediate piles there was a marked difference in efficiency, depending on head fixity conditions. For the fixed-head pile the equation is similar to that presented in Problem 1. For the free headed pile the data is represented by the following equation $y = 0.9573x + 0.0549$ with an R-squared value of 0.9982. Basically, the efficiency values were only slightly higher than the reduction factors.

The regression equations for all cases had R-squared values greater than 0.99. This implies that these equations could be used to determine the efficiency implied by the modulus of subgrade reaction procedure for any reduction value that the designer wanted to use with a high degree of accuracy. However, it is not appropriate to use the relationship developed for $L/T < 4$ (intermediate pile) because of sensitivity to L/T itself.

It is important to note that these calculated efficiencies are simply efficiency values predicted by the method. Field measurements of actual full-scale drilled shaft responses would be necessary to determine the accuracy of these predictions. The primary point of this computational exercise was to illustrate that, when the coefficient of lateral subgrade reaction reduction method is used, the efficiencies (as defined by Equation 2.45) do not come out numerically equal to the reduction factors, in general. However, in the case of free-head piles, they come out very close to the same.

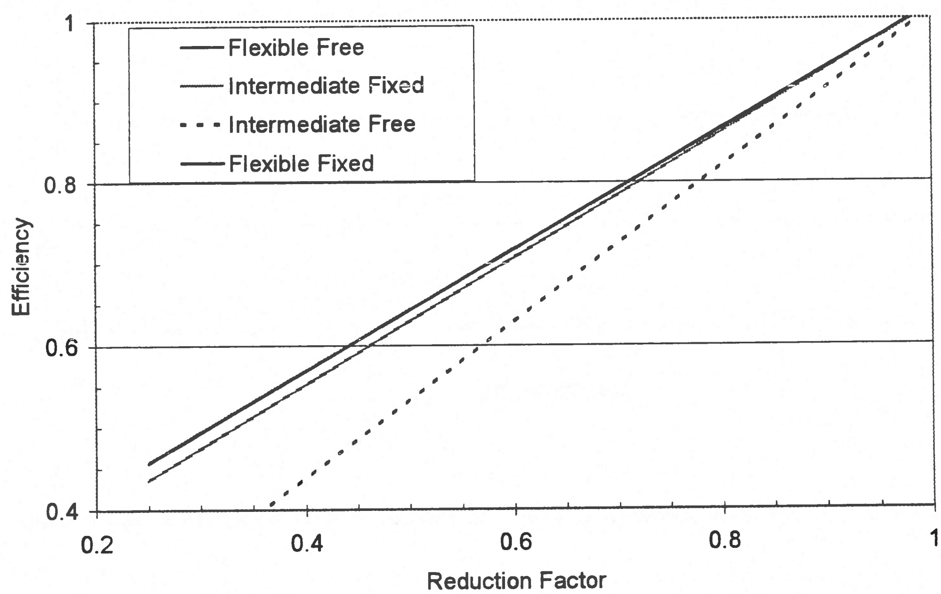


Figure 2.31: Results of Problem 3 using DM-7.2

2.3.5.4 Coefficient of Lateral Subgrade Reaction Reduction Using COM624P

A single analysis was performed for the coefficient of lateral subgrade reaction reduction method using COM624P. This analysis was done to determine the efficiency of a shaft in the group, similar to that used in the previous analysis, to that of a single drilled shaft. In this analysis the p-y curves were developed internally by the computer using a soil model based on Reese et al.'s (1974) criteria for sand. For the original shaft a value of k_{single} = 583 pci was used and for a shaft in the group a value of k_{group} = 146 pci (i.e. $0.25*k_{single}$) was used. P-y curves and load-deflection curves were obtained using both k values. The efficiency for the shaft group is the ratio of the loads for each curve at a given deflection. From these analyses, an efficiency versus deflection plot was generated. From the figure it is clear that as the deflection level approaches 0.30 in the efficiency approaches 1.0, and that the efficiencies calculated are significantly higher than the reduction factor itself (that is, efficiencies of 0.62 to 1.0 arose from a reduction factor of 0.25). These values of efficiency are greater than those obtained using the procedure outlined in NAVFAC.

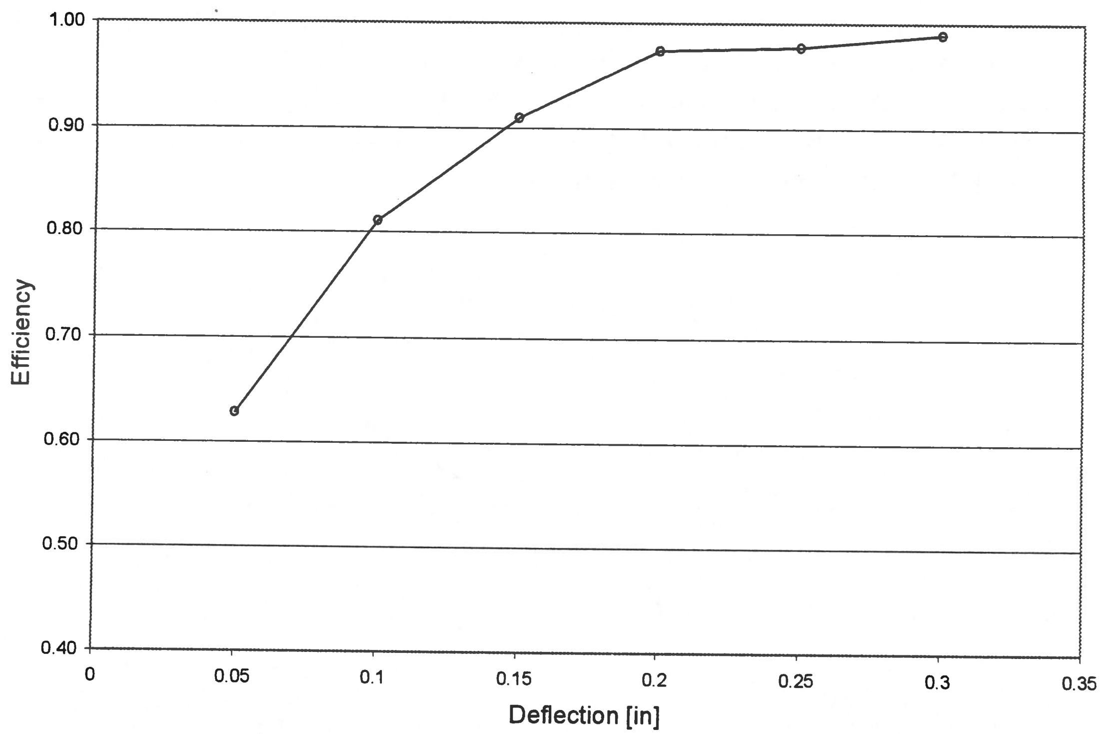


Figure 2.32: Efficiency versus Deflection for Coefficient of Lateral Subgrade Reaction Reduction using COM624P for Sand

2.3.6 P-multipliers

The p multiplier approach was explained previously. This section is concerned with how the application of p multipliers affect the efficiency of the pile group. That is, what is the relationship between the numerical values of the p-multipliers and the numerical values of efficiency. An analysis was performed using the procedure outlined in FHWA report FHWA-HI—97-013 (1996) to account for group effects (i.e. pile-soil-pile interaction).

The single pile capacity is determined by developing a p-y curve and running a computer analysis as discussed previously. To determine group capacity, the p-y curve is multiplied by the p-multiplier. The p-multiplier is a function of the row position of the pile within the group with respect to the direction of the applied load.

To determine the effect of each individual multiplier the following procedure was used:

1. Develop p-y curve based upon laboratory data using the Matlock (1970) procedure. The soil and pile data was defined previously.
2. Perform computer analysis and obtain deflection at the ground surface
3. Repeat Step 2 for various loads of interest and plot load-deflection curve
4. Multiply the p-y curves developed in step 1 by the appropriate p-multiplier. Only the p-axis is multiplied by the p-multiplier.
5. Perform computer analysis of single pile with modified p-y curve and obtain deflection at the surface under a given load.
6. Repeat step 5 for various loads of interest and plot load-deflection curve for a given p-multiplier.
7. Enter the load-deflection curve from step 2, original single pile, at a given deflection and obtain the corresponding load.
8. Enter the load-deflection curve from step 6, modified single pile for group, at the same deflection as the single pile from step 7.
9. Calculate the efficiency of the modified single pile:

$$E = \left(\frac{P_{mult}}{P_{\sin gle}} \right)_{\Delta}$$

10. Repeat steps 7 through 9 for interesting values of deflections.
11. Repeat steps 4 through 10 for various p-multipliers of interest.

It is important to note that the efficiency generated is not for the entire group. Instead it is for a pile in a particular row within the group. The efficiency of the group can be calculated using Equation 2.42.

For the particular case corresponding to these example calculations, the results were rather insensitive to the level of deflection; therefore, no designation of the deflection at which the efficiency was calculated was made. The only exception was the case for the intermediate or short pile with a fixed head.

2.3.6.1 Problem 1: Initial Conditions

The following analysis was performed for a pile group at a spacing of 3 diameters (3D). Table 2.8 displays recommendations of p-multipliers for this spacing. The multipliers recommended by the FHWA are used for comparison purposes in this report. However, it is expected that a relationship between p-multiplier and efficiency will be obtained that can apply to any p-multiplier recommendations. Results of Problem 1 are:

Table 2.17: Results of Problem 1 using p-multipliers

P-Multipliers	Efficiency
0.8	0.86-0.82
0.4	0.53-0.45
0.3	0.44-0.34

In this case, as pile head deflection increased the efficiency of the pile decreased. It was not deemed proper to select a certain value of deflection to perform an analysis because there was no logical reason to select one deflection level over another. It was not possible to perform a regression analysis on this set of data because the efficiency values were not constant.

2.3.6.2 Problem 2: Effects of Pile Slenderness (L/T)

The effects of the parameters that affect pile slenderness were evaluated and are presented in Table 2.18. The results are plotted in Figure 2.33. The values of efficiency presented were independent of the deflection level under consideration.

Table 2.18: Effect of Slenderness Ratio using p-multipliers (Problem 2)

	Efficiency	
P-Multipliers	ΔT	ΔL
0.8	0.85	0.84
0.4	0.53	0.50
0.3	0.43	0.40

The equations for the regression lines were as follows. For the ΔT data the equation is $y = 0.8084x + 0.1973$ with an R-squared value of 0.9988. The regression equation for ΔL is $y = 0.8534x + 0.1516$ with an R-squared value of 0.9993. There is little difference between the two equations and therefore it could be concluded that the important variable is the pile slenderness ratio and not the variables of which it is comprised.

To compare an intermediate pile with a flexible pile under fixed head conditions the values presented in Table 2.17 and Table 2.18 were used. From the tables there is little difference between the two piles at low deflections. However, as deflection increases the efficiency of the intermediate pile decreases while the efficiency of the flexible pile is constant.

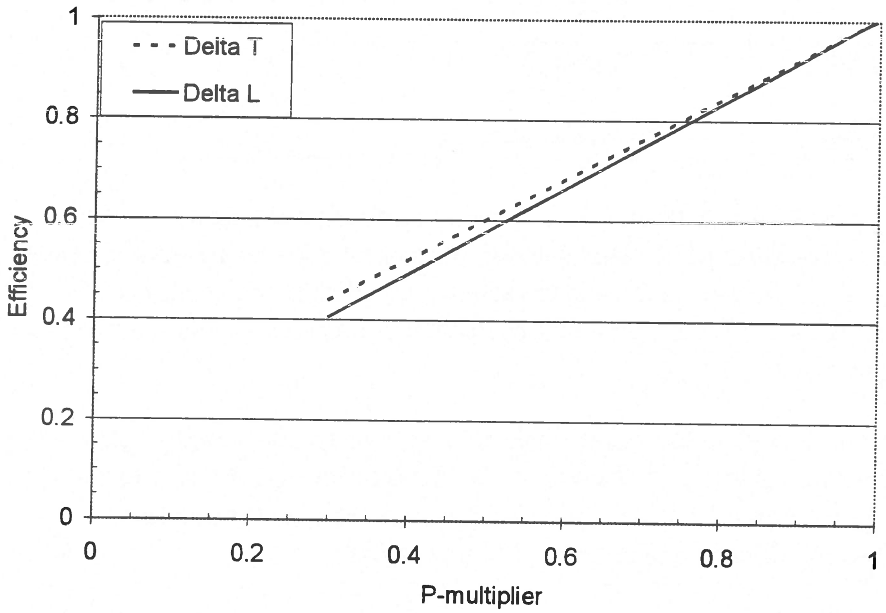


Figure 2.33: Effect of Pile Slenderness Ratio using p-multipliers (Problem 2)

2.3.6.3 Problem 3: Pile Head Fixity

An analysis of pile head fixity was conducted similar to that done for the coefficient of subgrade modulus. The results have been tabulated in Table 2.19 and Table 2.20, free and fixed respectively and have been presented in Figure 2.34. The intermediate pile resulted in an equation of y = x with an R-squared value of 1.0. This is what would be expected because the lateral pile system gets its resistance from the soil. The intermediate pile was close to being a rigid pile. Therefore, it would fail by deflecting in the direction of the load at the pile head and by kicking out at the pile bottom. A reduction in the soil stiffness results in an equal reduction in efficiency.

Comparing the flexible piles, L/T > 4, the free head and fixed head results were similar. The equation representing the flexible free pile is y = 0.8567x + 0.1588 with an R-squared value of 0.9948. The flexible fixed pile is represented by the equation for the flexible pile with Delta L.

Table 2.19: Free Head using p-multipliers (Problem 3)

P-Multipliers	Efficiency	
	L/T < 4	L/T >4
0.80	0.80	0.85
0.40	0.40	0.50
0.30	0.30	0.40

Table 2.20: Fixed Head using p-multipliers (Problem 3)

P-Multipliers	Efficiency	
	L/T < 4	L/T >4
0.80	0.86-0.82	0.85
0.40	0.53-0.45	0.50
0.30	0.44-0.34	0.40

The data plotted in Figure 2.33 and Figure 2.34 have an additional data point added, an efficiency of 1 with a p-multiplier of 1. This seemed necessary in order to obtain a more realistic equation that could be used to describe variation in efficiency, because theoretically efficiency cannot exceed one when boundary conditions are the same for the numerator and the denominator.

Again, the primary purpose of this computational exercise was to show numerical relationships between p-multipliers and efficiency values. To the extent that the p-multiplier values are derived empirically from full-scale load tests with boundary considerations, the ensuing efficiency factors should be realistic.

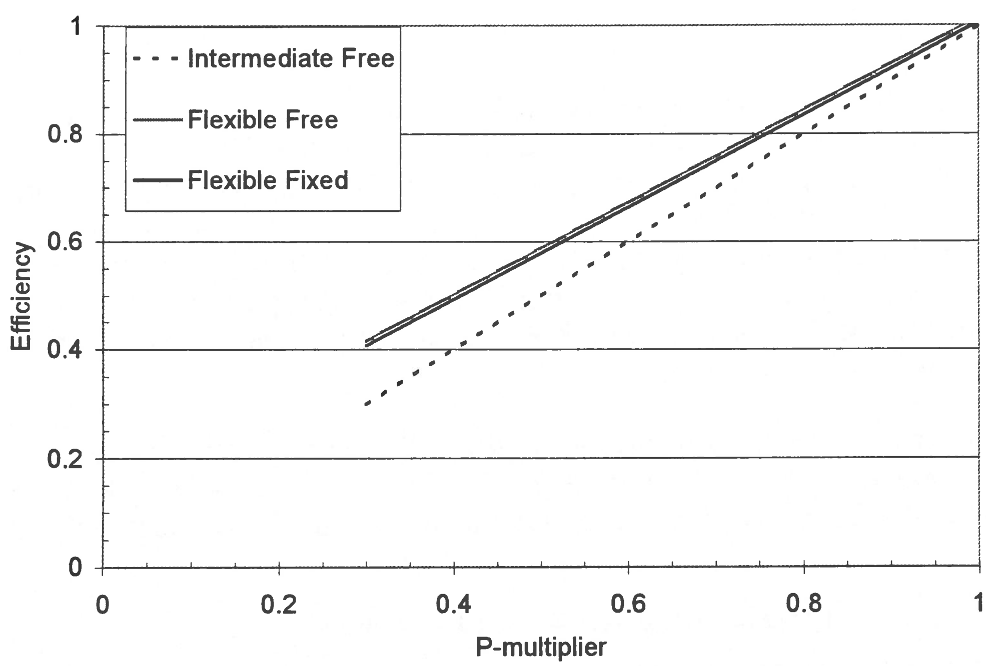


Figure 2.34: Results of Pile Head Fixity using p-multipliers (Problem 3)

2.4 Finite Element Modeling

The finite element method has become an increasingly useful tool for modeling physical systems as the power of computers has increased and the cost of processing power and time has decreased.

In a finite element analysis, a physical system is discretized into a finite number of elements having finite boundaries. These elements are defined by nodes at their corners. In some cases, nodes are also located at intervals along the edges of and within the elements. These higher order elements are much more powerful and yield better results than elements with nodes only at the corners.

The finite element method as applied to a simple load-deformation problem is based on the solution of the following matrix equation:

$$\mathbf{KQ} = \mathbf{F} \qquad \text{Equation 2.48}$$

where:

K = stiffness matrix
Q = displacement vector
F = load vector

The stiffness matrix is formulated using the material properties of the elements. The displacement vector contains the displacements of all of the nodes in their appropriate degrees of freedom. For example, a two-dimensional node would have three degrees of freedom, translation in two directions and rotation. Generally, the matrix equation is being solved for Q, the vector containing the unknown nodal point displacements. The load vector contains all of the applied loads at each node. The solution to this matrix equation, or system of equations, is obtained using one of a number of numerical, iterative techniques. This section will provide a review of the application of the general technique to the laterally loaded pile problem, and a review of available solution methods.

2.4.1 Elements

Laterally loaded piles must be modeled in three dimensions because although the physical system of a single pile is axisymmetric, the applied loads are not. In general, three-dimensional solid elements are sufficient for modeling the pile and soil in which it is installed. The shape of these elements is not critical, though some shapes are more efficient than others for different types of loading. The laterally loaded deep foundation problem does not require any special element formulations such as pore fluid pressure or heat transfer, but elements with these and other special properties are available in most finite element computer codes.

The physical bodies being modeled do not require special elements, but the interaction between the pile and the soil does. Figure 2.35 shows a laterally loaded pile in both plan and profile views. As the pile is loaded laterally, it tends to move away from the soil behind it. This gap may not always form, but its formation is common. This soil does not necessarily fail in tension or even have a tensile load applied to it. This behavior can be modeled with gap elements. These are often formulated as one-dimensional connectors between nodes of the pile and soil. They may have tensile stiffness, and usually have zero length before the loads are applied. As the pile moves in the direction of the applied load, there is a tendency for sliding or relative displacement between the pile and the soil. This can be modeled with zero thickness slip elements, which usually include frictional resistance to movement.

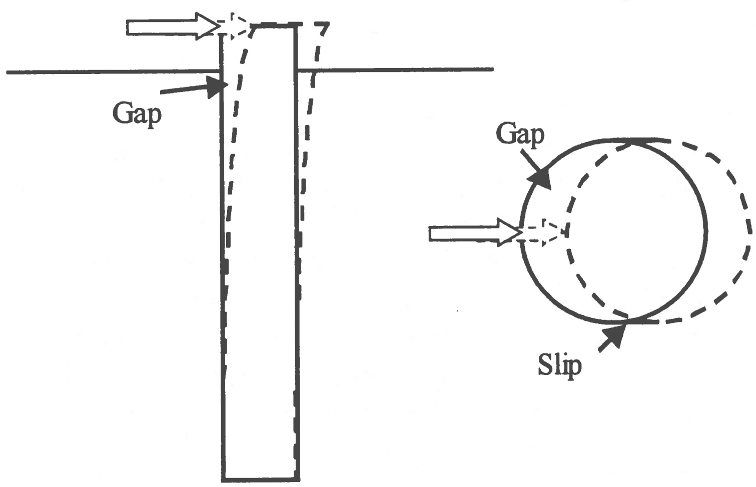


Figure 2.35 Laterally Loaded Pile with Gap and Slip Regions

2.4.2 Material Behavior

The stress-strain relationship of the materials is the single most important factor in finite element analyses. The relationship describes how the nodal point deformations are calculated from the applied loads (or the loads and stresses are calculated from the applied nodal point deformations). Modeling the concrete of the foundation itself is relatively simple, as long as cracking and crack propagation do not become an issue. In almost all cases, it is sufficient to model the concrete as a linear elastic material and check the internal stresses at the end of the analysis to ensure that they did not exceed the concrete strength. The load deformation behavior of soils is usually modeled in one of two ways: as elastic, or elastic-plastic. Linear elastic is by far the most common method of modeling pile group behavior (Chow 1987, El Sharnouby and Novak 1986, Kagawa 1983, Randolph 1981, Selby and Poulos 1983), but often includes modifications such as variation of soil properties with depth. When applied to a problem such as that of laterally loaded pile groups, a linear elastic treatment does not accurately reflect the "shadowing" effect on trailing piles, which reduces the overall efficiency of the pile group (Brown and Shie 1991).

2.4.2.1 Non-Linear Elastic Material Behavior

Non-linear elastic soil models are usually applied by using either a hyperbolic or a piecewise linear stress-strain curve. The model stress-strain curve is usually obtained as a best fit of laboratory strength test data, for example, a series of triaxial tests at different confining pressures. From these test results a hyperbolic or piecewise linear approximation of the stress-strain behavior can be obtained for each confining stress. The dependence of this relationship on confining stress can be approximated by using either a mathematical relationship between total mean stress and the stress-strain curve, or by interpolating between the individual curves obtained from the laboratory tests. This type of model can also account for volume change using Poisson's ratio. The input parameters are usually E (or G) and ν, sometimes varying with mean stress (ABAQUS 1998).

A non-linear elastic model has the advantage that elastic calculations are much less computationally intensive than elastic-plastic calculations. The main disadvantage of this type of model is that it does not accurately reflect the true soil behavior. By definition, elastic deformations are totally recoverable. This means that the material returns to its undeformed shape when the load is removed. If only very small deformations are being investigated, this approximation can provide reasonable results. The nature of the stress-strain relationship used in a non-linear elastic analysis appears to allow larger strains than normally considered elastic.

The reason a non-linear elastic material model can be somewhat effective is that in most geotechnical engineering problems the loads are applied monotonically and not removed. Because of this, when examining gross (large-scale) behavior, it does not appear important that the soil deformations are treated elastically. Many geotechnical problems, especially those involving soil-structure interaction, include relative movements between the soil and the structure. In some cases, this even includes total relief of contact between them. Even if that is not the case, the relative movement usually provides some stress relief to the soil as either the load applied through the structure is removed or surrounding soil elements take on more of the load. In these cases, some of the soil elements do experience unloading although the overall load on the structure is monotonically increasing.

2.4.2.2 Elastic-Plastic Material Behavior

A much more accurate description of soil stress-strain behavior is obtained using an elastic-plastic model. This type of model is required to accurately predict large-strain (greater than a few percent) behavior, and has been used successfully to model pile group behavior (Brown and Shie 1991). The stress-strain relationship of this type of model consists of two parts; an initial elastic portion followed by plastic deformation following failure. There are four elements required to define an elastic-plastic material model: an elastic stress-strain relationship, a yield function or surface, a plastic flow rule, and a hardening law.

When describing an elastic-plastic material model it may be helpful to temporarily set aside the traditional geotechnical definitions of some terms. This is necessary because the yield surface, no matter what name it goes by, is **not** necessarily the same as the failure surface usually defined by geotechnical engineers. For example, the Mohr-Coulomb failure envelope is usually described using c (cohesion intercept) and ϕ (friction angle) obtained from triaxial testing. The points used to define the Mohr's circles are usually taken from the peak of the stress-strain curve. Figure 2.36 shows a typical family of stress-strain curves obtained from triaxial testing at three different confining pressures. The triangles show the values that would usually be taken to define Mohr's circle, and subsequently, the failure envelope in terms of c and ϕ. If these curves are examined from a materials perspective, where yielding is critical, one would choose entirely different points to define the "failure" of the specimens. The yield point on a stress-strain curve is that point which defines the separation between fully recoverable elastic deformation and nonrecoverable plastic deformation. The rectangles in the figure denote points on the stress-strain curves where it is reasonable to assume this change takes place. Even these rectangular data points correspond to some plastic strain, but perhaps little enough to be neglected. It is possible to use either the rectangles or the triangles, but the yielding points are more accurate if the rest of the curve can be modeled because using the peak values makes use of elastic behavior for strains which are not recoverable.

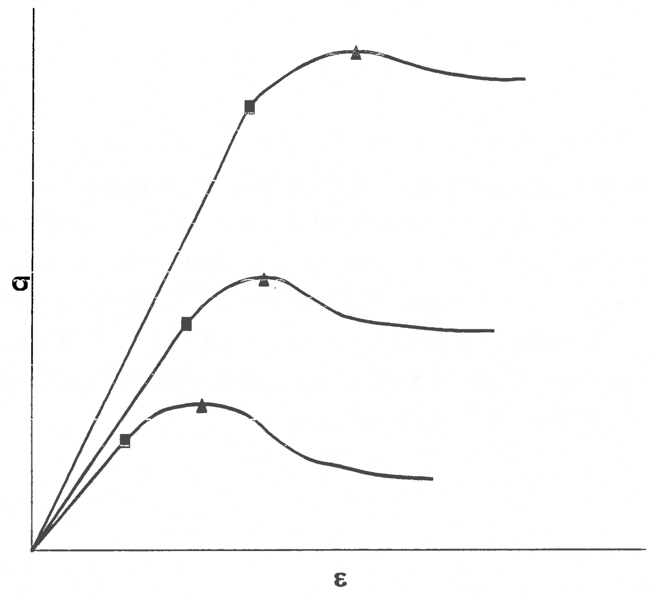


Figure 2.36 Stress-Strain Curves from Triaxial Tests

2.4.2.2.1 Elastic Deformations

The elastic portion of an elastic-plastic material model is usually modeled as linear, with strains calculated using Hooke's Law (Plaxis 1995, ABAQUS 1998). Truly elastic deformations in soil are very small, less than a few percent, so a linear approximation is usually adequate if the yield function has been defined as depicted by the squares in Figure 2.36. If the peak values are used, the modulus (slope of the stress-strain curve) can be determined in a number of ways. Three common modulus definitions are the initial tangent, 50% secant, and 100% secant. The initial tangent modulus is the initial slope of the stress-strain curve. The 50% secant modulus is usually defined as the secant modulus measured at 50% of the yield deviator stress. The 100% secant modulus is defined as the slope of a straight line drawn from the origin to the yield point on the stress-strain curve. If plastic deformations are expected, use of the 100% secant modulus is reasonable, since it predicts the correct strains at yield. The tangent and 50% secant moduli will underestimate elastic strains at yield. If plastic deformations and yielding are not expected, the 50% secant modulus is probably the best choice, but if plastic behavior is expected it may be possible to model the problem as entirely elastic, in which case a changing modulus can be used. Plaxis (1995) recommends using the 50% secant modulus when plastic strains are expected. It is also possible to use non-linear elastic stress-strain curves with most elastic-plastic material models, but that is neither necessary nor realistic. No matter which modulus calculation method is used, it is important to remember that there will be some error in the elastic region when the yield surface is defined from the peaks of the stress-strain curves.

2.4.2.2.2 Yield Functions

Most finite element programs define stresses in terms of the stress invariants p and q, instead of σ and τ which are most commonly seen in geotechnical engineering. The definitions of p and q are given in terms of the principal stresses (σ_1, σ_2, and σ_3) in equations below (Parry 1995). These are not the definitions used by many finite element programs, but the idea is the same. For example, ABAQUS (1998) defines p and q in terms of σ_{xx}, σ_{yy}, and σ_{zz}. The discussion of failure criteria usually involves the meridional and deviatoric planes. The meridional plane is simply the plane defined by the p and q axes. The deviatoric plane is the plane normal to the p axis.

$$p=\frac{1}{3}(\sigma_1+\sigma_2+\sigma_3) \qquad \text{Equation 2.49}$$

$$q=\frac{1}{\sqrt{2}}\sqrt{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2} \qquad \text{Equation 2.50}$$

The yield function for a material describes where purely elastic behavior ends and plastic behavior begins. This is accomplished by writing the yield criterion, or yield surface, in equation form. If the current stress state of the soil element falls outside the yield surface, the soil deformations are governed by the plastic flow rule. Mohr-Coulomb and Drucker-Prager material models are generally applicable to soils since they contain pressure-dependent yield criteria, which are analogous to the friction angle ϕ used in the Mohr-Coulomb failure envelope.

Both Drucker-Prager and Mohr-Coulomb yield surfaces are approximately cone-shaped when viewed in three-dimensional stress space. Drucker-Prager yield surfaces are circular when viewed in the deviatoric plane (looking down the axis of the cone along the p axis), while Mohr-Coulomb surfaces are hexagonal. Three of the corners of the Mohr-Coulomb surface are moved in toward the center of the cone to account for the lack of dependence on the intermediate principal stress which is usually assumed in geotechnical engineering applications (ABAQUS 1998). ABAQUS (1998) includes an input parameter which forces the yield surface to more closely match the Mohr-Coulomb model. Figure 2.37 shows these two surfaces, along with the Tresca and Rankine surfaces, which are special cases of the Mohr-Coulomb for ϕ=0° and ϕ=90°, respectively. The Mohr-Coulomb surface has sharp vertices, which makes it impossible to find a normal vector at the corners of the yield surface. This is usually handled by rounding the corners of the yield surface. The Mohr-Coulomb model has an interesting feature because of these vertices. If two principal stresses are equal, the flow direction can change with little or no change in stress. This could lead to problems when flow localization is important (ABAQUS 1998). The Drucker-Prager model can lead to inaccuracies for high friction angles (Plaxis 1995).

Mohr-Coulomb yield surfaces look like the standard Mohr-Coulomb failure envelope when viewed in the meridional plane. They are defined by straight lines. Drucker-Prager material models are usually much more flexible than Mohr-Coulomb models. They often include options for curved surfaces, either hyperbolic or exponential, in the meridional plane. Brown and Shie (1991) used a hyperbolic formulation of a modified Drucker-Prager yield surface to model the behavior of a pile group in sand with ABAQUS.

2.4.2.2.3 Flow Potentials

Plastic deformations are governed by the plastic flow potentials. The plastic strain increment is a function of the partial derivative of the plastic flow potential functions with respect to stress. These functions are very similar to the yield functions because the direction of the strain increment is related to the orientation of the yield surface. If the inelastic deformation is in a direction normal to the yield surface, it is known as an associated flow rule. Associated flow assumes that volume does not vary as a function of plastic deformation. Non-associated flow means that the plastic deformations are not in a direction normal to the yield surface. The direction is often associated with the dilatancy angle of the soil. Non-associated flow rules have been used to successfully model lateral loading of piles and pile groups (Brown and Shie 1991).

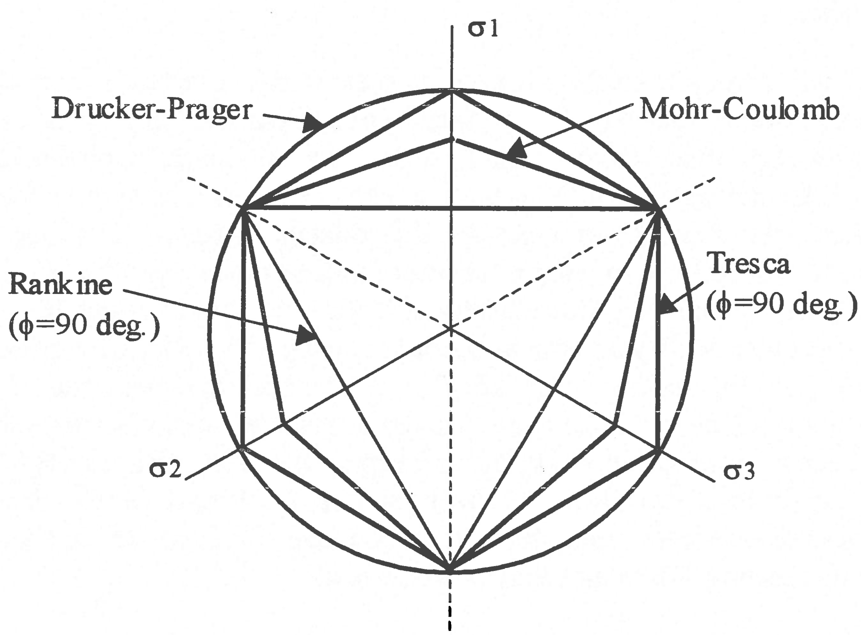


Figure 2.37 Yield Surfaces in the Deviatoric Plane

2.4.2.2.4 Hardening

Hardening can occur in one of two ways in soils. The first is through compression of the soil due to consolidation-type loading, which is not an issue in modeling laterally loaded piles. The second way hardening can occur is through shear. A soil which undergoes shear-induced hardening will usually not exhibit the typical stress-strain curve consisting of a steep initial portion with a relatively sharp peak followed by a gradual decrease in stress which eventually levels off at the residual strength. Instead, strain-hardening soils often have stress-strain curves that increase continuously, sometimes, but not always, approaching a limiting value asymptotically. Hardening can be modeled by raising the yield surface as plastic deformation occurs. Softening behavior is sometimes modeled by lowering the yield surface. Figure 2.38 shows an example of the treatment of hardening behavior and its relationship to the dilation angle, Ψ, including the direction of the plastic strain increment. The peak and post-peak behavior of a soil stress-strain curve can be modeled in one of two ways. The first, used in ABAQUS (1998) and most other programs, is to use an initial linear elastic relationship followed by hardening to the peak, then softening. The other method utilizes a series of nested yield surfaces (Prevost, 1996)

2.4.2.2.5 Dilatancy

Dilatancy is defined as volume increase caused by shear under conditions of no change in compressive stress, and is caused by the interaction of soil particles as they move apart, trying to slide past each other. Both plastic flow direction and hardening behavior are functions of the dilatancy angle, but the actual volume change may have to be included separately in the model. Even if that is not the case, dilatancy does not continue without bound in real soils. There are two ways to prevent limitless dilatancy. The first method is based on the volume of soil being sheared (Plaxis, 1995). A critical volume beyond which the soil cannot expand is usually entered as a soil parameter. This may be expressed as a critical void ratio. For this method to be effective, the original void ratio, the critical void ratio, and the volume of the soil shear zone must be established. Another method for handling the dilatancy cutoff problem is placing a cap on the yield surface (ABAQUS 1998). This option is usually available only with the Drucker-Prager yield surface. Figure 2.39 shows a yield surface with cap in the meridional plane. The capped yield surface stops dilatancy by creating softening behavior in the soil.

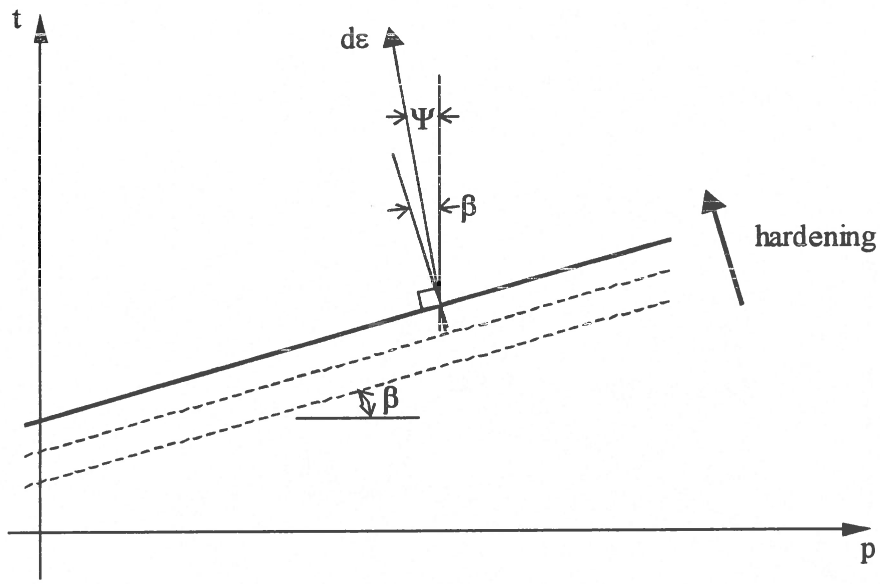


Figure 2.38 Yield Surface in the Meridional Plane with Hardening

2.4.3 Computer Program Requirements

Finite element computer programs can be extremely complex. The actual solution of the system of equations is only part of the programming task. The most complex part of a finite element program is the creation and assembly of the stiffness matrix, and then organizing it in a form that is easy to solve. Because of the complexity, it is extremely uncommon to develop a program oneself. Instead, it is common to rely on finite element software packages developed by others. In evaluating potential programs for use in this project, we have considered a number of factors, including the material models available in the program.

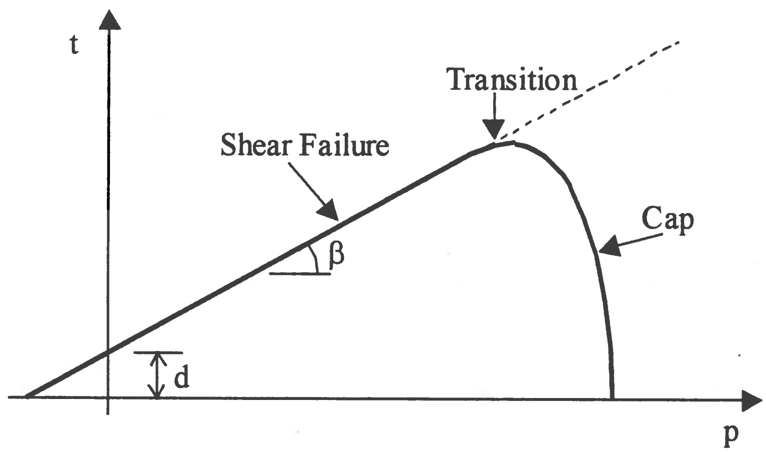


Figure 2.39: Yield surface with cap in meridional plane

2.4.3.1 Elements and Material Models

The program selected for the finite element modeling portion of this research must obviously be able to model the physical system. This includes the special gap and slip interface elements described above. These elements are fairly common in general-purpose and mechanical engineering-based finite element codes. The material models available with the program must be able to adequately model the behavior of soil when subjected to lateral pile loading. Either the Mohr-Coulomb or Drucker-Prager yield criteria should be sufficient. The Drucker-Prager is more common in non-geotechnical engineering programs, and is usually available with more of the special behavior requirements such as hardening and dilatancy cutoff. These factors were considered essential features of the program we select.

2.4.3.2 Operating System

Many finite element programs were originally created for use on mainframe computers with card input. Most are still designed to run on supercomputers or workstations, but there have been major improvements in the methods of input. With the recent advances in computer technology, personal computers are finally adequate to model at least some problems without being prohibitively slow. It is possible to get access to a workstation on which to run the program finally selected, but it would be preferable to get one that is designed for use on a computer running the Windows NT operating system. This is one of the more minor considerations in choosing a finite element program for this project. Most of the more powerful general-purpose finite element programs have versions available for Windows NT.

No matter what operating system is selected, there is a cost issue involved. If a workstation is required, it should be possible to purchase time on one of the workstations available. If a PC program is used, it may be necessary to purchase a more powerful PC than what is currently available. This issue is particularly relevant to follow-up modeling that might be performed by the Department or by local practitioners; clearly, the lower the computational power required, the better.

2.4.3.3 Pre-Processing and Mesh Generation

The most time-consuming and difficult part of finite element modeling is mesh generation and refinement. Since the program selected for analysis will be new to the project team members it would be very inefficient to spend weeks just learning the syntax required to create an input file. There are pre-processors with graphical user interfaces available with, or at least for, most finite element programs. Most of these pre-processors come with some form of three-dimensional automatic mesh generators, so the user is not required to discretize the mesh and enter hundreds or thousands of element corner nodes as x, y, and z coordinates. With the automatic mesh generating pre-processors, it is usually just a matter of a click-and-drag with the mouse to modify the location of nodal points.

2.4.3.4 Solution Methods

The basis of the finite element method is the solution of a very large system of equations, which describes the behavior of the model elements. All finite element programs are designed to solve this system of equations using numerical matrix solution methods. Many models have very sparse, low bandwidth, symmetric matrices. A matrix is sparse if it contains mostly zeros. The bandwidth of the stiffness matrix is determined by the interconnection between nodes. If an element is shared by many nodes, the bandwidth of the matrix increases. Since the system we are modeling is a continuum, the bandwidth of the matrix will be relatively high. Some programs, such as FLPier, treat only the pile as a collection of finite elements and use p-y curves to model the soil-structure interaction. In most cases, including most plasticity problems, the matrix being solved is symmetric about its diagonal. If hardening is not specified and the flow direction is perpendicular to the yield surface, the matrix will be symmetric. If hardening is specified so that the flow direction is not perpendicular to the yield surface, the matrix will be asymmetric. Special, less efficient, procedures are used to solve matrices that are not sparse or not symmetric. A choice of solvers would be a good feature for the selected program to include, since some solution methods may be more efficient for our particular problem.

2.4.4 Computer Programs Investigated

A total of 96 "finite element" computer programs were examined as part of this investigation. They were evaluated based on the four criteria described above, with the material model and element criteria being the most important. GROUP, a commonly used program to solve a laterally loaded pile problem, was not considered because it is not a finite element solution plus it assumes the response of the soil through p-y curves.

2.4.4.1 Programs Rejected

Table 2.22 lists the programs that were investigated and rejected, along with a reference and reason for rejection. The reasons for rejection were greatly simplified and abbreviated, so Table 2.21 lists additional explanations and descriptions of the reasons for rejection.

Table 2.21 Description of Reasons for Rejecting Programs

Reason for Rejection	Description
2D	The computer program could only model two-dimensional problems.
Linear	The computer program could only model linear elastic problems.
Structural	The program is designed for analysis of structures (buildings). This usually means a sparse matrix solver, linear elastic materials, beam elements, and truss elements. This also includes programs that model the pile as a finite element, but the interaction with the soil is treated almost like a boundary condition. In this case, there are often many assumptions made regarding the soil-structure interaction, which would not allow solutions at a sufficient level of detail.
Material	From the information available it did not appear promising that the program would include all of the material behaviors required for this project.
Not FE	This category includes boundary element and infinite element programs.
Earthquake	The program is designed for structural response or soil-structure interaction in response to earthquake loading.

Dynamic	The program is apparently designed for high-speed event simulation, or time-dependent problems.
Tools	These were not finite element programs, but equation solvers, material databases, etc.

Table 2.22 Programs Rejected

Program	Reference	Reason for Rejection
Aladdin	http://www.isr.umd.edu/~austin/aladdin.html	Tools
Algor (Accupak/NLM)	http://www.algor.com/	Event Simulation (Dynamic)
Altair	http://www.altair.com/	Linear
ANSR-1	http://www.eerc.berkeley.edu/software_and_data/software/ansr1.html	Structural
ANSYS/LS-DYNA	http://www.ansys.com/	Material
AVwin98	http://www.avansse.com/AVwin98.htm	Structural
BEASY	http://www.compmech.com/beasy/	Not FE (Boundary Element Method)
BEFE	http://www.cis.tu-graz.ac.at/ifb/soft/	3D FE/BE
CADRE	http://www.cadreanalytic.com/Cadrepro.htm	Structural
CAEFEM	http://www.caefem.com/	Material
CANSAFE	ftp://ftp.emr.ca/mets/mrl/readme.htm	2D, Linear
CESAR	http://www.lcpc.fr/LCPC/English/Service/CESAR/html/cesar_gb.html	All information in French.
COSAR	http://www.femcos.de/esoftware.html	Material

Program	Reference	Reason for Rejection
COSMOS/M, NSTAR	http://www2.ctceng.com/ctceng/	Material
CRISP-90	http://www.ggsd.com/programs/crisp.htm	2D
CSA/NASTRAN	http://www.csar.com/nastspec.html	Material
CSTRAAD	http://www.eaglepoint.com/structural/cstradd.htm	Structural
DEFPIG	Selby & Poulos 1983	Elastic soil (Mindlin) with failure
DIE	http://www.xfemily.com/english.htm	in German
DrFrame, DrBeam	http://www.DrSoftware-home.com/	Structural
Elfen	http://rsazure.swan.ac.uk/	Material?, not able to contact
ENERCALC	http://www.enercalc.com	Structural
ESAComp	http://www.esacomp.hut.fi/	Tools
FE/PIPE, BOS Fluids	http://www.paulin.com/	Pipes, Fluid Flow
FE2D	http://www.ce.vt.edu/faculty/kuppuhome/fem1.htm	2D
FEACPP-3D	http://www.netmagic.net/~indus/cpp3d.html	Linear
FEAP	http://www.eerc.berkeley.edu/software_and_data/software/feap.html	2D Plastic
FEECON	http://www.geocomp.com/	2D
Felt	http://www.cse.ucsd.edu/users/atkinson/FElt/	Limited Materials, Compiling Required
FEMAP	http://www.entsoft.com/	Pre/Post

Program	Reference	Reason for Rejection
FEMLAB	http://www.femlab.com/	Math
FlexPDE	http://www.pdesolutions.com/	Equation Solver
FLPIER	Flpier Version NT 1.33 User's Manual/Help Files	Structural
GBW32	http://www.grapesoftware.mb.ca/	Structural
GENESIS	http://www.vma.com/	Linear
GeoFEAP	http://www.eerc.berkeley.edu/software_and_data/software/geofeap.html	2D
GEOnac	http://www.sintef.no/	Material
GT STRUDL	http://shell5a.best.com/~solvers/gtstrudl.html	Structural
HI/DYNA3D	http://www.hydrosoft.com/	Dynamic
HyperVAST	http://www.martec.com/HyperVAST/	Material
I-DEAS	http://www.sdrc.com/pub/catalog/ideas/simulation.html	Linear
INERTIA	http://www.meridian-marketing.com/INERTIA/	Material
LapFEA	http://home.earthlink.net/~lapcad/lapfea.htm	Pre/Post
LARSA	http://larsausa.com	Structural
LifEst	http://www.somat.com/software/lifest.html	Fatigue
LUSH2	http://www.eerc.berkeley.edu/software_and_data/software/lush2.html	Earthquake
Macsyma	http://www.macsyma.com/	Math, 2D

Program	Reference	Reason for Rejection
MARC	http://www.industry.net/c/mn/08fqj	info requested
MECANO	http://www.samcef.com/07-mecanl.html	Material
MI/NASTRAN (ME/NASTRAN)	http://www.macroindustries.com/	Linear
Microstran	http://dial.pipex.com/engsys/ms.htm	Structural
mTAB*STRESS	http://www.sai-mtab.com/software/index.htm	Linear
Multiframe	http://www.formsys.com/	Structural
NE/NASTRAN	http://www/noraneng.com/	Material
NISA II	http://www.emrc.com/	info requested
NISA/CIVIL	http://www.emrc.com/	Structural
NISA/P-ADAPT	http://www.emrc.com/	Linear
PERMAS	http://www.intes.de/	Material
PHASE2	http://www.rockeng.utoronto.ca/Phase2.htm	2D
PHLEXsolid	http://www.comco.com/	Material
PIGLET	Selby and Poulos 1983	Elastic, Uses results of FE analyses, Interaction from Poulos 1971
PISA	http://www.pisa.ab.ca/	2D
Plaxis	http://www.plaxis.nl/, Plaxis (1995)	2D
PROKON	http://www.prokon.com/	Geotech, Structural
QuickField	http://www.quickfield.com/	Electromechanical

Program	Reference	Reason for Rejection
RISA-3D	http://www.risatech.com/	Structural
ROBOT97	http://robot97.com/usa/	Structural
RSTAB	http://www.dlubal.de/softwree.htm	Structural
SAFI	http://www.safi.com/safi3d.htm	Structural
SAP2000, ETABS	http://www.csiberkeley.com/	Structural
SASSI	http://www.eerc.berkeley.edu/software_and_data/software/sassi.html	Earthquake
SESAM	http://www.dnv.com/dnvsoftware/products/products_body.html	Shipbuilding
S-FRAME	http://web.idirect.com/~softek/sframe.htm	Structural
SODA	http://www.acronym.on.ca/soda/features.phtml	Structural
SOLVIA	http://www.solvia.se/	Material
SPACE GASS	http://www.spacegass.com/sg_oview.htm	Structural
Spectrum	http://www.centric.com/products/solver/index.html	Material
STAAD-III	http://www.reiusa.com/	Structural
STARDYNE	http://www.reiusa.com/	Material
STRAND6	http://www.strand.aust.com/	2D, Material
STRAP	http://www.atir.com/aboutstrap.html	Structural
StressCheck	http://www.esrd.com/documentation.htm	Material
TEDDS	http://www.csc-leeds.co.uk/csc/news.htm	Structural

Program	Reference	Reason for Rejection
TOCHNOG	http://www.tm.wb.utwente.nl/~roddeman/tn_release/tnhome.html	Free program, requires compiling, etc.
Toolbox:PDE (Comsol)	http://www.comsol.se/Matlab/pde_tb.e.html	Math
UAI/NASTRAN	http://www.uai.com/nastran.html	
Z_SOIL	http://www.zace.com/	2D
Zebulon	http://www.nwnumerics.com/Zebulon/	Elements, Material

2.4.4.2 Programs Not Rejected

The programs described in this section were not initially rejected based on the criteria described in section 2.4.3. Based on the information available at this time, these programs should be able to meet our modeling requirements. All of the companies, except ABAQUS (which was not contacted because we already have access to their manuals) and DYNAFLOW (which provides the entire manual on the internet), were sent email requesting more information and only ADINA has responded as of this writing.

2.4.4.2.1 ABAQUS

ABAQUS is an extremely versatile finite element program designed for the analysis of a wide variety of problems ranging from simple linear elastic stress/displacement analysis to transient mass diffusion and heat flow analyses. This is the only program that has been used for fully non-linear analysis of laterally loaded pile group interaction (Brown and Shie 1991). All information was taken from the ABAQUS/Standard User's Manual (ABAQUS 1998). ABAQUS includes both Drucker-Prager and Mohr-Coulomb yield surfaces. Its formulation of the Drucker-Prager yield surface allows simultaneous hardening and shear-induced volume change. There is also an option in ABAQUS for direct input of triaxial test data in the form of stress-strain values. This data is analyzed by the program to determine the appropriate parameters for use with the Drucker-Prager material model. The linear Drucker-Prager model includes a parameter which is used to model the dependency on the intermediate principal stress by moving in the "corner" of the yield surface as viewed in the deviatoric plane. ABAQUS includes all of the elements required for modeling the laterally loaded pile group problem, along with a variety of other elements. There is a pre-processor/mesh generator available for ABAQUS called ABAQUS/CAE, which may be useful for this project (http://www.hks.com). Without it, all input is done using input files. There are versions of ABAQUS available for Windows NT 4.0 and most workstation and mainframe environments. ASU has a license for ABAQUS, but not for the pre-processor. The main disadvantage of ABAQUS is that all input is through files, making it extremely time-consuming to create and troubleshoot the very large input files required for modeling laterally loaded piles.

2.4.4.2.2 ADINA

ADINA is a general-purpose finite element program used mainly in the structural and fluid flow fields. All of the information on this program was obtained from the company web site (http://www.adina.com/), and e-mail contact with their customer support department. ADINA includes a Drucker-Prager cap material model, which can include a zero tensile strength condition. One strength of the program is its ability to model contact with special interface elements. These include the gap and slip elements that are required for this analysis. The material models and interface elements are sufficient for our needs. ADINA has its own pre-processor, called the ADINA User Interface (AUI), which is a Windows-like program. Versions of ADINA are available for PCs, workstations, and supercomputers. The modeling capabilities appear to meet our requirements, but the pre-processor does not appear to be as easy to use as a full windows program. The user support would probably be adequate, since they immediately answered a request for information.

2.4.4.2.3 DIANA

DIANA is a general-purpose three-dimensional finite element analysis program that was developed by TNO Building and Construction, a civil engineering company, in cooperation with the major Dutch Technical Institute Universities. All of this information was gathered from the company web site (http://www.diana.tno.nl/). According to the company, the strengths of the program lie in analysis of concrete and soil. DIANA can be used for linear and nonlinear statics, frequency response analysis, linear and nonlinear transient analysis, fluid-structure interaction, and many other problem types. Both Mohr-Coulomb and Drucker-Prager plasticity models are included in the program. It also includes several types of interface elements. The details of the implementation of the material models and interface elements are not available yet. Information was requested, but not received. Two solvers are available in DIANA, a direct Gauss decomposition, and an iterative method. The program is designed to work with the pre- and post-processor FEMGV, available from Femsys Ltd. It is unclear what operating systems DIANA can operate under, but they do mention that FEMGV will work on everything from PCs to Cray supercomputers. The fact that DIANA requires a pre-processor from a third party is probably its greatest weakness. This could conceivably lead to problems when material behavior is more complex and not very common, such as strain-hardening followed by softening, or dilatancy cutoff.

2.4.4.2.4 DYNAFLOW

DYNAFLOW was developed by Professor Jean H. Prevost at the Princeton University Department of Civil and Environmental Engineering. It, like ABAQUS, is a very versatile general-purpose finite element program, designed to handle both static and transient nonlinear problems. All of this information was gathered from the Dynaflow Version 99 Release 0.1A user's Manual (Prevost 1999). DYNAFLOW includes interface and slide-line elements. The slide-line elements are available with perfect friction, no friction, or Coulomb friction. The Drucker-Prager model included with DYNAFLOW can use either an associative or non-associative flow rule. A cap model is also included. DYNAFLOW includes an option for nested yield surfaces, called a "multi-yield" model. This model includes options for isotropic hardening followed by softening which can be used to model the nonlinear portion of the soil stress-strain curve. DYNAFLOW is available for both workstations and PCs. There is no pre-processor available with the program, but it is designed to work with FEMGV, a graphical pre- and post-processing program, available from a third party. Solution of the finite element problem may be done using a variety of methods, depending on the particular situation. Overall, DYNAFLOW appears to meet all of the requirements for this project, but the documentation is limited. The user's manual contains descriptions of parameters and variables, but does not tell the user how to use material properties to obtain the desired results.

2.4.4.2.5 LS/DYNA3D (NIKE3D)

LS/DYNA3D is the Livermore Software Technology Corporation's version of DYNA3D which was developed by Lawrence Livermore National Laboratory. The program is designed for analysis of deformation of metal objects striking hard surfaces at high velocities. NIKE3D is the module of the program that performs static analysis, and was listed as a finalist, along with ABAQUS, on the Superpave research project. All of this information was gathered from the company web site (http://www.lstc.com). LS/DYNA3D and LS/NIKE3D are general-purpose finite element programs, capable of modeling a variety of problems. It may be possible to get NIKE3D without DYNA3D, but DYNA3D is the main product of the company. Gap and sliding elements are included as part of the element library. The information available on their Internet site says they have material models that are appropriate for "concrete and soils." They also have special automotive elements such as seatbelts, which may indicate that the program is too specialized for our use. There are versions of the program available for workstations and PCs and it appears to have more than one matrix solver. Additional information has been requested but not received. Since LS/NIKE3D is treated as a component of the main dynamic and impact analysis program LS/DYNA3D, it may be difficult to use on its own. User support may also be lacking, based on the fact that requests for information were ignored.

2.4.4.2.6 LUSAS

LUSAS is primarily a structural engineering program, but it includes the capability of performing full three-dimensional nonlinear plastic analyses. All of this information was gathered from the company web site (http://www.lusas.com/). It includes slide-line elements and special gap elements for rock mechanics that should work for our application. Drucker-Prager and Mohr-Coulomb failure criteria are included, but the dilatancy and hardening capabilities are not known. LUSAS runs under the Windows NT operating system and has a graphical pre-processor. LUSAS has several versions available for different applications ranging from mechanical engineering to bridge design. The fact that the versions are so specialized may mean that the program is not as flexible as we would like. The civil engineering version is intended for use in structural engineering.

2.4.4.2.7 MSC/NASTRAN

NASTRAN is a general-purpose finite element program that was originally developed for NASA in the 1960s and is being modified and distributed by a number of companies. This particular version is a product of the MSC Software Corporation, one of the original NASTRAN developers. All of this information was gathered from the company web site (http://www.macsch.com/). This program includes all of the required gap and slip element types. It also includes both Mohr-Coulomb and Drucker-Prager yield criteria. Isotropic hardening is also possible, but it is unclear whether it can be followed by softening and whether or not volume change can be included. Additional information on the available material models was requested from the company, but no response has been received. There are versions of the program available for a variety of computer platforms, including workstations and PCs. MSC/PATRAN is a graphical pre-processor that is available with MSC/NASTRAN. Several solver options are included with the nonlinear analysis option. A demo version of the program has been ordered, but not received. The user interface for this program appears to be very easy to use, but the lack of response to inquiries leads to questions about the quality of user support.

2.4.4.2.8 VISAGE

VISAGE is a three-dimensional finite element program designed for soil and rock mechanics applications. All of this information was gathered from the company web site (http://www.vips.co.uk). One of its greatest strengths is in reservoir engineering where drawdown and temperature changes are used to find the stress state in reservoir rock and then model the initiation and propagation of faults in the rock. Mohr-Coulomb and Drucker-Prager yield models are both included in the program. The included element types, dilatancy, and hardening capabilities are not known. More information was requested, but no reply received. VISAGE includes both iterative and skyline solvers, which are appropriate for dense and sparse matrices, respectively. There is a pre-processor available called FEMGEN, and the programs are available for both Windows and Unix operating systems. The specialization in reservoir engineering and rock mechanics is one drawback to the program. Another is the lack of response to information requests.

2.4.4.3 Recommendation or Selection

Three of the programs, ABAQUS, ADINA, and MSC/NASTRAN, appear to meet our requirements better than the others do. Operationally, it is very difficult to determine if one program is better than the others. It appears that ADINA has the analysis and modeling capabilities required, but their pre- and post-processing program appears to be less user-friendly than PATRAN, which comes with MSC/NASTRAN. The material model capabilities of MSC/NASTRAN are unknown at this time. At this stage in the investigation we believe it is not practical or wise to choose with finality. After some initial use it will be more clear. Based on the data available and the evaluations performed to date, our preliminary first choice would be ABAQUS, and our second choice would be MSC/NASTRAN, and then ADINA would be our third choice. However, all three computer programs appear likely to be satisfactory for the intended application.

2.5 Summary of Practice

The purpose of this section is to determine the state of the practice with regards to laterally loaded groups of drilled shafts. It was deemed that the best way to obtain this information was through directly contacting practicing engineers. This was accomplished by conducting a series of interviews with state departments of transportation (DOTs) and Maricopa County (i.e. local area) consultants. After identifying the two groups an interview protocol was tailored to each. A copy of the protocols can be found in Appendix F.

2.5.1 State Departments of Transportation

The main objective of conducting the interviews with state DOTs across the U.S. was to determine the design procedure for groups of laterally loaded drilled shafts. It is important to specify drilled shafts in the questions because AASHTO has different design procedures for driven piles and drilled shafts. The process of conducting the interviews with the DOTs was completed over the course of three months during which all 50 states were contacted. In an effort to question the most knowledgeable person on the subject at each DOT Mr. Frank McCullagh, of the Arizona Traffic Research Center (ATRC), contacted the DOTs via e-mail, requesting that he be notified of whom the researchers should contact. Approximately two-thirds of the states responded to Mr. McCullagh's e-mail. In order to have a contact at the DOTs that did not respond to the e-mail Mr. McCullagh provided names of senior bridge designers with hopes that they could direct the researchers to the correct person.

The interviews were conducted primarily by telephone with some written lists of questions sent to some DOTs upon their request. If the questions were e-mailed or faxed, a follow-up phone call was conducted with the respondent to verify their answers. The question and answer portion of the interview took anywhere from 5 minutes to 1 hour and 15 minutes. The average amount of time spent actually conducting the interviews was 20 minutes. Upon completing an interview, the researcher reviewed the notes from the conversation and completely filled out the answers that were abbreviated during the course of the phone call. To maintain the integrity and consistency of the process the same person conducted all of the interviews.

The primary focus of the interview was with regards to laterally loaded drilled shaft groups in full height abutments. However, a number of states either did not use drilled shafts in full height abutments or they did not use full height abutments in general. In these situations the survey was continued with the emphasis on any projects for which a drilled shaft group was designed to resist lateral loads.

To date 42 interviews have been completed. Of the 42 states responding to the interview there were only 27 states that design groups of drilled shafts. Of the 27 states, most indicated that driven piles were the most common pile type used, but had designed some groups of drilled shafts and they were beginning to use more drilled shafts and would continue to do so in the future. The interview results were divided into three different categories of drilled shaft design information: design procedure, computer programs used in design, and allowable deflections.

2.5.1.1 Design Procedure

After establishing which states design groups of drilled shafts to resist lateral loads it was necessary to determine the design procedure. Of the 27 states that design groups of laterally loaded drilled shafts some were unaware of what analysis method was used or how the reduction factors for that method should be applied in the design process. The survey results are tabulated in Table 2.23. From Table 2.23 it is clear to see that the two analytical methods that are used most by the various state DOTs are the AASHTO procedure and the p-multiplier approach.

Table 2.23: Design Procedures for State DOTs

<table>
<tr><td rowspan="2">Analysis Method</td><td colspan="3">AASHTO</td><td>P-multiplier</td><td>Elastic</td><td>Don't Know</td></tr>
<tr><td colspan="3">15</td><td>12</td><td>1</td><td>0</td></tr>
<tr><td rowspan="2">How Factors Applied</td><td>Capacity</td><td>Soil Prop.</td><td>Don't Know</td><td rowspan="2" colspan="3"></td></tr>
<tr><td>12</td><td>3</td><td>0</td></tr>
</table>

The AASHTO design procedure uses reduction factors to account for group effects. Of the 15 states that said they use the AASHTO design procedure, not all of them applied the AASHTO reduction factors in the same way. There were 8 states that applied the reduction factor to the single pile capacity, including Arizona. There were two states, Washington and Texas, that applied the reduction factors to various soil properties. The Texas DOT applies the reduction factor to the ultimate soil strength. The Washington DOT uses modified AASHTO reduction factors and uses a nomagraph to modify the friction angle (ϕ) of the soil. They also modify the modulus of subgrade reaction. The remaining 5 state DOTs who indicated that they follow the AASHTO design procedure were not sure how the reduction factors presented in AASHTO were applied in design. It is believed that these answers may result from a contact who was not, in fact, the most knowledgeable person in the DOT on the subject. To confirm answers from these DOTs and those that have yet to complete the survey, they will be contacted in the future.

The p-multiplier design procedure uses p-multipliers to account for group effects. There were seven states that reported that they use this design procedure and all of them apply the reduction factor in the same way, the p-values of the p-y curve. These two design methods have been explained in previous sections.

2.5.1.2 Computer Programs Used

A point of information of interest was how state DOTs design single piles or shafts subjected to lateral loading: computer program or a hand solution. In general, most DOTs reported use of a computer program that requires p-y curves. In those instances a follow up question was asked as to how the p-y curves are obtained, whether from geotechnical recommendations, single pile load tests, or generated from the program based upon site specific soil properties. Most DOTs responding primarily use program generated p-y curves, with only a few exceptions. The most notable exception was when dealing with rock.

2.5.1.3 Allowable Deflection

As previously discussed, reduction factors, for any method, are a function of the deflection level. Therefore, it was important to know the allowable lateral deflection of the pile head under service and extreme conditions. This question was applicable to drilled shaft groups and single drilled shafts. Of the 42 states responding, 40 indicated that they have designed single drilled shafts. Of the 40 states responding 14 (see Table 2.24) replied that the allowable lateral deflection under service loads was less than 1 in. There were 16 respondents who indicated that the allowable lateral deflection varied from structure to structure. The remaining 10 DOTs responding were not sure of the standard for allowable lateral deflection in their state.

With regards to allowable lateral deflections during extreme (typically seismic) events the responses fell into four general categories: deflections less than 2 in, not considered, varying structure to structure, and not sure. The results are presented in Table 2.24. State DOTs which fell into the not considered category were in states where the seismic classification is very low. The number of state DOTs unsure about the allowable deflection under seismic loading increased from 10 to 16 when considering static loading. It is believed that the increase in the number of state DOTs not sure is due to the complexities associated with seismic loading.

Table 2.24: Allowable Lateral Deflection for State DOTs

Static				Seismic				
< 1"	> 1"	Not Sure	Varies	< 2"	> 2"	Not Sure	Not Considered	Varies
19	3	2	21	7	2	7	17	12

2.5.2 Maricopa County Area Consultants

A portion of this research was to involve determining the practice of Maricopa County (i.e. local area) consultants. An effort was made to contact 20 consultants in the area. Of those contacted there were 12 respondents that were able to answer the interview questions. Those unable to answer stated that they had not designed any groups of drilled shafts to resist lateral loads nor did they have a design procedure. The protocol used for conducting these interviews was similar to that used for the DOT surveys. Upon examination of the results of the interviews, it is clear that there is a dividing line between the answers. The dividing line was based upon what type of engineer was interviewed: geotechnical or structural engineer. Most commonly, the geotechnical engineer provides the structural engineer with information about the soil at the site and specifically soil properties necessary for the structural engineer to perform a computer analysis of the single pile problem. It is typically up to the structural engineer as to how the reduction factors are applied in design, for closely spaced groups of laterally loaded shafts.

The majority of the interviews were conducted in person. The consultants were contacted and a meeting was scheduled. The questions were faxed in advance so that the engineer would have a chance to review the questions and gather his/her thoughts. Some of the respondents filled out the answers prior to the meeting. In these situations, the answers to the questions were reviewed and additional questions asked if any clarification was necessary. In cases when the questions were not completed in advance then the engineer was asked the questions and notes were taken during the meeting. Upon returning to the office the researcher typed up the answers to the questions and faxed them to the engineer for their review and approval. The remaining interviews with local area consultants were conducted over the telephone. In this case, the interview process was similar to the DOT interviews.

Of the 12 respondents 6 were geotechnical engineers and 6 were structural engineers. Due to the difference in answers between the two groups the results are presented in two tables. In Table 2.25 the results of interviews conducted with geotechnical engineers is presented.

Table 2.25: Survey Results for Geotechnical Consultants

Analysis Method for Groups of Drilled Shafts				
	Yes			No
Recommends Reduction Values	3			3
	AASHTO		Other	
What is Recommended	3		0	
	Capacity	Don't Know		
How Apply	1	2		
Programs				
	Computer Program			Other
How Solve Single Pile	6			0
	LPILE	COM624	Other	
What Program	6	0	1	
How Develop p-y curves	Programs with some exceptions			
Deflection				
	Static		Seismic	
	1 –1½"	Don't Know	Don't Know	
Allowable Deflection	5	1	6	

From Table 2.25 it is clear that the geotechnical engineer is primarily involved with providing soil properties and recommendations to the structural engineer. Of the 6 geotechnical engineers surveyed, all stated that they provide the structural engineer with the soil properties necessary to perform analysis using the LPILE program. There were 3 geotechnical engineers who stated that they commonly provide the structural engineer with recommended reduction factors to account for closely spaced piles in a group. All 3 of them said that they recommend the AASHTO reduction factors per ADOT directive. Of those that provide recommendations, only 1 provides a recommendation of how to apply the reduction factor. Table 2.25 shows that LPILE is the program that is used by all the geotechnical engineers and that they rely on the computer to generate the p-y curves with some exceptions. The most common exception was when dealing with rock. The allowable deflection for a drilled shaft was of little concern to the geotechnical engineer in most cases.

Table 2.26: Survey Results for Structural Consultants

Analysis Method for Groups of Drilled Shafts				
	AASHTO		Other	
What is Recommended	4		2	
	Capacity		Don't Know	
How Apply	6		0	
Programs				
	Computer Program			Other
How Solve Single Pile	6			0
	LPILE	COM624	Other	
What Program	4	3	0	
How Develop p-y curves	Programs with some exceptions			
Deflection				
	Static		Seismic	
	$^1/_2$"	Varies	varies	
Allowable Deflection	3	3	6	

The survey results for structural engineers are presented in Table 2.26. The structural engineers were asked what design procedure they used to account for group effects. Of the 6 respondents all stated that they used the AASHTO design method. When asked what reduction factors were used 4 respondents stated that they followed AASHTO or interim ADOT policy (Walsh etal, 1998), and 2 stated that they follow the recommendation from the geotechnical engineer. However, regardless of where the reduction factors came from the 6 structural engineers responding to the survey all applied the reduction factors in the same way, to the single pile capacity. This is the method that is currently recommended by ADOT. Table 2.26 shows that the computer programs used to solve the single pile problem were LPILE and COM624. These programs are very similar. LPILE is the commercial version and COM624 is the government public access program. When the structural engineer uses these programs he/she generally allows the computer to generate the p-y curves used in the analysis with some exceptions, similar to those for geotechnical engineers.

With regards to deflection, 3 of the respondents stated that the allowable lateral deflection should be less than ½". The other 3 respondents stated that it varied from structure to structure. All 6 respondents stated that the allowable lateral deflection under seismic loading varied with the structure.

Another aspect of the consultant interviews not presented in the tables was identifying projects in which groups of drilled shafts were used. This proved a valuable starting point in identifying projects in Arizona. This is dealt with in more detail in the next section. Along with identifying projects was determining the design procedure used for the projects. Although currently the design procedure uses AASHTO or ADOT interim reduction factors this has not always been the case. It was during the interviews that the consultants were asked if the researchers could obtain details as to what design procedure was used. Most of the respondents said that they would help in this stage of the research, however, file retention periods for many of the structures have subsequently expired. As a result, some uncertainty about design methods remains.

3.0 Summary of Historic Use

The purpose of this section was to determine how drilled shaft groups have been used in lateral load applications in Arizona, specifically with regards to shaft geometry, shaft group geometry, and soil conditions. It was thought that the primary application of laterally loaded drilled shaft groups was for full height abutments. However, the search for groups of drilled shafts was not limited to this application.

3.1 Project Identification and Plan Review

Prior to beginning the search for information about projects in which groups of drilled shafts were used to resist lateral loads, it was necessary to identify these projects. Project identification was done in two ways: meeting with local consultants, as was previously discussed, and looking at ADOT files. During interviews with local consultants they were asked to identify projects that they had worked on that met the requirements of the survey. The other source of project identification was the ADOT files. With the help of ADOT employees of the Bridge Group section a list was complied using a Microsoft Access database.

The next step was to review the plans of the projects identified on the list. It was the goal of the researchers to look at the plans to obtain information with regards to shaft and group geometry, and soil properties. The plans at ADOT's Engineering Records were used to gather this information. The results of the plan review are presented in complete detail in Appendix G. Subsets of the data are presented and discussed below.

3.1.1 Drilled Shaft Length

There were a total of 108 abutments in which groups of drilled shafts were used and for which the length of the shaft could be identified. The range of lengths for the drilled shafts was 10 to 74 ft. Rather than listing individual lengths to determine what is the predominant length that was used, the lengths were grouped into 10 foot ranges starting at 10 to 20 ft and ending with above 70 ft. The data was plotted against percent of projects identified (Figure 3.1). From Figure 3.1 it is clear that the predominant range of shaft length is between 21 and 50 ft with the largest number of projects being between 31 to 40 ft. In this range there were 30 projects identified or approximately 28 percent of identified projects.

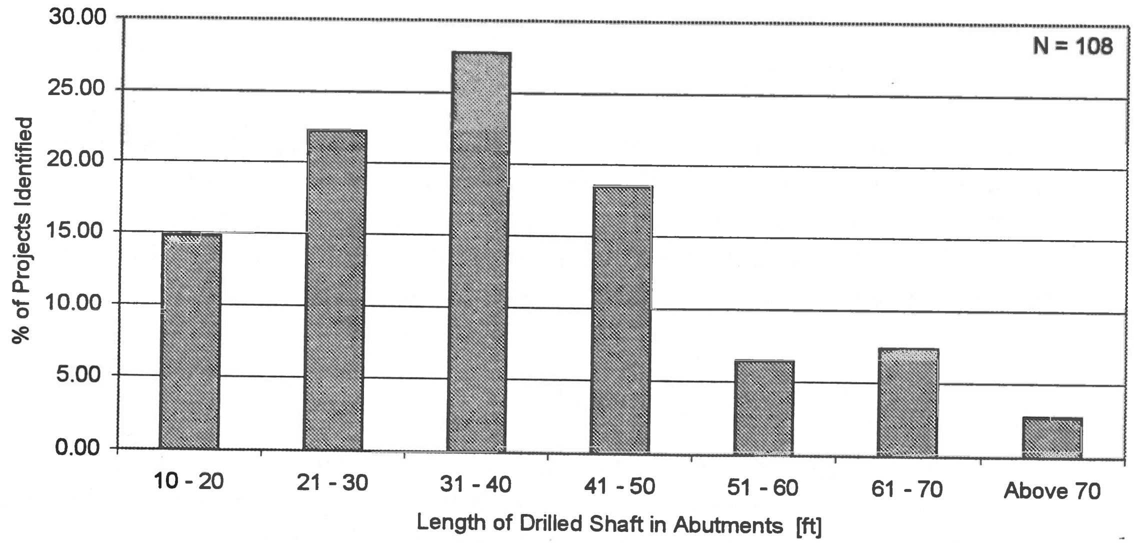


Figure 3.1: Drilled Shaft Length

3.1.2 Drilled Shaft Diameter

There were a total of 118 abutments in which groups of drilled shafts were used and for which the diameter of the shaft could be identified. The range of diameters for the drilled shafts was 20 to 48 in. The data obtained during the project identification and plan review phase was plotted against the percent of projects identified in Figure 3.2. From Figure 3.2 it is clear that the predominant shaft diameters used in Arizona are 24, 30 and 36 in with the largest percentage of projects using 36 in diameter drilled shafts. There were a total of 35 projects, or approximately 30 percent of the projects identified, that used 36 in diameter drilled shafts.

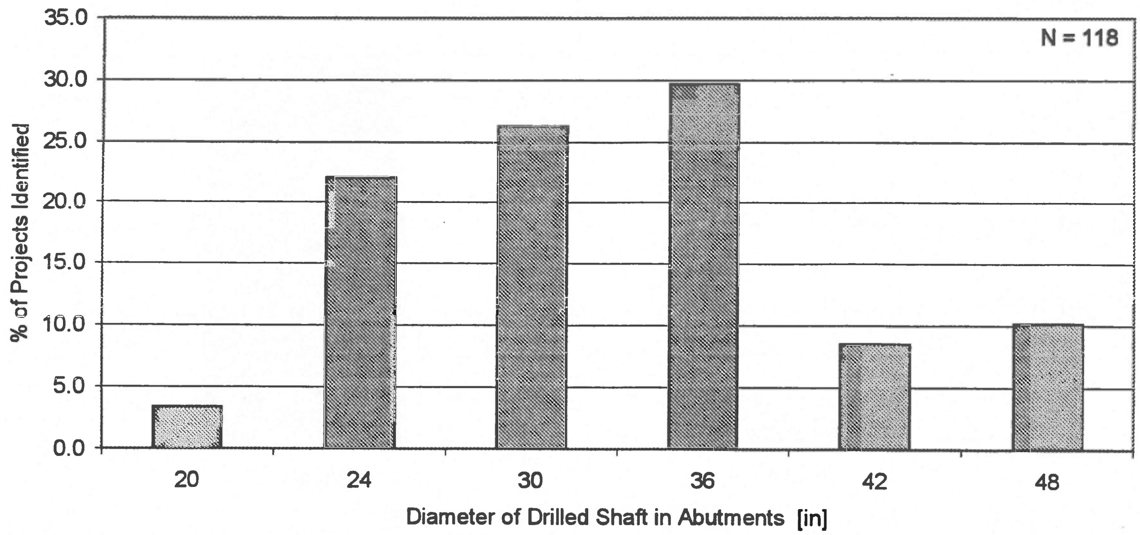


Figure 3.2: Drilled Shaft Diameter

3.1.3 Drilled Shaft Group Geometry

The pile group geometry is an important factor when determining the efficiency of a pile group. All of the drilled shaft groups found in Arizona during the plan review were composed of two rows of shafts. In Arizona there were two prominent types of pile group geometry: in-line and staggered. A graphical representation of what is meant by staggered and in-line is presented below in Figure 3.3.

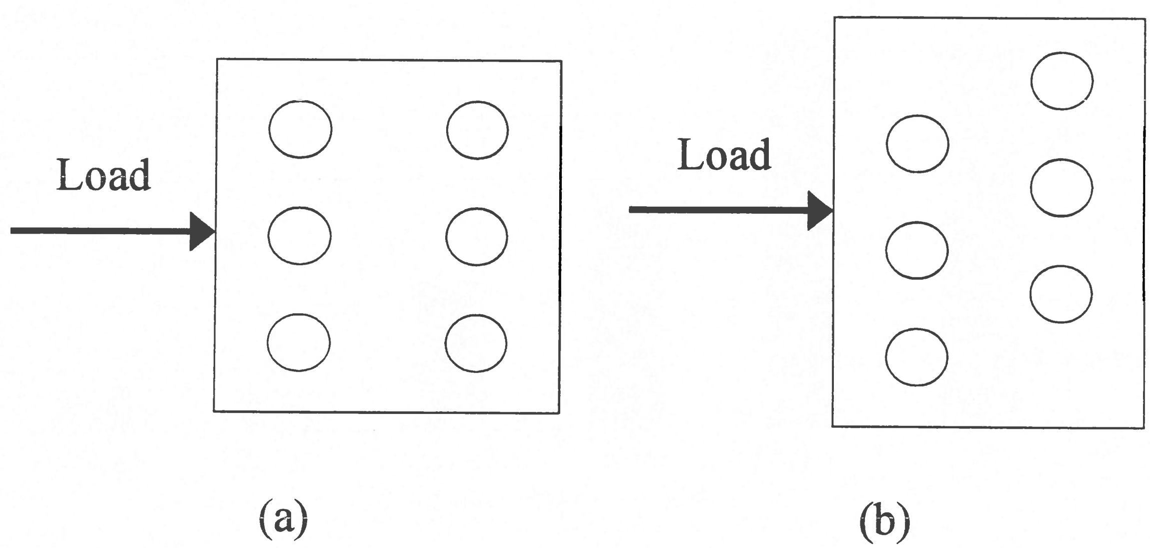


Figure 3.3: Pile Layout; (a) In-Line, (b) Staggered

The percent of projects that had each type of geometry is presented in Figure 3.4.

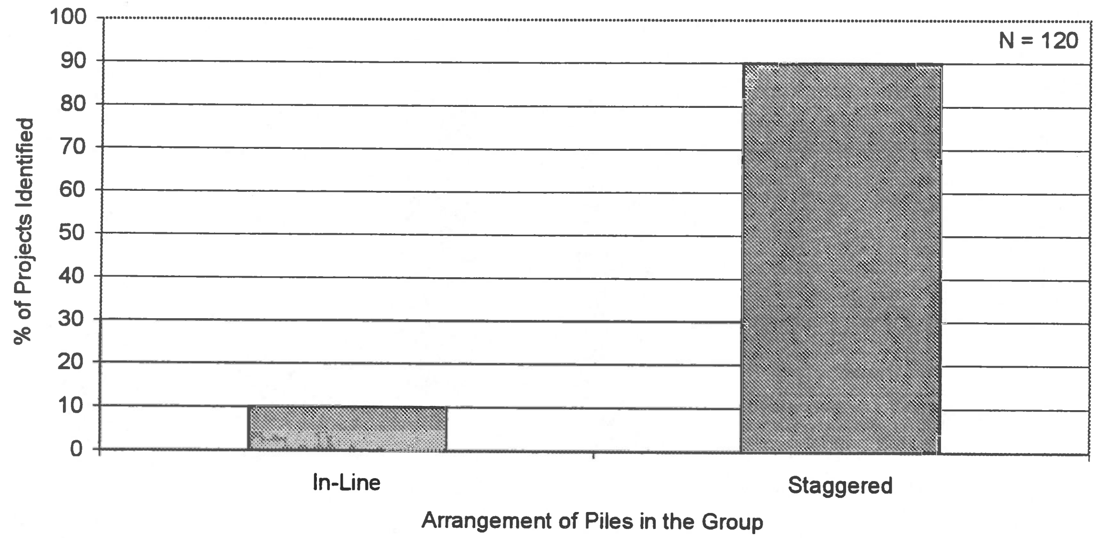


Figure 3.4: Staggered versus In-Line Geometry

After identifying whether the pile is arranged in a staggered or in-line pattern it is important to determine the number of piles in each group. For some procedures this is deemed important and it is taken into account (i.e. the elastic and hybrid analysis). For other procedures there is no means of accounting for the number of piles in the group. For example the p-multiplier method commonly uses multipliers based upon groups with only three columns of piles (i.e. 3 piles in a row). The number of shafts in drilled shaft groups in Arizona was examined and is presented in Figure 3.5.

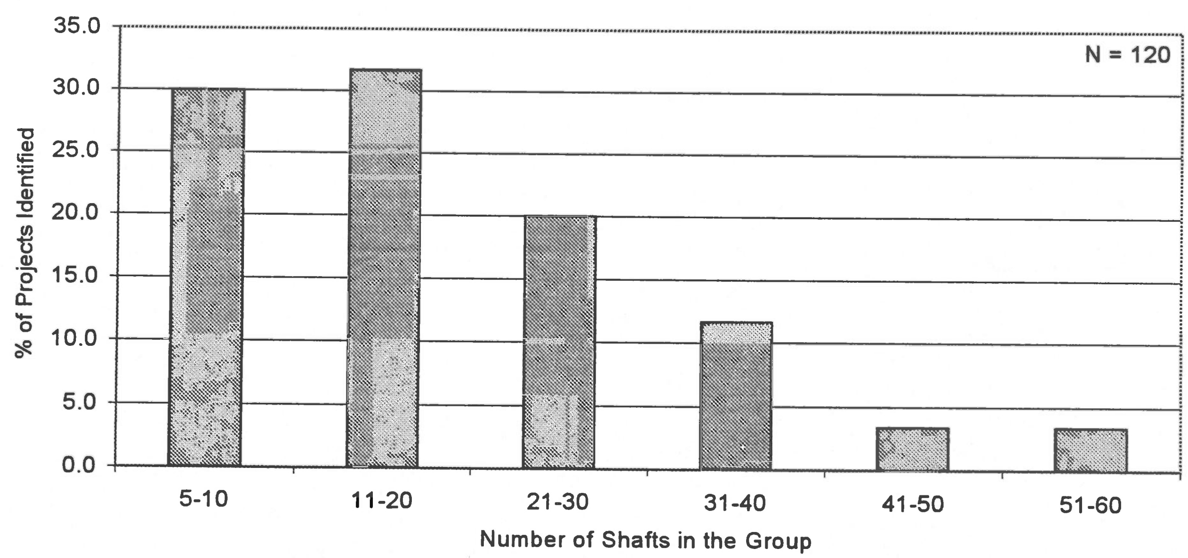


Figure 3.5: Number of Shafts in Group

3.1.4 Drilled Shaft Group Spacing

The drilled shaft group spacing is the most important factor affecting pile-soil-pile interaction. In general the closer the spacing of the shafts the greater is the amount of interaction between the piles. The drilled shaft spacing is normally the only criterion taken into consideration when developing reduction factors to account for group effects. All the spacings reported have been normalized by the diameter (D). Further, as already stated, a large number of projects identified used a staggered pattern of drilled shaft instead of in-line groups. There are two ways to present the results of group spacing: row to row spacing and diagonal spacing. Both types of spacing are presented. The diagonal spacing is calculated as the shortest distance between two piles in different rows.

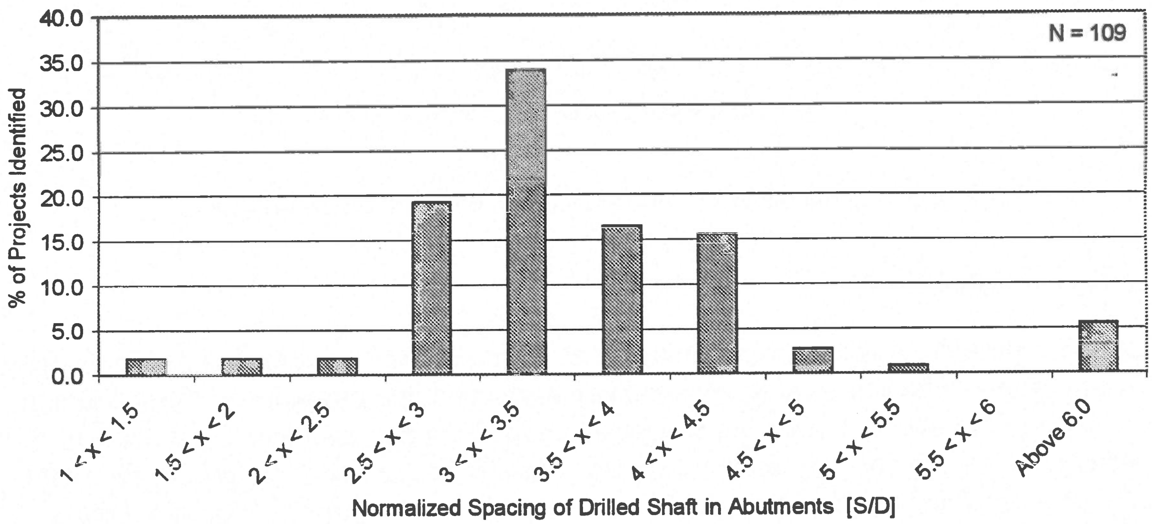


Figure 3.6: Normalized Diagonal Spacing of Drilled Shaft Groups in Arizona

Figure 3.6 shows a breakdown of the normalized spacing for the projects identified during the project identification and plan review stage. From this figure it is clear that the predominant range of normalized spacing was from 2.5 to 4.5 diameters. The largest group within this range is clearly 3 to 3.5 diameters. There were a total of 37 projects equaling 34 percent of all projects identified that used this spacing.

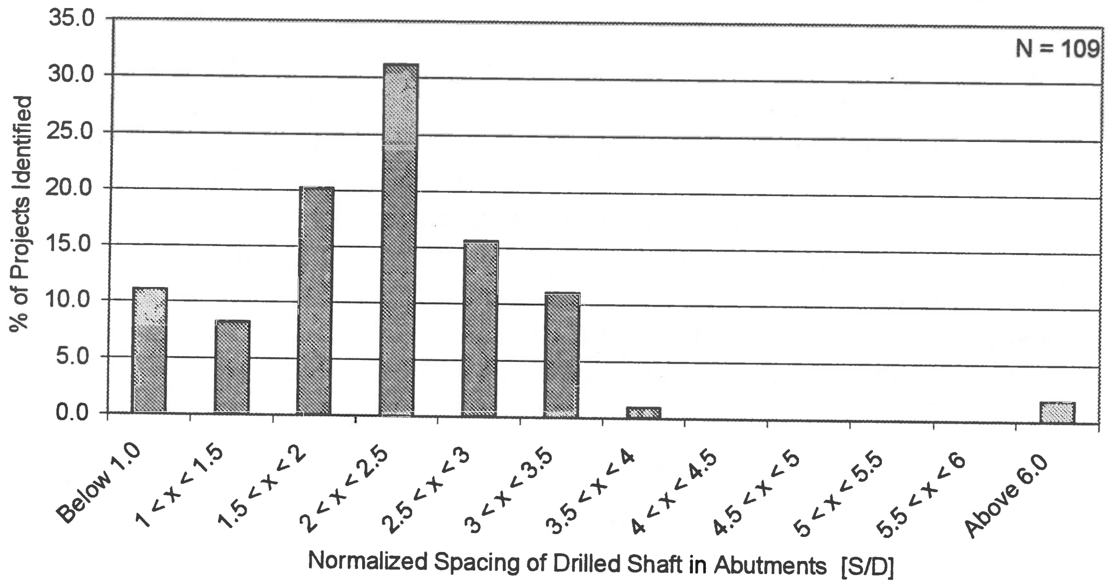


Figure 3.7: Normalized Row-Row Spacing of Drilled Shaft Groups in Arizona

3.1.5 Soil Conditions

Figure 3.8 provides a general representation of the percentage of sites that had the various soil conditions. The soil conditions at the site were classified primarily by the soil along the length of the pile with the most weight given to the upper portions of soil because this is where the majority of the lateral resistance comes from. Of the 105 projects for which soil data was available, 44 of them had cementation in the soil profile. For all projects identified there was no water table encountered in the boring logs.

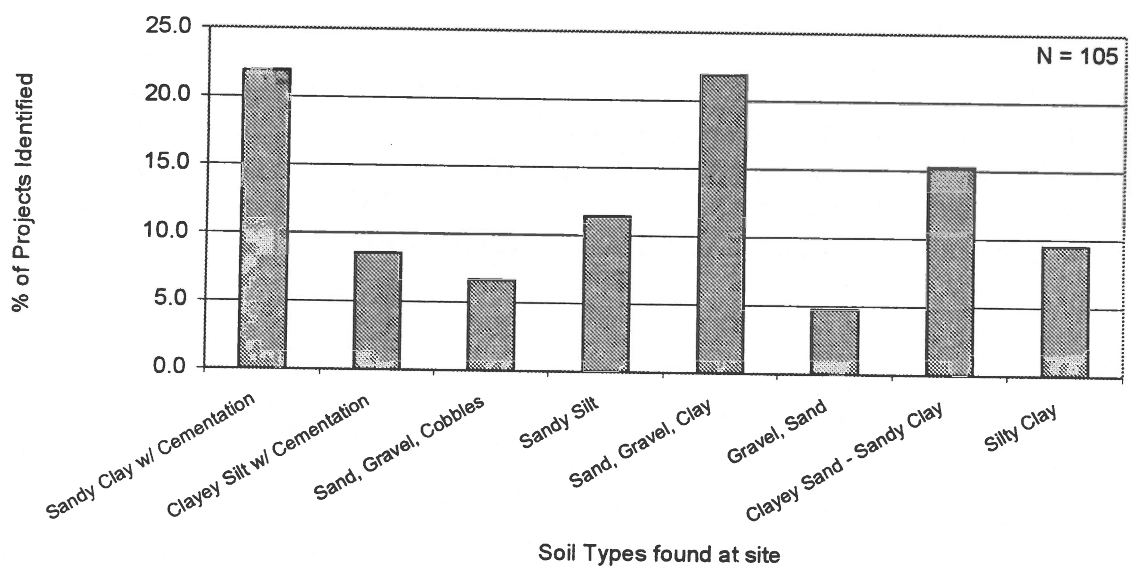


Figure 3.8: Percentage of Projects Identified versus various soil conditions

3.2 Performance of Structures

The remaining step in summarizing the historic use of drilled shafts in Arizona was to observe the performance of the structures using drilled shafts to resist lateral load. Obviously this portion of the information gathering could not begin until some projects were identified. To date, 106 abutments have been identified for inspection and 51 have been inspected by Western Technologies Inc. (WTI), What follows is a progress report on this effort.

3.2.1 Progress Report on Abutment Structure Inspections by WTI

3.2.1.1 Objectives

The objectives of these inspections were and are to look for signs of distress in the abutment structures. Excessive deformations of the abutment walls, excessive rotation of the walls, and pronounced opening of construction joints were all features that were used as signs of distress. If signs of distress were to be observed, there could be multiple causes, including under-design in particular. If, however, no significant signs of distress were observed, then it could be inferred that the design methodology and assumptions, whatever they were, did not result in under-design.

3.2.1.2 Observations

Detailed descriptions of the abutment observations are located in Appendix I, along with photographs of the abutments. The performance of the inspected full height abutments was, in general terms, adequate. No significant sign of distress was observed in any of the abutments. Most of them showed horizontal and vertical cracks, ranging in width from hairline to a few millimeters, due mostly to shrinkage of the concrete. A few of the abutments exhibited a wider opening of the construction/control joints at the top than at the bottom. It was also observed in two abutments that a vertical displacement of an adjacent retaining wall, relative to the abutment, existed. This relative vertical displacement is believed to be due to differential settlement of the walls, given that the adjacent retaining walls are most likely not supported by drilled shafts. Only one abutment showed a slightly visible rotation/inclination, with the opening in the construction joint between the wing wall of the abutment and the adjacent wall being about 60 [mm] at the top and about 40 [mm] at the bottom. The responses observed may be related to the design methods, design parameters, or construction sequences adopted for each structure. However, without extraordinary effort these dependencies cannot be explored, as this information is not readily retrievable from ADOT records.

A majority of the abutments were protected by an embankment, which provided additional lateral resistance to the abutments. Some of the embankments exhibited an " exposed aggregate" surface, which provided protection against erosion. An indication of inadequate performance would have been a bulging or deformation of the embankment surface. No significant deformation of the embankment surfaces was observed during the WTI inspections.

With respect to the overall objectives of the research project, it appears that these abutment structures have been performing satisfactorily. The field observations by WTI indicate that, although these structures might have been designed conservatively, they were very likely not designed unconservatively.

To the extent that these structures were designed with a group reduction factor of one, these field observations tend to support the contention that a group reduction factor of one is not unconservative in the Arizona environment. Although the design value is known to be one for some of these abutments, the value actually used is not known for all structures. The design value is very likely to have been one in any particular case, however, because the "state of the local practice" survey indicates that a value of one is essentially the only value used prior to the adoption of the AASHTO Guidelines, which prompted this research project.

WTI will continue its inspections and a summary of their findings will be included in the Final Project Report for this research project. However, in view of the fact that essentially none of the first 51 abutments inspected showed signs of significant distress, it seems unlikely that a large number of the remaining abutments will show signs of significant distress.

4.0 Data Gaps

After obtaining information on groups of drilled shafts in Arizona it was necessary to compare them to published results in the literature in order to evaluate any overlap or data gaps in the literature. Eight test programs on drilled shaft groups were identified in the literature review (Zhang 1999, Zafir and Vanderpool 1998, Matsui 1993, Kimura et. al, 1993, Ismael 1990, Schmidt 1985, Schmidt 1981, and Agarwal and Prakash 1967). In these 8 test programs, a total of 12 groups were tested. Information was gathered from these studies similar to that presented in section 3.1. In this chapter, the information from the summary of historic use by ADOT and the literature review is presented for the various categories for comparison.

4.1 Drilled Shaft Length

There were a total of 108 projects in which groups of drilled shafts were used to resist lateral loads in AZ for which the length of the shaft could be identified. The range of lengths for the drilled shafts was 10 to 74 ft. The 12 full-scale tests on drilled shafts in the literature included a range of lengths of 8 to 110 ft, with most of the tests being on shafts over 70 ft long. The Arizona data and the literature data are shown together in Figure 4.1. From Figure 4.1 it is clear that the most common lengths in Arizona usage are not represented in the literature. The most common length range in the literature, above 70 ft, is not representative of Arizona conditions.

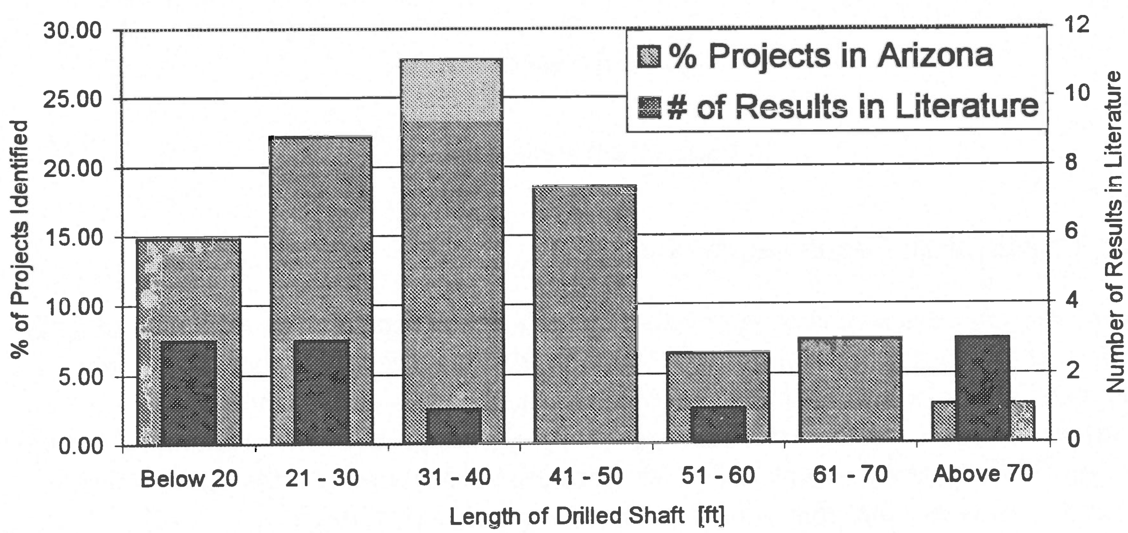


Figure 4.1: Drilled Shaft Length

4.2 Drilled Shaft Diameter

There were a total of 118 Arizona abutments in which groups of drilled shafts were used and for which the diameter of the shaft could be identified, with diameters ranging from 20 to 48 in in diameter. The predominant shaft diameters used in Arizona are 24, 30 and 36 in with the largest percentage of projects using 36 in diameter drilled shafts. The distribution of diameters used in Arizona is presented along with those encountered in the literature on Figure 4.2. The drilled shafts in the literature ranged from 8 to 59 in in diameter with the largest group, 5 tests, being between 43 to 48 in diameter. Clearly the most common drilled shaft diameters that have been used in Arizona applications are not well represented in the literature. However, ADOT's policy in the future will be to use drilled shafts with diameters of 42 in and 54 in. This needs to be considered in the remainder of the study.

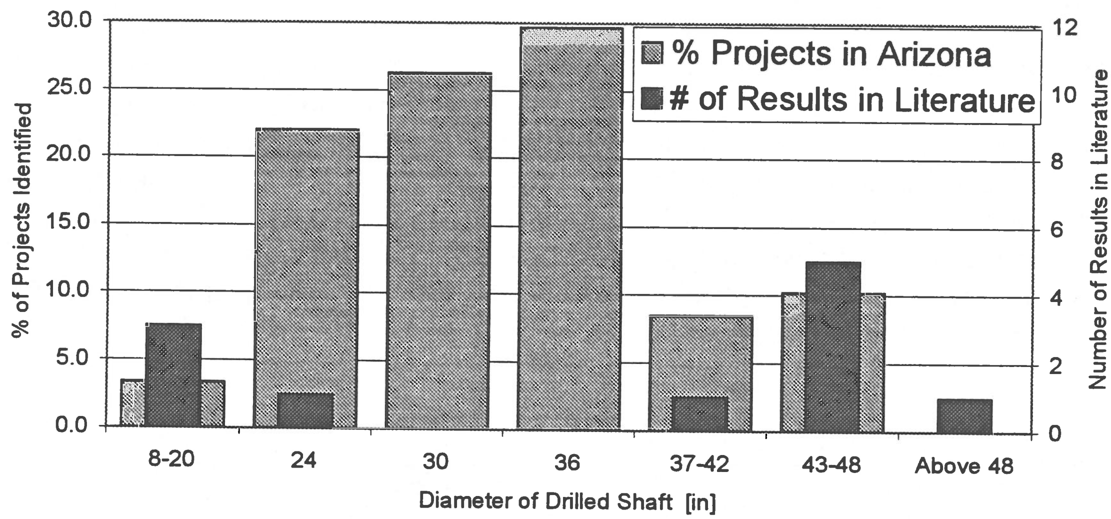


Figure 4.2: Drilled Shaft Diameter

4.3 Drilled Shaft Group Geometry

The pile group geometry is an important factor when determining the efficiency of a pile group. All of the drilled shaft groups found in Arizona during the plan review were composed of two rows of shafts. In Arizona usage both in-line and staggered arrangements were encountered, although the staggered pattern was much more common. All the full-scale tests encountered in the literature used an in-line arrangement. From Figure 4.3 it is clear that this is not representative of Arizona usage.

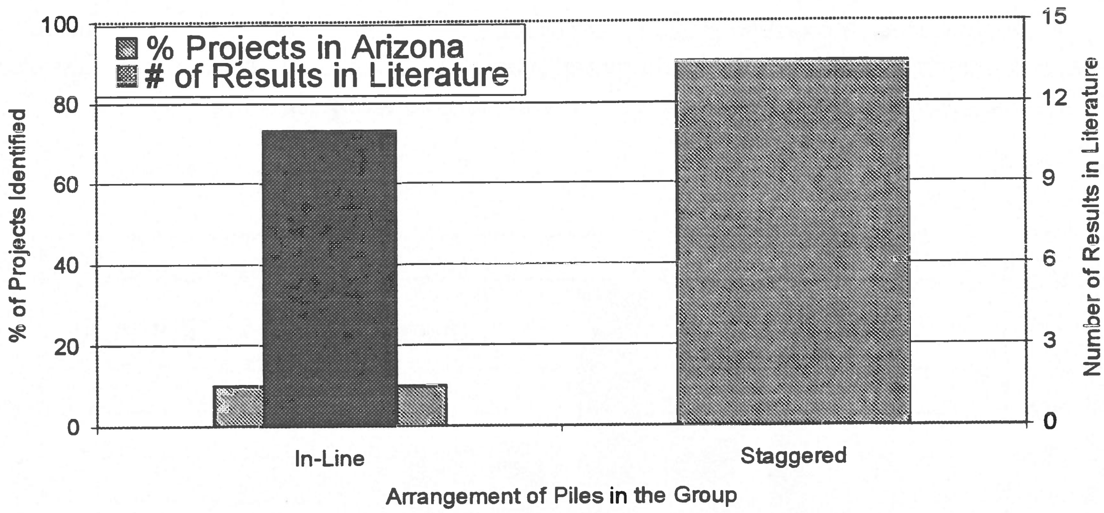


Figure 4.3: Staggered versus In-Line Geometry

The number of shafts in drilled shaft groups in Arizona and in the tests found in the literature is presented in Figure 4.4. Clearly, tests conducted and reported in the literature typically use much smaller groups than are common in Arizona practice. This result can no doubt be explained by the difficulties in generating test loads sufficient to deflect a group composed of a large number of shafts.

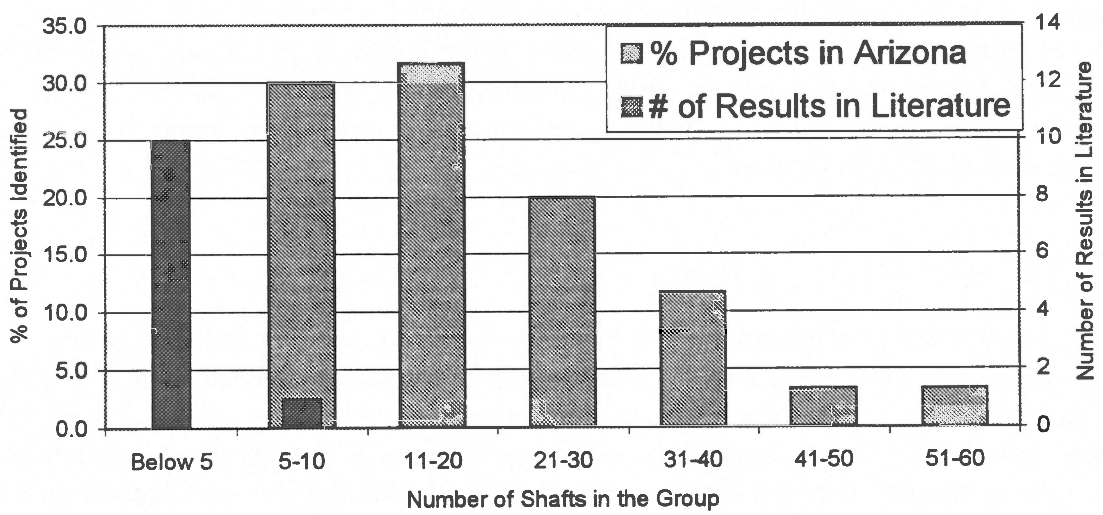


Figure 4.4: Number of Shafts in Group

4.4 Drilled Shaft Group Spacing

As previously stated, the drilled shaft group spacing is the most important factor affecting pile-soil-pile interaction. All the spacings reported have been normalized by the diameter (D), with spacings for staggered arrangements calculated along the diagonal.

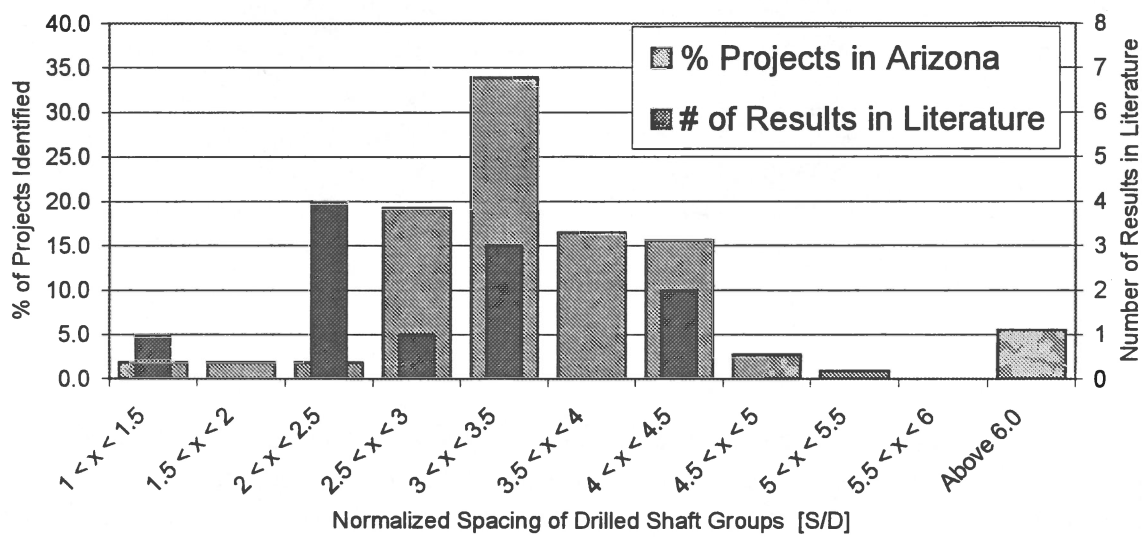


Figure 4.5: Normalized Diagonal Spacing of Drilled Shaft Groups in Arizona

Figure 4.5 shows a breakdown of the normalized spacing for the projects identified in Arizona along with the tests from the literature. From Figure 4.5 it is clear that the tests in the literature cover the conditions in Arizona reasonably well, although the highest concentration of results found in the literature arise from more closely spaced groups than are common in Arizona practice.

4.5 Soil Conditions

The soil conditions in which groups are founded in Arizona and in tests found in the literature are presented in Figure 4.6. The soil conditions were classified primarily by the soil along the length of the pile, although for simplicity when layering was present more weight was given to the upper portions of the pile form which the majority of the lateral resistance is derived. Figure 4.6 shows that while the data in the literature represents some of the conditions in Arizona there are still a number of soil conditions that are not represented. The soil conditions that are not dealt with include any soil including gravel as a major component, clayey silt with cementation, and a clayey sand – sandy clay. Combined, these conditions represent over 50% of the existing drilled shaft groups in Arizona.

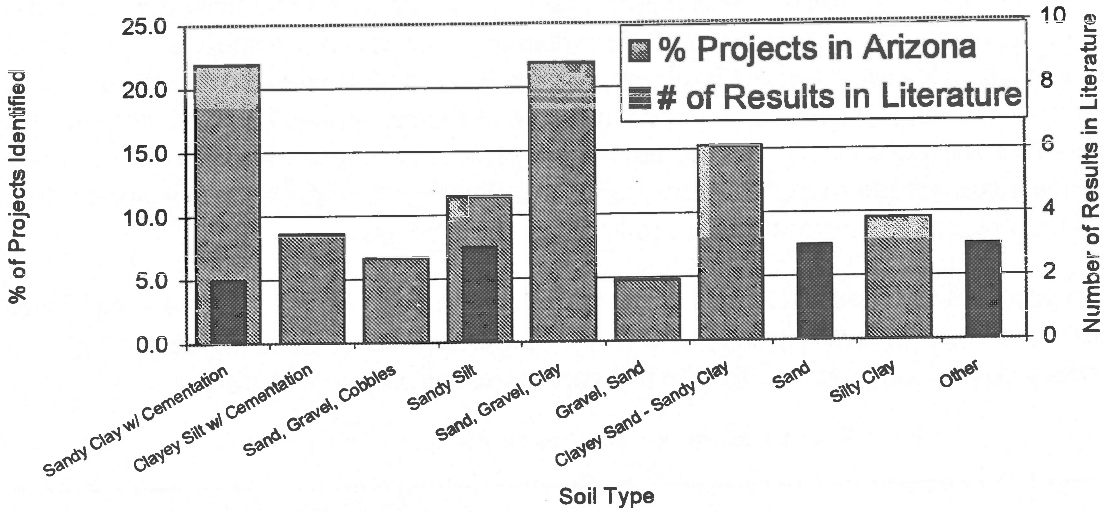


Figure 4.6: Soil conditions at testing site locations

Groundwater conditions have also been evaluated for comparison. As previously mentioned all of the projects encountered in Arizona were entirely located above the groundwater table. A number of group load tests identified in the literature were conducted on drilled shafts located above the groundwater (Figure 4.7).

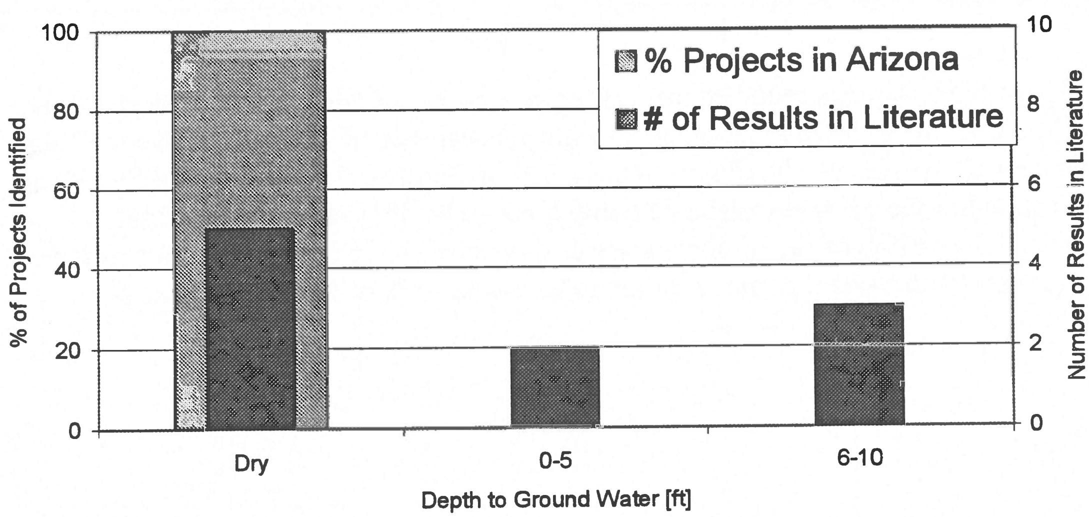


Figure 4.7: Depth to Water Level

4.6 Boundary Conditions

In the discussion of group reduction values observed in the literature, it was pointed out that the pile cap boundary conditions are very important to group behavior (see Section

2.2.3.6 and Figure 2.17). Due to its importance, the type of boundary conditions present for the pile caps in Arizona transportation usage and in the load tests from the literature have been presented. In Arizona, the overwhelming majority of the abutments reviewed used a buried pile cap. Of the 120 abutments for which plans were reviewed 101 were found to have buried pile caps, while the position of the remaining 19 relative to the final ground line was unclear. Of the 101 buried pile caps, it is believed that four of them are potentially susceptible to exposure by scour, meaning that it might be reasonable to count on passive resistance in front of the cap in 97 of the 101 cases.

The group load tests identified in the literature, by contrast, were conducted with pile caps located above the ground surface, with only a few exceptions. Table 4.1 details the boundary conditions observed for the pile caps in tests from the literature.

Table 4.1: Boundary Conditions for Pile Caps in Literature

	Pile Cap	
Source	In contact with soil (friction)	Below soil surface (passive resistance)
Agarwal and Prakash (1967)		
Ismael (1990)		
Kimura et al (1993)		
Schmidt (1981)		
Schmidt (1985)		
Zhang (1999)	X	
Zafir and Vanderpool (1988)	X	X

Very few of the load tests reported in the literature were conducted with friction on the base of the pile cap or passive resistance on the vertical face of the cap. However, these components are extremely significant as indicated by Figure 2.17, Rollins and Sparks (1999) estimated the passive resistance component to be 40%, and the frictional component to be 10%, of the total capacity of their group of piles. In the tests reported in Zhang (1999) the frictional component provided some 15% of the total capacity.

5.0 Summary and Recommendations for Finishing Study

5.1 Summary

The objective of this project was to examine the problem of laterally loaded groups of drilled shafts. This was done through a literature review and interviews with practitioners in the consulting and DOT communities and prominent researchers, along with a review of historic use of drilled shaft groups in Arizona. The purpose of the literature review was to identify design procedures for laterally loaded single piles and pile groups, to identify load tests, model tests, and analytical methods relative to pile groups, and to describe the information uncovered. A preliminary literature review was completed by the authors in 1997 (Walsh, et al, 1998). The findings from this literature study included the following: (a) Group reduction factors at a pile spacing of 3-D from small scale model tests performed in the lab tended to be low, usually in the 0.45 to 0.65 range. (b) Likewise, group reduction factors derived from solely analytical methods which emphasize the effects of overlapping stress fields but do not rest heavily on treatment of the pile group as a structural system were also low, in the same range as the lab model tests. (c) Full-scale pile load tests on single piles and pile groups in the field generally yielded higher efficiency factors, typically in the range of 0.65 to 0.9. A limited number of 3-D finite element analyses were uncovered, and the group reduction factors from these studies were also near 0.7 to 0.75. (d) When full-scale tests on pile groups involved a pile cap firmly in contact with the underlying soil, group reduction factors well above 1, up to 1.4 in fact, were obtained. All of the preceding group reduction factors cited were evaluated at moderately low deflections, about 3% of the pile diameter or slightly less. As a part of Task 2 completed for the current project, these original citations were revisited and were augmented with numerous additional citations. These additional citations enhanced the database, of course, but did not change the overall conclusions cited above. It seems reasonable to contend that full-scale field tests on pile groups whose boundary conditions match, as closely as possible, those typically imposed in Arizona would yield group reduction factors most appropriate for drilled shafts in Arizona. Therefore, it is probably fair to say that the database from the literature survey strongly suggests that application of the AASHTO factors to load resistance (rather than to modulus of subgrade reaction) produces a result which is too low for use in typical Arizona abutment structures.

The most commonly used design methods for single piles were discussed, including p-y analysis, single pile load tests, and elastic analysis. Of these the most common method used in practice is a p-y analysis, usually performed using a computer program such as COM624 or LPILE, with p-y curves developed from site-specific soil data. After presenting the most common single pile methods, we went on to examine the pile group problem. A number of methods were found during the literature search and presented in this report, including: (1) the elastic method, (2) the Focht and Kocht hybrid method, (3) the group reduction factor method (use of an average efficiency for the whole group), (4) the coefficient of lateral subgrade reaction reduction method, (5) the p-multiplier method,

(6) the group amplification procedure, (7) the strain wedge method, (8) load and resistance factor design, and (9) the finite element method. The procedures most commonly used in practice, (3) (4) and (5), were compared for a variety of field conditions. Many states implement the AASHTO specifications as load reduction factors. However, the sources cited in AASHTO recommend reducing the modulus of subgrade reaction.

An in-depth presentation of the finite element method (FEM) was presented, providing background information on developing the elements or mesh used in the method. This was followed by a discussion of material behavior and the various models that can be used to define the material behavior. For this particular problem, an FEM solution can only be obtained using a computer program capable of handling 3-dimensional geometries and having appropriate element and material models. Other factors that were considered were the operating system, pre- and post-processors, and the solution methods. A total of 96 finite element programs were investigated. Of the 96 programs investigated, 8 were determined to be able to meet the requirements of the task and were presented in more detail. After careful consideration, the top three programs were ranked as follows: (1) ABAQUS, (2) MSC/NASTRAN, and (3) ADINA. The capabilities of these three programs do not appear to be significantly different, but ABAQUS appears to have better response time to inquiries. We propose to begin modeling with this program, and to move on to the other choices in the order presented only if ABAQUS proves unsatisfactory.

Interviews with practitioners in the local consulting community provided us information about design procedures used in Arizona. These interviews showed that the local procedure in use is consistent with that recommended by ADOT, namely the application of the AASHTO reduction factors to the single pile capacity. Additional interviews were conducted with engineers at state DOTs around the country, in order to develop a national perspective on this problem. Through these interviews, we discovered that there are a number of design procedures used to account for group effects for laterally loaded drilled shafts. The most common procedures included applying the AASHTO reduction factors to the single pile capacity, applying the AASHTO reduction factors to the modulus of subgrade reaction, and use of the p-multiplier method. Clearly the AASHTO recommendations are being applied in two ways.

Several other researchers in the field of deep foundations were contacted to determine their thoughts on the AASHTO recommendations and which design method they recommend for the design of laterally loaded drilled shaft groups. Those researchers familiar with the AASHTO procedure stated that the intended interpretation was to apply the reduction factors to the modulus of subgrade reaction. With regards to which design procedure would be recommended for design of a group of laterally loaded drilled shafts, the majority of the researchers recommended the p-multiplier method.

Finally, a summary of historic use of laterally loaded drilled shafts in Arizona was performed. This was divided into two phases: project identification and plan review, and performance of structures. During the project identification and plan review phase, projects in Arizona, particularly full height abutments, which used groups of drilled shafts were identified and the soil and pile characteristics were tabulated. The information obtained from the plans included soil type and pile length, pile diameter, pile spacing, and group geometry. The performance of structures was determined by site visits conducted by Western Technologies, Inc. Through their site visits it was determined that there were no significant structural problems at the existing abutments. This finding is particularly important because it is believed that nearly all of the structures were designed with a group reduction factor of one.

The characteristics of the shafts and soil present for projects in Arizona abutments were compared with the data collected during the literature review. There were 12 full-scale tests in the literature in which drilled shaft groups were tested; these were used for comparison. From the comparison there were some clear omissions or data gaps between Arizona conditions and the literature. These data gaps are listed below:

1 Diameter - In Arizona over 50% of abutments using drilled shafts to resist lateral loads had a diameter of either 36 in or 30 in. Neither of these diameters are represented in the literature. However, some data are available for larger and smaller drilled shafts. However, ADOT's policy in the future will be to use drilled shafts with diameters of 42 in and 54 in.

2 Length – There are two ranges of length, 41 to 50 ft and 61 to70 ft, which together represent approximately 25% of abutments in Arizona that are not represented in the literature. However, lengths are represented on either side of these ranges, which could be used to estimate the effect of length on behavior. Furthermore, it is likely that only the uppermost portions of the pile are important to the lateral resistance.

3 Group Geometry – Approximately 90% of abutments in Arizona had a staggered pile group geometry. There is no representation of this type of group geometry in the tests from the literature.

4 Number of Shafts in Group – All but one of the literature tests had less than 5 shafts in the group. There were no groups in Arizona that had less than 5 shafts in them.

5 Group Spacing – The tests from the literature represented the conditions used in Arizona reasonably well.

6 Soil Conditions – Abutments are located in a number of soil conditions in Arizona. Those not represented in the literature were predominantly fine grained materials exhibiting cementation, and materials with significant amounts of very coarse particles.

7 Depth to Groundwater – Several of the tests presented in the literature were performed on foundations located entirely above groundwater, which was the

only condition encountered during the study of Arizona abutments. Therefore, Arizona conditions were matched well in the literature.

8 Boundary Conditions – The overwhelming majority of the pile caps used in Arizona are located below the ground surface, where friction on the base of the cap and passive resistance on the front of the cap are likely to be significant. This condition has not been matched often in the literature. This fact tends to make one expect the overall efficiency of pile groups in Arizona to be somewhat higher than the results from most full-scale tests reported in the literature.

5.2 Recommendations for Further Study

This interim report was intended to provide a review of relevant information in order to guide decisions about proceeding with Task 6, Analytical Results, and Task 7, Design Field Load Tests. Based on the information presented herein, we proffer the following recommendations for each task.

<u>Task 6, Analytical Results</u>

The objective of this task is to use analytical approaches to develop a set of recommendations for use in pile group design based on fundamental principles. From the review of analytical methods, it is clear that most methods in common use are largely empirical, and are based on results which may not be highly relevant to Arizona conditions. We believe the only analytical method which can overcome this problem is the finite element method (FEM). We propose to implement an FEM model with ABAQUS (with MSC/NASTRAN or ADINA as potential backups).

As a first step, we propose to develop a model of the field tests conducted by Zafir and Vanderpool (1999) and Ismael (1990). These tests were located in Las Vegas, Nevada, and Kuwait, respectively, in materials that bear at least some similarities to Arizona soils. We will use our ability to match the observed results of these tests as a measure of confidence in the method. This process will be augmented with the results of the instrumentation installed at the Warner Road structure on the Price Freeway and testing of the adjacent single piles (referred to as Task 6A in the Statement of Qualifications). Based on our current understanding of the construction schedule for this structure, we believe that this testing should be possible within the next six months.

Once the model has been calibrated to our satisfaction, we will use it to model more generic conditions which are representative of Arizona transportation applications. We propose to model a 20 pile group, arranged in a staggered pattern, with a pile cap located below the ground surface. The most interesting soil conditions for the modeling appear to be predominantly fine-grained soils with significant cementation (ϕ-c soils with rather brittle σ-ε curves) and coarse granular materials (characterized by fairly high ϕ-values and dilatancy). The second case is likely to be the more challenging. With the aid of available geotechnical reports for the existing abutments and the experience of the Steering

Committee, we will choose a range of properties for each soil, to allow parametric evaluation.

An FEM model will be developed for the two soil conditions, and used to develop load-deflection curves for the pile head (cap) and single-pile load deflection curves. These two curves will allow the calculation of group reduction factors and p-multipliers for each case modeled. Because a range of properties will be used for each of the two soil conditions, the dependence of the E and p_m values on properties can be determined parametrically. Finally, we will consider the impact of a pile cap located above ground in a similar fashion.

From this analysis, we will be able to propose a set of values for E and p_m appropriate for Arizona conditions. This interim report provides the means to develop an accompanying description of the use of these values, including the relevant assumptions and the computer programs used. This material will form the majority of the content of the Technical Presentations (Task 10).

<u>Task 7. Design Field Load Test</u>

The procedure outlined for the completion of Task 6 will allow the development of recommendations for design values of group reduction factors and p-multipliers in which we can place considerably more confidence than the group reduction factors described in the Interim ADOT policy. However, given that these results will arise from an analytical model, even though that model will be carefully calibrated, we believe that validation of the results through field testing will be important to the development of full confidence on the part of the consulting community, the FHWA, and ADOT. Furthermore, the practice would be well served by these studies, which are focused on the data gaps identified in Section 4, a conclusion which was shared by Professor O'Neill in our interviews with him. To this end, we believe that completion of load testing for the two soil conditions outlined under Task 6 is appropriate, and recommend that we proceed with design of this effort in Task 7.

The complete design should include the following components:

1. The pile group geometry: Based on Chapter 4, we believe that the most useful results would be obtained from a pile group of about 8 to 10 piles, with each pile having a diameter of 3.5 to 4.5 feet and a length of about 40 feet. The piles should be arranged in two rows with a staggered pattern, at a row-to-row spacing of 3D.

2. The pile cap boundary conditions: The most applicable condition would be a pile cap for which friction and passive resistance are present. However, clearly these components will not always be present, and so some testing or instrumentation to allow the removal of these terms should be considered. For economy, it may be best to repeat a test on the same pile group after removal of the soil ahead of and then underneath the cap. In addition, the horizontal load to the pile cap will be applied well above the level of the cap, to simulate the abutment loading condition wherein the abutment wall applies both horizontal load and moment to the cap.

Furthermore, a serious attempt will be made to apply vertical load to the cap, concurrently with the horizontal load, as does the abutment wall.

3. The loading and reaction system: Fairly high loads will be required in order to produce displacements of a group on the scale proposed. We will devote some rather significant effort to the design of hydraulic jacking devices to apply these loads. Multiple hydraulic loading rams are anticipated. A related issue is the provision of a reaction system for the applied load. We believe that the best design may be to install two pile groups a moderate distance apart, and load both of them in opposite directions. In essence, this creates two load tests, and it may be possible to have slightly different geometries in each pile group. Alternatively, a retaining wall or deadman may be considered.

4. The instrumentation and data acquisition system: The piles and the pile caps should be well instrumented to allow researchers to "peer inside" and calculate load transfer within the pile group, and to evaluate the load transferred through friction and passive resistance on the pile cap. At least, we anticipate the instrumentation system will include strain gages within the drilled shafts and the pile cap and load cells for measuring the applied loads. Specific selections and locations of instruments will be a part of the design effort. There will be a large number of measurements to obtain at each load increment, so we believe that a computer data acquisition system will be essential. At least the broad outlines of this system will be developed in Task 7. In addition, some manual measurements will be made as a confirmation of the electronic results; we will make recommendations for these measurements. Instrumentation recommendations will be developed based on the experience of the research team, what is learned from the measurements at Warner Road and the Price Freeway, and some additional interviews with others in the general field.

5. The locations: We believe that the most appropriate soil conditions for field testing match those outlined under Task 6, and would seek potential locations within the ADOT property inventory for which those conditions would exist. This will require the assistance of ADOT geotechnical engineers and planners. One option to consider is the potential use of a prototype pile group for future ADOT work. This option will require additional investigation, but may be viable if the test loads are applied in the opposite direction to the anticipated load on the pile group. The advantage of this approach is that the pile group construction cost may be largely absorbed (neglecting instrumentation and any additional structural allowances for the test use) in the construction budget. The chief disadvantage is reluctance to use the pile group in this way, perhaps due to concerns about softening of the soil adjacent to the pile.

6. Potential partners: The soil conditions and the general problem appear to be regional in nature, in that similar issues arise in surrounding states. A number of the DOT personnel we contacted for the interviews expressed uncertainty about their current practice and an interest in the results of this study. As a result, we

believe it may be possible to identify partners among the DOTs for nearby states. Other additional partners may also be possible; the Association of Drilled Shaft Contractors (ADSC) has expressed tentative willingness to provide drilled shafts for these studies, at least partially as an in-kind contribution. We will explore the extent to which surrounding DOTs may wish to participate, and the possibility of some federal funding sources, with the help of ADOT Research personnel.

APPENDIX A

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