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CHEMICAL AND PHYSICAL PROPERTIES OF ASPHALT-RUBBER

Phase II - Product Specifications and Test Procedures

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16. Abstract

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A single asphalt-rubber formulation consisting of 75 percent AR-1000 asphalt and 25 percent crumb rubber from automobile tire treads (sized from No.16 to No.25) was used. Independent variables were three levels of reaction temperature [350, 375, and 400 F (177, 191, and 204 C)], and three levels of reaction time (0.5, 1.0 and 2.0 hours).

Dependent variables included ring and ball softening point, absolute viscosity by capillary viscometer at 140 F (60 C), ductility and force-ductility at 39.2 F (4 C), elastic rebound, and low temperature viscosity with the Schweyer rheometer at 39.2 F (4 C).

Force-ductility was the only single test that detected the effect of reaction time and temperature. Variability of results was several times greater than is experienced for conventional asphalt cements and strongly suggests that conventional asphalt tests and criteria should not be used to specify this asphalt-rubber formulation.

Recommendations are made for new tests, modifications of conventional tests and improved laboratory mixing and formulation techniques. An additional experiment is discussed that will use improved techniques to evaluate properties of other formulations, compare properties of field and laboratory produced mixtures, and to compare certain mixture properties with performance of experimental field test sections.

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CHEMICAL AND PHYSICAL PROPERTIES OF ASPHALT - RUBBER MIXTURES - PHASE II PRODUCT SPECIFICATIONS AND TEST PROCEDURES

by

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ABSTRACT

Asphalt-rubber was tested with conventional and non-conventional methods to begin development of testing methods for specification purposes and to find methods for studying the reaction between asphalt and reclaimed crumb rubber.

A single asphalt-rubber formulation consisting of 75 percent AR-1000 asphalt and 25 percent crumb rubber from automobile tire treads (sized from No. 16 to No. 25) was used. Independent variables were three levels of reaction temperature [350, 375, and 400 F (177, 191, and 204 C)], and three levels of reaction time (0.5, 1.0 and 2.0 hours).

Dependent variables included ring and ball softening point, absolute viscosity by capillary viscometer at 140 F (60 C), ductility and force-ductility at 39.2 F (4 C), elastic rebound, and low temperature viscosity with the Schweyer rheometer at 39.2 F (4 C).

Force-ductility was the only single test that detected the effect of reaction time and temperature. Variability of results was several times greater than is experienced for conventional asphalt cements and strongly suggests that conventional asphalt tests and criteria should not be used to specify this asphalt-rubber formulation.

Recommendations are made for new tests, modifications of conventional tests and improved laboratory mixing and formulation techniques. An additional experiment is discussed that will use improved techniques to evaluate properties of other formulations, compare properties of field and laboratory produced mixtures, and to compare certain mixture properties with performance of experimental field test sections.

1.0 PROJECT DESCRIPTION

1.1 This project had the following objectives:

A. To evaluate the problems encountered when asphalt-rubber mixtures are tested with current asphalt test methods and to determine what changes in equipment and/or procedures would be necessary to test these mixtures with present apparatus.

B. To develop new test methods to specifically measure the unique viscoelastic properties of asphalt-rubber mixtures, including the evaluation of new apparatus commercially available which show capabilities of testing asphalt-rubber mixtures.

C. To develop asphalt-rubber performance specifications for highway applications using procedures developed in this study.

D. To develop requirements as needed, to control those manufacturing, transportation and application procedures found to be critical to the final properties of the asphalt-rubber mixture.

1.2 Considerable success was realized for items A and B. Items C and D were not as successful but a good deal of useful information was generated. Each item will be considered individually and will be discussed in detail later in the report.

1.3 A series of laboratory experiments were conducted to find the effect of production variables on properties of asphalt-rubber. Scope of the work limited the production variables to a single asphalt type, a single rubber type and gradation, three levels of reaction temperature, and three levels of reaction time.

1.3.1 Material properties measured as part of the study include:

- Ring and Ball Softening Point
- Absolute Viscosity
- Ductility
- Force-Ductility
- Elastic Rebound
- Rheologic properties (apparent viscosity and shear rate sensitivity) by the Schweyer Rheometer.

1.4 This study measured properties outlined above as part of a full factorial experiment with three levels of independent variables (time and temperature). The reason for using these independent variables was twofold. First, of the four major production variables of asphalt type, rubber type, reaction temperature and reaction time; the latter two are considered to be the most probable to be subjected to uncontrolled variability due to job conditions. Secondly, the scope of the project was not large enough to consider the many possible combinations of asphalt and rubber types that are available. This was of special importance in view of the uncertainty of the validity of conventional asphalt tests and procedures when used for asphalt-rubber mixtures. Furthermore, field experience has shown that both temperature and holding time affect the asphalt-rubber reactions and, subjectively at least, affect properties and performance of the material. These effects include breakdown of rubber with consequent reduction of viscosity and the addition of excessive oil fractions to the base asphalt. This results in reduced adhesion and durability. This study examined the sensitivity of several tests to detect changes in material properties resulting from variations in reaction time and temperature.

2.0 ASPHALT-RUBBER REACTIONS AND MECHANISMS

2.1 The reaction between asphalt and rubber has been studied and reported by Green and Tolonen (13). A generalization of the process, while far from being fully understood, is a combined chemical and physical reaction that produces a two phase composite material.

2.1.1 The original asphalt-rubber as developed by McDonald (5) used approximately 75 percent of relatively low viscosity (AR-1000) asphalt cement and 25 percent reclaimed rubber. Source of the rubber was reclaimed tread from conventional automobile tires that was pulverized and graded to a single size (No. 16 to No. 25 sieve). Rubber and asphalt are reacted in a single batch operation before incorporation into construction work. For this process, the finished material consists of readily visable rubber particles in a bituminous binder.

2.1.2 More recently, asphalt-rubber systems have been developed that use different types and sizes of rubber, different reclaiming methods that utilize the full tire carcass, different base bituminous materials, and certain additives. This study considered the system briefly described in the previous section. Properties of asphalt-rubber systems more recently developed are considered in another study that is presently being conducted by the Arizona Department of Transportation.

2.2 Physically, during the reaction, rubber particles in hot asphalt swell to two to three times their original volume (5). These swollen particles cause an enormous increase in mass viscosity of the system as compared to conventional asphalt or to unswollen rubber and asphalt early in the reaction process.

2.3 While not fully proven, it is hypothesized that rubber and asphalt participate in an exchange of components (13). Automobile rubber contains vulcanized rubber and significant amounts of extender oils that can be characterized by the Rostler parameters (14). The quantity of these oils is, of course, dependent on the particular rubber formulation and the extent of weathering of the rubber before reclaiming or recycling. New, unused tires will contain more oil fractions than tires that have extended exposure to the service environment.

2.3.1 Asphalt also contains components with more or less the same Rostler characteristics as the extender oils of rubber. It is, therefore, hypothesized that under the influence of elevated temperature and/or time, that components can react or be exchanged from rubber to asphalt or vice versa. Laboratory studies have shown that an original chunky appearing asphalt-rubber with high mass viscosity will, with heat and time, degenerate to the point where rubber particles become "dissolved" and a more homogeneous material with a substantially reduced mass viscosity is produced.

This aspect has some connotations for field performance inasmuch as asphalts with certain component combinations could, perhaps, have some of the peptizing fractions unbalanced because of reacting with rubber components with consequent asphaltene precipitation with reduced system durability. Needless to say, a good deal more work needs to be done on fundamental aspects of asphalt-rubber chemistry in order to produce a balanced composition with predictable performance.

2.4 Parameters (independent variables) for this study were selected to include levels that reflect service extremes of reaction temperature and holding times. Response tests were selected for the purposes of determining the effect of reaction time and temperature on physical properties as well as to examine test variability and sensitivity to these changes.

3.0 MATERIALS

3.1 Rubber

3.1.1 Rubber utilized in this study was processed by Atlos Rubber Reclaiming, (Los Angeles, California). The material consisted of ground (crumb) rubber produced by mechanically grinding passenger car tread. No additional processing or depolymerization was used on the ground rubber product to alter material properties prior to mixing with asphalt. However, the introduction of no more than 4 percent calcium carbonate powder to the crumb rubber prior to packaging is permitted to prevent the rubber particles from sticking together during shipment. The Atlos designation for the crumb rubber product is TP044 which is a ground rubber produced as described above having a gradation conforming to the limits shown below:

Sieve	Percent Passing
No. 16	95-100
No. 25	0-10

3.1.2 Texture and shape of the individual rubber particles appears smooth and cylindrical when viewed without magnification. However, with magnification, the notion of smooth cylindrical shapes vanishes. With magnification, an irregular surface texture appears with particles that are longer than they are wide. Processes used during reclaiming are responsible for texture. Magnification also reveals what appears to be particles of calcium carbonate used during packaging.

3.2 Asphalt

3.2.1 AR-1000 viscosity graded asphalt cement was used to produce the asphalt-rubber mixtures. Asphalt was supplied by Sahuaro Petroleum and Asphalt Company, Phoenix, Arizona.

3.2.2 Physical properties of asphalt were measured in accordance with the test procedures described by The Asphalt Institute in the Pacific Coast Division (PCD-7) Specifications. All tests in the characterization process were performed at the beginning of the study and also at the end. These results can be compared by referring to the tabulated results in Table 3-1. The characterization process was conducted twice to identify any asphalt property changes which may have occurred during storage.

3.3 Laboratory Preparation

3.3.1 Rubber was delivered to the ETL laboratory in six (6), fifty (50) pound paper bags. All the rubber was thoroughly mixed prior to storing in 30 gallon polyethylene drums. Additional mixing of the rubber immediately prior to making each asphalt-rubber batch was conducted to insure that a representative sample was obtained.

3.3.2 AR-1000 asphalt was pumped directly from the Sahuaro Asphalt storage tank into a 55-gallon drum, filling it to approximately 90 percent capacity. Retrieval of asphalt for testing was accomplished, without applying heat, by pumping compressed air into the drum forcing the AR-1000 out the opened drain spout.

Retrieval of the asphalt using compressed air required several hours due to the relative high viscosity of the AR-1000 at 77 F. Therefore, several six liter quantities were obtained in this way and frozen and stored at approximately 0 F (-18 C). The asphalt needed for each asphalt-rubber batch was then obtained by pulverizing frozen AR-1000 with a chisel in the six liter can and removing the necessary quantity of fractured material required for mixing with rubber.

Table 3-1

ASPHALT CHARACTERIZATION

Test	AR-1000 / AR-1000	May, 1978 / RTFO 1000	AR-1000 / AR-1000	Aug., 1978 / RFTO 1000
Pen (Std)	134	86	138	94
Abs. Vis., P (140 F, 30 cm)	613	1280	642	1166
Kin. Vis., cst (275 F)	159	230	155	215
Duct., cm (Std)	150+	--	134+	134+
Solub, % (TCE)	99.6	99.8	99.7	99.2
Softening Pt., F	104	113	--	--
Specific Gravity	--	1.0155	--	--

4.0 SPECIMEN PREPARATION

4.1 Specimen testing and preparation were based on complete randomization of all elements of the factorial including replications.

4.2 In order to facilitate testing, mixing, and fabrication, it was decided to mix batches of sufficient asphalt-rubber to provide material for an entire replication of all tests. After the mixture was prepared, it was frozen at 0 F (-18 C) and remained frozen until testing.

4.2.1 Statistical analysis of force-ductility, absolute viscosity, and ring and ball softening point showed no significant effects of freezing for periods of up to 14 days.

4.3 Mixing was accomplished in an open 3000 ml. stainless steel beaker with a three bladed propeller (3 inch, 45° pitch) turning at a constant speed of approximately 750 rpm. Mixer motor speed was controlled by a power-stat to maintain a constant voltage. Heat was applied by a Meeker burner and was controlled by a gas flow-meter. Temperature was monitored by an armored immersion thermometer. A view of the mixer is shown in Figure 1.

4.4 Mixing procedure was as follows:

4.4.1 Weigh 750 grams of asphalt into the beaker. Preweigh 250 grams of rubber.

4.4.2 Heat asphalt with thermometer placed ¼ inch from bottom and ½ inch from side of beaker.

4.4.3 Manually agitate asphalt to prevent local overheating. Begin stirring with propeller as soon as possible. Maintain constant propeller speed by adjusting voltage to 115 volts with Powerstat.

4.4.4 When temperature is 25F below prescribed mixing temperature, lower gas flow and stabilize without overshoot. A maximum of five minutes is allowed to stabilize mixing temperature.

4.4.5 When temperature has stabilized, add rubber to hot asphalt. Rubber is at room temperature and is added within five seconds.

4.4.6 The mixture temperature will drop to approximately 50F (28.8C) below prescribed mixing temperature. This will occur in approximately five minutes. When temperature begins to rise, increase gas flow and propeller speed. In approximately 30 minutes, the temperature will stabilize at the prescribed mixing temperature. At this point, begin timing for prescribed holding time.

4.4.7 Begin manually scraping asphalt-rubber from sides of beaker, continue scraping for prescribed holding time.

4.4.8 At conclusion of holding period, remove burner and propeller and transfer mixture with heated spoon to five 8-ounce penetration tins.

4.4.9 Allow tins to cool for 60 minutes, mark with cell and replication designation and place in freezer at 0 F (-18 C).

5.0 EXPERIMENT DESIGN

5.1 The experiment was designed as a completely randomized full factorial with two replications. Analysis of data was by conventional two way analysis of variance and by the Newman-Keuls multiple range test when independent variables were found to be statistically significant (15).

5.2 Fixed factor model for analysis is as follows:

$$Y_{ijk} = \mu + T_i + H_j + (TH)_{ij} + \varepsilon_{(ij)k}$$

Where:

Y_{ijk} = Response of material to the i^{th} temperature and j^{th} holding time.

μ = Effect on response of the overall mean.

T_i = Effect on response of the i^{th} temperature (fixed).

H_j = Effect on response of the j^{th} holding time (fixed).

$(TH)_{ij}$ = Effect on response of the interaction of the i^{th} temperature and j^{th} holding time.

$\varepsilon_{(ij)k}$ = Experimental error (random).

5.3 Levels of independent variables:

5.3.1 Temperature at three levels of 350, 375 and 400F (177, 191, and 204C).

5.3.2 Holding times at three levels of 0.5, 1.0 and 2.0 hours.

5.3.3 Factorial layout and cell designations are shown in Table 5-1.

Table 5-1
FACTORIAL AND CELL DESIGNATIONS

		TEMPERATURE (T_i)		
		i=1 350	i=2 375	i=3 400
TIME (H_j)	j=1 0.5	1,1	2,1	3,1
	j=2 1.0	1,2	2,2	3,2
	j=3 2.0	1,3	2,3	3,3

5.4 Dependent (response) variables are briefly described in Section 1.3.1.

Additional information on the test and modifications to standard test methods is included in the specific section dealing with the particular property measured.

6.0 RING AND BALL SOFTENING POINT

6.1 Testing was performed in accordance with ASTM D36-76 (part 15).

6.2 Considerable difficulty was encountered in performing the test because of the nonhomogeneous nature of the asphalt rubber mixture and the relatively small size of the specimen used for the test. Testing of conventional asphalt cement results in the ball and asphalt sinking to the bottom plate. Asphalt rubber, on the other hand, forms a structural "basket" that usually breaks in the side allowing the ball to fall from the mixture. Figure 2 shows the completion of a typical test.

6.3 Preliminary testing showed that for cases where the ball did not fall through the side, softening points were essentially the same as for the more usual occurrence of a side break in the structural basket.

6.4 Data obtained for softening point testing are shown in Table A-1 (Appendix A).

6.4.1 Overall mean for all data is 158F (70C). Softening point for unaged asphalt is 104F (40C) and for the asphalt after aging in the rolling thin film circulating oven is 113F (45C).

6.4.2 ASTM D36 states that values obtained for each ball should not vary by more than 1C (1.8F). The precision statement requires replicate tests (average of left and right specimens constitutes a single test) should not vary by more than 1C (1.8F).

6.4.3 The top data matrix of Table A-1 (all data) shows that three runs (350F, 0.5 hr., replicate 2), (400F, 0.5 hr., replicate 1) and (400F, 2.0 hr., replicate 1) exceed the limitation of 1.8F between sides.

6.4.4 Table A-4 is data using the mean of left and right sides as a single test. All data exceed the precision requirement of ASTM D36.

6.4.5 In view of the relatively low variability "between sides" (or "within tests"), the cause for the large variability between tests can be attributed to material nonhomogeniety, sample size, sample preparation (ring filling), or some combination.

6.5 Analysis of Variance (ANOVA) for all data (right and left sides not averaged) is shown in Tables A-2 and A-3. Table A-3 shows that no factors of the model (temperature, holding time or interaction) are significant in affecting softening point of asphalt-rubber mixtures.

6.5.1 When the mean of left and right sides are used as a single test, the same conclusion of the significant factors is drawn. Analysis of these data are in Tables A-4 through A-6. Plots of softening point versus holding time for each reaction temperature are shown on Figure A-1. This plot shows means but not variability.

6.5.1.1 Some discussion regarding Table A-6 F values is warranted inasmuch as in all cases, calculated F values are less than unity. It is generally accepted that several circumstances can cause low calculated F values. Some possibilities are:

A. Some uncontrolled factor may have affected the mean square error term.

B. Proper experimental randomization may not have been employed.

C. Insufficient number of measurements may not establish normality as was assumed for use of the model and calculations.

D. The assumption of homogeneous variance has been violated.

6.5.1.2 In reviewing the above possibilities, the following may point to improved future testing and evaluations:

A. Temperature and time factors were held constant (see Section 4, "Specimen Preparation"). It is quite possible that nonhomogeneity of material and relatively small specimen size (D36 rings) may be the uncontrolled factor responsible.

B. Since the experiment was completely randomized, operator bias, learning curve effects, or equipment bias are probably not causes.

C. Insufficient measurements may be a cause. If additional work is undertaken, increased replications, rejection of data that does not comply with D36 precision requirements, and modifying the apparatus may improve precision and reliability of the test.

D. The assumption of homogeneous variance was not violated in the analysis. Table A-5 shows results of the Foster and Burr q test for homogeniety of variance as described by Anderson and McLean (15). This check and appropriate transformation of data, if necessary, were standard analysis procedure for this study.

6.6 Conclusions

6.6.1 This experiment suggests that asphalt-rubber provides a substantially higher softening point material than does the conventional base asphalt. The mean value of softening point of 158F (70C) approaches maximum service temperatures for many Arizona pavements.

6.6.2 Variability of asphalt-rubber, as probably should be expected, is greater than conventional asphalt cement. If softening point is going to be used, additional work should be done to reduce or at least evaluate and set reasonable limits on variability. Steps that should be taken include increased replication testing, modification of ring and ball geometry to accommodate rubber particles, or the development of a flow index test procedure as suggested by L. C. Krchma (18).

6.6.3 The experiment did not detect any differences in softening point due to reaction temperature or reaction time. If the assumption (based on field experience) that time and temperature have an effect is considered, it can be concluded that the conventional test is not sufficiently sensitive or that softening point is, in fact, not affected.

6.6.3.1 Additional work and experimentation could resolve the question. However, in view of the usefulness of softening point for paving purposes, the expenditure probably cannot be justified.

7.0 ABSOLUTE VISCOSITY AT 140F (60C)

7.1 The standard method of ASTM D2171-66 (Part 15) was modified as outlined by Green and Tolonen (13). Modifications of the procedure include:

- Use of a large bore capillary viscometer (Asphalt Institute No. 400) to accommodate rubber particles.
- Reduction of applied vacuum from the standard 30 cm. Hg. to 10 cm. to produce acceptable flow times.

7.2 Use of reduced vacuum required a correction of tube constants that are provided by the manufacturer. The correction is based on the following:

$$K = \frac{M_H}{(H-h)}$$

Where:

K = Instrument constant in poises/sec. per cm. Hg.

M_H = Multiplier constant for vacuum head of test (or calibration).

H = Applied vacuum head (cm. Hg.).

h = Average liquid head (cm. Hg.). Provided by manufacturer.

7.2.1 Viscosity is, of course, flow time in seconds, t, multiplied by the constant, M_H.

7.2.2 As an example for the tube no. 12-X522, (bulb B):

$M_H = K\ (H-h) = 18.30$ (Mfgrs. data)

$H = 30$ cm. Hg.

$h = 0.24$ (Mfgrs. data)

$$K = \frac{M_H}{(H-h)} = \frac{18.30}{(30-0.24)} = 0.6149$$

For 10 cm. Hg., the multiplier constant, M_H, is:

$M_{10} = K\ (H-h) = 0.6149\ (10-0.24) = 6.00$

7.2.3 A series of tests were undertaken using asphalt cement at the two levels of vacuum. Analysis showed no significant difference in viscosity due to the vacuum head, hence the concept of multiplier correction was considered valid and was used for viscosity experiments with asphalt-rubber.

7.3 Viscometer tubes were loaded by means of a syringe consisting of a glass tube with a wood plunger. The syringe tube outside diameter is such that it loosely fits inside the viscometer tube and the wood plunger, of course, snugly fits inside the syringe tube.

7.3.1 Procedure for filling the viscometer tube is as follows:

7.3.1.1 Asphalt-rubber is heated to 275F (135C) on a hotplate.

7.3.1.2 Viscometer, viscometer clamp stand, syringe tube, and plunger are preheated in an oven to 275F.

7.3.1.3 When the asphalt-rubber mixture reaches 275F, the syringe is loaded by suction from the wood plunger.

7.3.1.4 Transfer from the syringe to the viscometer is by the plunger forcing material from the syringe tube while the tube is being withdrawn from the viscometer. This operation takes place in the hood at room temperature.

7.3.1.5 Immediately after filling, the viscometer is placed in the 140F bath. After 60 minutes in the bath, temperature equilibrium is assumed and the test is conducted.

7.4 Data for viscosity tests are shown in Table B-1. Entries (poises) shown are averages of six timing marks for each viscometer section for each replication.

7.4.1 Overall mean for all data is 146,267 poises. Viscosity of unaged base asphalt is 613 poises and is 1280 poises for the asphalt after aging in the rolling thin film circulating oven.

7.4.2 ASTM D2171 does not specify a precision for values obtained between different sets of timing marks for a single tube but experience indicates that variability not exceeding 5 to 10 percent should be expected for conventional asphalt cement. The mean difference for this experiment is about 25 percent. This increased variability of asphalt-rubber is about the same as was observed in softening point testing.

7.4.2.1 D2171 repeatability precision statement requires that two values must agree to within 7 percent of their mean. If data of Table B-1 is used to compute a test value based on the mean of the two readings from a single tube, none of the data will comply with the repeatability requirement for asphalt cements.

7.4.2.2 One source of this variability could be that a clean meniscus does not form at the surface as is the case for asphalt cement. This is shown on Figure 3. Timing between marks is difficult because the shape of the surface plane does not remain constant between marks. Another source of variability is, undoubtedly, related to material nonhomogeniety resulting from methods that were used to charge asphalt-rubber into the viscometer.

7.4.3 Logarithmic transformation of data was necessary to comply with the homogeniety of variance requirement for analysis of variance. Transformed data are shown in Table B-3. The analysis is shown in Table B-5. Analysis of variance shows holding time to be significant at the 0.05 but not at the 0.01 level. Temperature and interaction were not significant.

7.4.3.1 A Newman-Kuels test for multiple contrasts was conducted in an attempt to determine which reaction time(s) were responsible for an optimum (or pessimum) viscosity, but this analysis (Table B-7) shows no significant differences.

7.4.3.2 Figure B-1 is a plot of holding time versus transformed viscosity and Figure B-2 is a plot of mixing temperature versus transformed viscosity. Both plots are of mean cell values and do not, for clarity, show variability. Both of these plots suggest that holding time may have an effect and that viscosity may maximize at about one hour.

7.4.4 Another analysis was made using transformed data where the replicate measurement is the mean viscosity based on the two timing marks of a single tube. These data are shown in Table B-8. Analysis of variance is shown in Table B-10. For this analysis no factor contributes significantly to a viscosity effect.

7.4.4.1 This result is probably a reflection of the variability of the test as well as some masking of information as a result of the logarithmic transformation and averaging process. This conclusion is additionally reinforced when it is remembered that the analysis of all data showed time significance at the 0.05 level but not at the 0.01.

7.5 Conclusions

7.5.1 Asphalt rubber exhibits viscosities at 140F (60C) approximately two orders of magnitude greater than the base asphalt used for the mixture.

7.5.2 Variability of asphalt rubber greatly exceeds that of conventional asphalt cement. Sources of variability are of concern because of the effect of mass viscosity on production and application. Viscosity affects mixing, pumping, nozzle size and application temperature. Theory and experience shows that viscosity is an indicator of the extent of reaction between asphalt and reclaimed rubber.

7.5.2.1 Sources of variability in viscosity measurement using capillary viscometers includes:

- Nonuniformity of material (asphalt and rubber separation in the tube) due to the method of loading or charging the viscometer.

- Uneven surface of the meniscus.

- Extremely long flow times between timing marks. Flow times in this study were on the order of 30 minutes whereas the standard method for asphalt cement (D2171) suggests times much shorter.

- Shear rates that are not constant along the length of the tube. See Section 7.6.

7.5.2.1.1 Laboratory experience obtained as part of this study indicates capillary viscometry with tube sizes used (The Asphalt Institute No. 400) are not adequate for asphalt-rubber from either a precision nor a time economy point of view. There are several alternative systems that should be considered for further study. These include:

- Vacuum capillary viscometers with bore sizes considerably larger than those used for this study. A No. 16 sieve size rubber particle will have a length of up to 3.5 mm. If the rule of bore diameter of 3 times the particle size to prevent interference is accepted, a minimum bore size of about 11 mm. (0.4 in.) will be necessary.

- Falling coaxial viscometers of the type described by Frobel, Jimenez, and Cluff (10).

- Rotating vane viscometers are a possibility. If properly calibrated, these devices could be incorporated into the laboratory mixer and could monitor the rate of reaction.

7.5.3 This experiment did not adequately detect a difference in viscosity of finished materials due to mixing temperature but did suggest that holding (reaction) time may have an effect. This experiment shows that viscosity, as measured in this part of the study, is more sensitive to the effect of holding time than is softening point, but is not more sensitive to the effect of mixing (reaction) temperature.

7.5.3.1 At this point of the study, the question of material changes due to reaction time and temperature are unresolved. Based on the variability of measured values and on field experience, a reasonable conclusion is that test sensitivity deserves additional study and that other properties should be considered.

7.6 Discussion

7.6.1 Since the test was intended to be run using as near "conventional" techniques as possible, corrections were not made to account for shear rates that were not 0.05 $sec.^{-1}$. This correction, or the lack of it, could, of course, affect results.

7.6.1.1 A study is presently underway, in conjunction with development of another mixing technique, that will evaluate shear rate effects of asphalt-rubber in capillary viscometers. A separate report of these findings will be prepared and presented in the near future.

8.0 DUCTILITY AT 39.2F (4.0C)

8.1 Ductility values were determined according to ASTM D113-77 at a testing speed of 1 cm. per min.

8.2 Data obtained for ductility are shown in Table C-1.

8.2.1 Overall mean for all data is 17.0 cm. Ductility for the unaged asphalt used for the mixture was 100+cm. and for the asphalt after aging in the rolling thin film circulating oven, the measured value is 100+cm. It should be noted that when aged base asphalt cement is tested at 5 cm. per min. that ductility values drop significantly. Unaged asphalts retain the 100+cm. values. Beaker aged asphalts showed values from 2 cm. to 41 cm. and rolling thin film oven residues produce ductilities of about 6 cm.

8.2.2 Logarithmic transformation of data was necessary to comply with the homogeneity of variance requirement for analysis of variance. Transformed data are shown in Table C-3. The analysis is shown in Table C-5.

8.2.2.1 For the case of all data, mixing temperature is highly significant; holding time and the interaction are not significant.

8.2.2.2 Newman-Keuls test for multiple contrasts were run on the mean of all data with mixing temperature as the single factor. This analysis (Table C-8) shows ductility for a mixing temperature of 400F is significantly greater than 350F but not greater than 375F and ductility for a mixing temperature of 375F is significantly greater than 350F. Stated another way, ductility of materials mixed at 350F are lower than those mixed at 375 or 400F and there is no significant difference between 375 and 400F. This effect is shown graphically on Figures C-1 and C-2 where transformed ductility is plotted versus temperature and holding times.

8.2.3 Another analysis was made using the mean of three ductility measurements as a single test or replicate. These data are shown in Table C-9.

8.2.3.1 Analysis of these data is shown in Table C-11. Results are essentially the same as were found when all data were used in the analysis, with some information being masked by the averaging process. For this case, temperature is significant at the 0.05 level but not at the 0.01.

8.3 Effect of Temperature on Ductility

8.3.1 A report by Jimenez (19) shows that asphalt-rubber ductility is relatively unaffected by temperatures in the range of 77-33F (25-0.6C) as compared to the well known pronounced effect of temperature on the ductility of straight asphalt cement. Jimenez shows ductility of straight asphalt cement (tested at 5 cm. per min.) at 55F (13C) to be 150+cm. and a value of 0 cm. at 33F (0.6C) whereas asphalt-rubber exhibited a constant ductility value of approximately 24 cm. for the temperature of 77, 55 and 35F (25, 13 and 0.6C).

8.3.2 To further consider this property, a small experiment was designed to produce asphalt-rubber using the same mixing times and temperatures as the previous experiment and to test for ductility at 1 cm. per min. at 77F

(25C). The data are shown in Table D-1.

8.3.2.1 Overall mean for all tests was 16 cm. (Overall mean for 39.2F was 17 cm.)

8.3.2.2 Since the experiment was not replicated, two-way analysis of variance could not be used to analyze main effects and interactions. A one-way analysis was made using each combination of time and temperature as a single treatment. This analysis is shown in Table D-2. Analysis of the data shows there is no significant difference due to treatment, hence 77F ductilities are statistically the same regardless of reaction temperature and time combinations.

8.3.2.3 An analysis was made to compare ductility at 77F with ductility at 39.2F. Data from Tables C-1 and D-1 were compared with a t-test of means.

	39.2F	77F
Σ	916.0	380.5
$\bar{x}$	16.96	15.85
s	2.25	2.87
n	54	24

$$F = \frac{8.24}{5.06} = 1.63 \quad (F_{0.05,\ 53,\ 23} = 1.88)$$

$$\therefore \sigma^2_{39.2} = \sigma^2_{77}$$

$$S^2 = \frac{[(54-1)(2.25)^2 + (24-1)(2.87)^2]}{54 + 24 - 2} = 6.02$$

$$S^2_{\bar{x}_1 - \bar{x}_2} = \frac{(6.02)}{54} + \frac{(6.02)}{24} = 0.36$$

$$t = \frac{16.96 - 15.85}{0.602} = 1.84$$

$$t_{0.975,\ 76} = 1.99$$

Since the calculated t value of 1.84 is less than the critical value, it is concluded that there is no significant difference between ductilities at 77 and 39.2F.

8.4 Conclusions

8.4.1 Ductility of asphalt-rubber at 39.2F (4C) at test rates of 1 cm. per minute is considerably less than aged or unaged asphalt (AR-1000) that was used to prepare the asphalt-rubber mixture.

8.4.2 Ductility of asphalt-rubber at 39.2F (4C) at test rates of 1 cm. per minute is considerably more than the ductility of thin film circulating oven aged base asphalt if the base asphalt is tested at 5 cm. per minute.

8.4.3 The ductility test is sensitive to the effects of mixing temperature but not sensitive to the effect of reaction (holding) time nor to the interaction between holding time and temperature.

8.4.4 Ductility of asphalt-rubber is essentially constant for the temperature range of 77-39.2F (25-4C).

9.0 FORCE-DUCTILITY

9.1 This test is a modification of the standard ASTM D113 inasmuch as a force ring is added to the briquet clip in somewhat the same fashion as described by Anderson and Wiley (16). Reason for using this device was to increase the amount of data generated during ductility testing. Strength as well as a measure of energy (by area under the force-elongations curve) to deform or fail the specimen are easily obtained. Furthermore, it has been suggested that, by modifying the cross-section and longitudinal geometry of the specimen to provide a constant cross-section, the method can be used to evaluate creep compliance of asphalt-rubber.

9.1.1 For this study, deformation of a proving ring was measured with a LVDT. LVDT movement was recorded on a timed strip chart. Figures 4 and 5 show the force-ductility set up used for this study.

9.2 Maximum Ductility Load at 39.2F (4C), 1cm. per minute.

9.2.1 Data for the experiment are shown in Table E-1. Forces shown are in Newtons (1 lbf = 4.448222N).

9.2.1.1 Data were transformed by $x' = \sqrt{B} - \sqrt{B-x}$ to provide homogeneity of variance required for analysis of variance. Analysis is shown in Table E-4.

9.2.1.2 Analysis of these data show that holding time is significant at the 0.05 level but not at the 0.01 level. The interaction term is highly significant which can also be seen by the intersecting or crossing curves of Figure E-1 and E-2.

9.2.2 Since data from other tests is not available, variability comparisons cannot be made.

9.2.3 Conclusions

9.2.3.1 Of all tests and experiments described thus far maximum ductility load (and, as will be shown later, force-ductility) are the only methods that are sensitive enough to detect the interaction between reaction time and reaction temperature. This may be important in the production of asphalt-rubber if it can be shown that lower temperature combined with longer holding times will produce the same material as the reverse combination of higher temperature and shorter holding time.

9.3 Force-Ductility Energy

9.3.1 Energy is calculated as the area under the curve of elongation versus force expressed in Newton-Metres (N-M). Data for the experiment is shown in Table F-1.

9.3.1.1 Analysis of these data show (as did maximum ductility load) that reaction time and the interaction between reaction time and temperature to be highly significant.

9.3.2 Conclusions

9.3.2.1 Force-ductility energy is probably more sensitive to the effect of holding time than is the maximum ductility load because energy is significant at the 0.01 level whereas maximum load is not. The test also suggests that one hour may be the optimum mixing time, at least 375 and 400F (191 and 204C) as can be seen on Figures F-1 and F-2.

9.3.2.2 Based on experience gained during this study, the force-ductility test appears to show promise as an analytical tool and perhaps as a specification test if a correlation can be generated between force-ductility parameters and asphalt-rubber field performance. Furthermore, energy calculations based on area under the load-deformation curve can be related to toughness in much the same manner used for the analysis of metals. This will provide a means of comparing different asphalt-rubber formulations, particularly if performance can be correlated with this property.

10.0 Schweyer Rheometer

10.1 The recently available Schweyer rheometer is described as a constant stress rheometer (17) that produces a rheogram of apparant viscosity (η_a) versus shear rate ($\dot{\gamma}$). A schematic of the device is shown on Figure 10-1 and overviews are shown on Figures 6, 7 and 8.

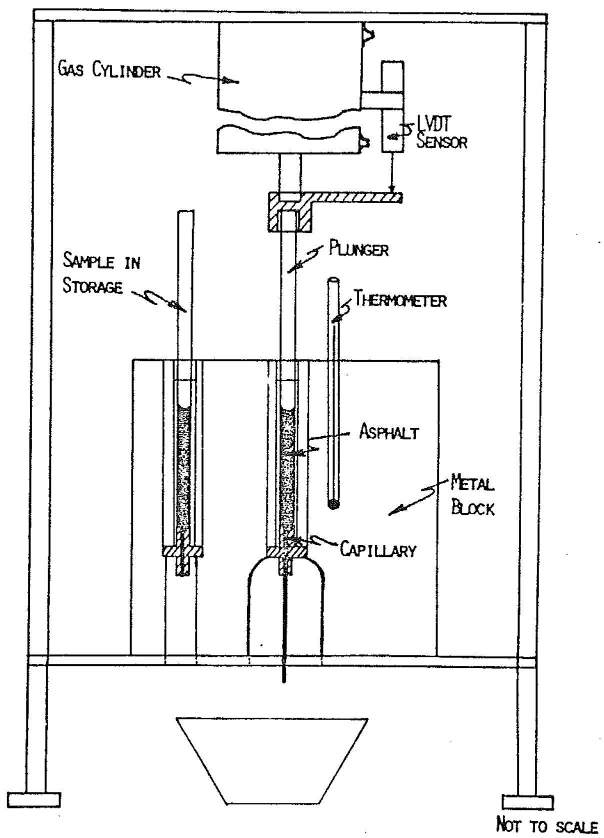

Figure 10-1

Schweyer Rheometer Schematic

10.2 Principal of operation is that a sample of material is forced through a precision capillary by means of a constant load. Load is applied to the specimen by means of a plunger and downward movement of the plunger is monitored by a LVDT.

10.2.1 A timed strip chart is produced that records plunger movement as a function of time. The corrected plunger movement is equated to specimen flow through the capillary tube. Figure 10-2 shows a typical output curve.

10.2.2 Movement of the plunger begins as nonlinear with time until equilibrium (constant) flow is established when the curve becomes linear. Velocity measurements are made on the linear portion of the curve.

10.2.3 Calculations are straightforward and simple.

10.2.3.1 Pressure on the loading ram is read directly from a gage on front of the instrument. Pressure is supplied by pressurized gaseous nitrogen and can be controlled to suit test conditions and material characteristics.

10.2.3.2 Gage pressure (P) is multiplied by a machine constant (M) to calculate force applied to the specimen.

10.2.3.3 Force is then multiplied by a shear stress tube constant ($K_{\dot{\tau}}$) to calculate shear stress (τ).

10.2.3.4 Hence: $\tau = (P \times M)\ (K_{\dot{\tau}})$

10.2.3.5 Velocity (V) is read from the linear portion of the recorded curve of movement versus time.

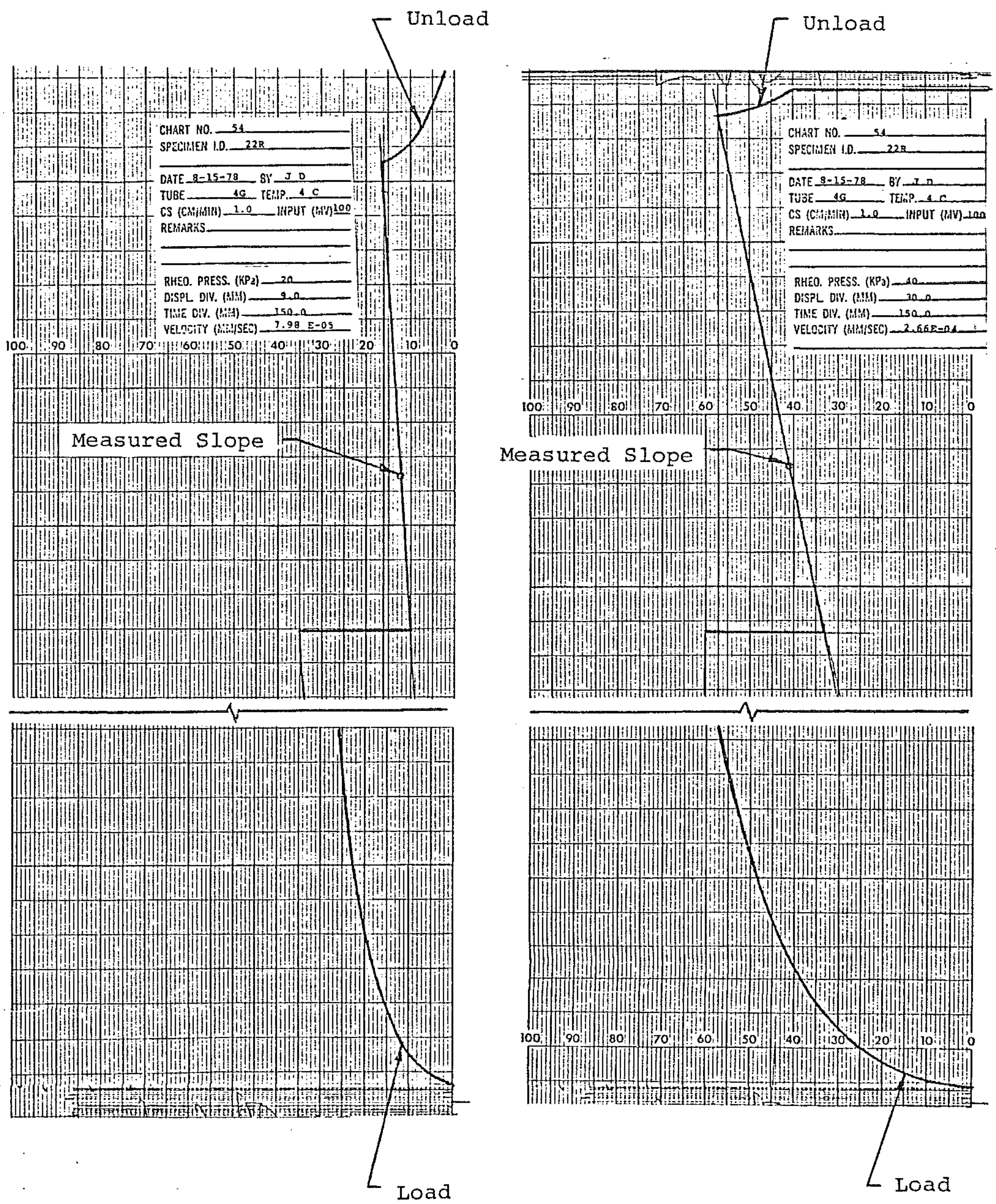

Figure 10-2

Rheometer Traces

10.2.3.6 Shear rate ($\dot{\gamma}$) is calculated by multiplying velocity by a shear rate tube constant (K_τ).

10.2.3.7 Hence: $\dot{\gamma} = VK_{\dot{\gamma}}$

10.2.3.8 Apparent viscosity (η_a) is given as:

$$\eta_a = \frac{\text{shear stress}}{\text{shear rate}} = \frac{\tau}{\dot{\gamma}}$$

and is expressed in Pascal-seconds (Pa.s)

10.2.3.9 After the test load is removed and the specimen returns to rest equilibrium, a different load is applied and a new apparent viscosity is calculated for the new shear stress and shear rate.

10.2.3.10 Several runs are made and a rheogram is developed that is a plot of log of apparent viscosity (log η_a) versus log of shear rate (log $\dot{\gamma}$) where each point on the curve represents one run. These points form a straight line amenable to linear regression analysis and, theoretically, allow calculation of apparent viscosity for any shear rate. In this study, viscosities are reported for shear rates of 0.05 and 1.0 sec.^{-1}.

10.2.3.11 A typical rheogram and data used to generate it are shown on Figure 10-3.

10.3 Straight lines on a log scale are presented by the power law equation:

$$y = a\,x^b$$

or for the rheogram:

SCHWEYER RHEOMETER

IDENTIFICATION 22R

CHART NO. 54 TEMP. 4C TUBE 4G

(L), LVDT Chart Dsp., mm	4.2	9.0	13.4	17.0	30.0	
(T), Time Chart Dsp., mm	150.0	150.0	150.0	150.0	150.0	
(CS), Chart Speed, cm/min	1	1	1	1	1	
(P), Pressure, KPa	15	20	25	30	40	
τ Pa	1.1E04	1.4E04	1.8E04	2.2E04	2.9E04	
V mm/sec	3.72E-05	7.98E-05	1.19E-04	1.51E-04	2.66E-04	
$\dot{\gamma}$ sec^{-1}	3.07E-05	6.58E-05	9.80E-05	1.24E-04	2.19E-04	
η_a Pa-sec	3.5E08	2.2E08	1.8E08	1.8E08	1.3E08	

Regression ($\eta_a = a\dot{\gamma}^b$): a = 1.961E06 b = -4.956E-01 R^2 = 0.982

Rheologic Constants : $\eta_{0.85}$ = 8.9E06 $\eta_{1.0}$ 2.0E06 c = 5.1E-01

Plotting Points:

$\dot{\gamma}$	1.0E-05	1.0E-04	1.0E-03
η_a	5.9E08	1.9E08	6.1E07

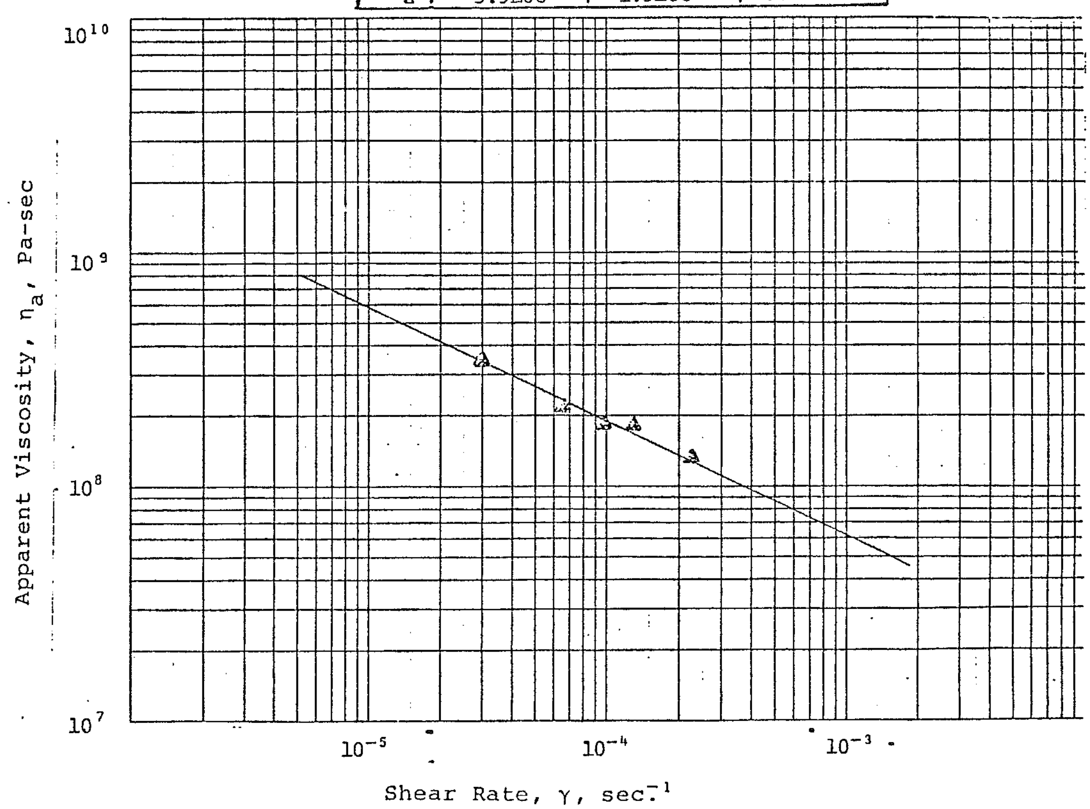

Figure 10-3

Rheogram

$$\eta_a = a(\dot{\gamma})^b$$

where b is the slope of the straight line of the log plot.

10.3.1 Materials with horizontal rheogram plots ($b = 0$) are Newtonian fluids (20) and hence, are not shear susceptible. That is, apparent viscosity is constant over a range of shear rates.

10.3.2 Materials with slopes down to the right ($b < 0$) are termed pseudoplastic. These materials are shear susceptible inasmuch as viscosity decreases when shear rate increases. Paint that becomes "thinner" during stirring is an example.

10.3.3 Materials with slopes up to the right ($b > 0$) are termed dilatant. Shear susceptibility of these materials is exhibited by increased viscosity with increased shear rate. Certain flocculating agents produce dilatant fluids. This property may be of interest in the asphalt-rubber field since observations have been reported that performance in runway touch-down areas (high shear) is better than in areas where traffic is relatively slow moving. If the observations are valid, this performance is the opposite of what would be expected from pseudoplastic, and probably, Newtonian materials.

10.4 Shear susceptibility, then, is a function of the slope of the rheogram. Newtonian materials have zero slope, pseudoplastic materials have negative slopes, and dilatant materials have positive slopes. The rheogram is described by a characteristic apparent viscosity at a given shear rate (say 0.05 sec^{-1}) and the power law parameters a and b (slope).

10.4.1 Shear susceptibility in this study is reported by the parameter c as described by Schweyer (17). The shear susceptibility index is defined as:

$$c = 1 + slope = 1 + b$$

10.4.1.1 Newtonian materials (b = 0) have a shear susceptibility index of 1.

10.4.1.2 Pseudoplastic materials (b < 0) have shear susceptibility indices of less than 1.

10.4.1.3 Dilatant materials have shear susceptibility indices greater than 1.

10.5 Specimen tubes used for the constant stress rheometer (Figure 8) consist of a specimen or flow tube of 9.47 mm. (approximately 3/8 in.) diameter. The capillary is threaded onto the end of the flow tube and is much smaller in diameter (down to less than 1 mm.) than the flow tube. Selection of capillary size is based on relative viscosity of the material under test conditions. Thinner or low viscosity materials would, of course, use smaller capillaries.

10.5.1 The pressurized ram or plunger forces the large plug of material down the flow tube (50 mm.) and through the capillary. The capillary controls flow rate and hence shear rate and shear stress. There is some question as to size effects on the capillary on measured viscosity. It should be expected that, for a given material under constant environmental conditions, different tubes would give the same viscosity if capillary size effects were not present. In the case of asphalt rubber, the question of particle interference arises inasmuch as rubber particles could clog a capillary opening smaller than the rubber particle. If interference exists, measurements would be confounded since, instead of representing asphalt rubber mass viscosity, elasticity of rubber and/or flow of asphalt around rubber particles could be the response.

10.6 To investigate the size effect and to aid in selecting a tube size for the study, a small experiment was conducted on asphalt cement. The data are shown in Table G-1. Rheograms of the two asphalts (stock and aged) for three test temperatures are shown in Figures G-1 through G-9. These figures show the strong possibility of an effect due to capillary diameter on both apparent viscosity and slope of the rheagram (which indirectly indicates shear susceptibility). Discussions with Dr. Scheweyer indicate that this is not necessarily unexpected and that sensitivity to capillary size may, indeed, be characteristic of a given material.

10.6.1 It is possible to use a single capillary for a given combination of material and temperature but not necessarily for all materials nor for a given material at all temperatures. This is because ram movements are excessively fast when relatively low viscosity materials are tested in large bore capillaries and conversely, ram pressures are not great enough to force high viscosity materials through small capillaries in reasonable amounts of time.

10.6.2 A series of tests were run on asphalt-rubber using the E, F, and G tubes. Capillary diameter of these tubes are 3.186, 4.650, and 9.700 mm. respectively. The same parameters of mixing temperature and holding time that were used in previous experiments were used for this factorial. In addition, three tube sizes were included. Data generated include a parameter of the rheogram, shear susceptibility index (c), R^2 (correlation coefficient of data), apparent viscosity at a shear rate of 0.05 sec.$^{-1}$ ($\eta_{0.05}$). Data of the experiment are shown in appendix G.

10.6.2.1 The fixed factor model used for this analysis is:

$$Y_{ijkl} = \mu + T_i + C_j + H_k + (TC)_{ik} + (CH)_{ik} +$$

$$(TCH)_{ijk} + E_{(ijk)1}$$

where:

Y_{ijkl} = response measured (a, c, R^2, N_a, etc.)

μ = Effect of the overall mean.

T_i = Effect of i^{th} mixing temperature.

c_j = Effect of j^{th} capillary size.

H_j = Effect of n^{th} reaction time.

$(TC)_{ij}$ = Interaction of temperature by capillary.

$(TH)_{ik}$ = Interaction of temperature by time.

$(CH)_{ik}$ = Interaction of capillary by time.

$(TCH)_{ijk}$ = Three way main effect interaction of temperature, capillary and reaction time.

$\varepsilon_{(ijk)1}$ = Experimental error.

10.6.2.2 Analysis was made to determine the effect of capillary size, mixing temperature, reaction time, and all interactions. For all cases, capillary size is highly significant at the 0.01 level. Each parameter will be discussed below.

10.6.3 The a parameter of a linear system in cartesian coordinates represents the intercept of the curve with the y-axis when x equals 0. For the case of logarithmic or "power functions" as is found for rheograms ($\eta_a = a\ \dot{\gamma}^{\,b}$), the a parameter serves as a shift factor for linear functions on a log log plot. The b parameter is, of course, the slope of the functions on a log log plot.

10.6.3.1 Analysis of variance, see appendix G, shows the following effects on the a parameter.

A. Capillary diameter (tube) is significant at the 0.01 level. The level of significance (or effect) of capillary size is greater than that of mixing temperature or reaction time.

B. Mixing temperature is significant at the 0.05 but not at the 0.01 level.

C. Reaction time is significant at the 0.05 level but not at the 0.01 level.

D. The only interaction term that is significant is the three way interaction of the main effects (temperature by capillary by holding time). This term is significant at the 0.01 level and should be expected in view of the strong effect of capillary size and the significance of time and temperature.

10.6.4 Shear susceptibility index (c) is defined as:
$c = (1 + \text{slope}) = 1 + b$
analysis of the effect of experimental factors on b were not made because the same factors that affect the slope (b) will be reflected in the shear susceptibility index (c).

10.6.4.1 Analysis of variance, see appendix G-21, shows that the only factor that affects the shear susceptibility index is capillary size. No other main effects (temperature and time) have a significant effect. The three way interaction is highly significant, probably due to the strong influence of capillary size.

10.6.5 Correlation coefficient (R^2) is defined as:

$$R^2 = \frac{\text{Sum of Squares Due to Regression}}{\text{Sum of Squares About the Mean}}$$

This value is a measure of the "goodness to fit" of the data to the calculated regression model (a and b parameters). It can be interpreted as a measure of the total variability that is explained by the model. Values close to unity represent a good fit and values approaching zero indicate a poor fit and hence, probably the selection of an incorrect model to represent the data.

10.6.5.1 Analysis of variance, see appendix G-21, shows tube size to be highly significant.

10.6.6 Viscosity at 0.05 reciprocal seconds ($\eta_{0.05}$).

10.6.6.1 Values for this property are obtained from the rheogram by plotting calculated values of apparent viscosity (η_a) versus shear ($\dot{\gamma}$) on a logarithmic scale. Since the shear rate of exactly 0.05 is rarely obtained during the test, a best fit straight line is drawn through the data and the viscosity value is read from the curve or extrapolated for a particular shear rate.

10.6.6.2 For this study, a least square regression (power curve fit) was used to plot the curve and obtain the viscosity for a given shear rate. It was this regression that generated a,b, and R^2 that were analyzed earlier.

10.6.6.3 In order to satisfy homogeneity of variance requirements, the data was transformed as follows:

$$N^1_{0.05} = \log_{10} (N_{0.05} + 1.00\ E + 07)$$

Where:

$N^1_{0.05}$ = transformed viscosity.

$N_{0.05}$ = measured viscosity.

E+07 = scientific notation for powers of 10.

10.6.6.4 Analysis of variance, see appendix G, of transformed data shows capillary size to be highly significant and that holding time is significant at the 0.05 level but not at the 0.01 level.

10.6.6.5 Data (see appendix G) show a range of values from 0.3134 E+07 (tube E, 375F, 0.5 hr.) to 94.05 E+07 (tube F, 375F, 9.5 hr.) Pascal-Seconds (Pa•S). Since 1 Pa•S is 10 Poises, the range is 3.134 E+07 to 9.405 E+09 Poises. It should be noted that, at 32.9F, viscosity values are approaching the maximum (or asymptotic) values of 10^9 to 10^{10} Poises that are sometimes used to define the change of state of fluids to solids.

10.6.6.6 Viscosity measurements on the base asphalt used for the asphalt-rubber after the asphalt was heated to 375F for 1 hour show ranges of values from 5.0 E+07 to 95 E+07 Pa•S. (All measurements and data are not included in this report due to space limitations, but are available upon request.)

11.0 Elastic Rebound

11.1 Asphalt rubber mixtures were prepared at Engineers Testing Laboratories and sent to Mr. R. L. Dunning of Petroleum Sciences, Spokane, Washington for testing. Testing was performed at 77F (25C) by the method described by Green and Tolonen (13). The experiment was not replicated, hence there is a single data point for each temperature - holding time combination. Rebound data (in percent) are shown in Table 11-1.

Table 11-1
Elastic Rebound

		Temperature		
		350	375	400
Time	½	78.2	74.0	76.0
Time	1	81.6	81.8	95.6
Time	2	81.4	90.2	82.0

11.2 Since the data was not replicated, two way analysis of variance was not performed.

11.2.1 Grouping the data by temperatures and performing one way analysis of variance showed no significant effect due to treatment (temperature).

11.2.2 Grouping the data by holding time and performing one way analysis of variance showed no significant effect due to treatment (holding time).

11.2.3 Elastic rebounds ranged from a high of 95.6 to a low of 74.0 percent. Mean for the 9 data points is 82.3 percent and standard deviation is 6.77 percent.

12.0 Conclusions

12.1 Conclusions are, of course, based on results of tests and experiments conducted during this study. Experiments were designed to be completely randomized fixed model factorials with two replications. Independent variables were three levels of mixing temperature (350, 375, and 400F(176.7, 190.6, and 204.4C), and three levels of reaction time (0.5, 1.0, and 2.0 hours).

12.1.1 Tests performed as part of this study can be classified as either:

- Modifications of standard or conventional asphalt tests.
- Non-standard tests.

12.1.2 Standard (modified) or conventional tests include:

- Ring and Ball Softening Point
- Absolute Viscosity with a Capillary Viscometer.
- Ductility at Low Temperatures.

12.1.3 Non-Standard tests include:

- Force-Ductility
- Apparent Viscosity by Schweyer Rheometer
- Elastic Rebound by Sliding Plate Microviscometer.

12.2 A tabulation of results of testing and analysis of variance is shown in Table 12-1.

12.3 In general, conventional tests, with precision constraints that are applied to asphalt testing, are not applicable for testing asphalt-rubber without modification of both the test method and the precision statement.

Table 12-1

Summary of Results

Measurement	Analysis of Variance Significance		
	Temp	Time	Interaction
Softening Point			
All Data	No	No	No
Mean of Runs	No	No	No
Absolute Viscosity (140F)			
All Data	No	Yes	No
Mean of Runs	No	No	No
Ductility at 39.2F			
All Data	Yes	No	No
Mean of Runs	Yes	No	No
Max. Ductility Load			
All Data	No	Yes	Yes
Force - Ductility			
All Data	No	Yes	Yes
Schweyer Rheometer			
a	Yes	Yes	No
c	No	No	No
R^2	No	No	No
$\eta_{0.05}$	No	Yes	No
Elastic Rebound	No	No	-

12.4 There appears to be a measurable effect of mixing temperature and reaction time on properties measured by the Force - Ductility test.

12.5 Force - Ductility (utilizing elongation at failure and either maximum load at failure or area under the load elongation curve) is the most sensitive to detection of effects of time and temperature of all tests conducted during this study.

12.6 The Schweyer Rheometer is effective and efficient for determination of viscosity at low temperatures. There is, as was expected, an effect due to capillary size (tube effect) used for the test. Furthermore, there is a strong possibility that measurements obtained with the device can be used to calculate properties such as stiffness and relaxation time.

12.7 Laboratory mixing procedures may not adequately simulate field techniques and hence, laboratory prepared materials may not be the same as those manufactured in the field. This aspect will be discussed further in the following section.

12.8 Without correlation of field performance to specific properties measured in the laboratory, materials specifications should be based on a combination of manufacturer's recommendations, experience with producing and placing the material, and strict control of material components and quantities used for production. This is not to say that laboratory testing is not effective. Tests considered during this study show promise, with additional development, of providing both quality control and behavior information.

13.0 Discussion and Recommendations

13.1 Test Methods and Variability

13.1.1 As was briefly discussed previously, sources of variability of tests can most probably be attributed to interference or sidewall effects caused by rubber particles in the asphalt rubber mixture. These effects are observed in all the tests of this study with the possible exception of ductility. Ring and ball softening point and absolute viscosity with capillary viscometers are especially affected.

13.1.1.1 Two recommendations are offered for softening point determinations if softening point is, indeed, a property that should be measured. The first is to develop a larger ring and ball that would have the same relative geometry and volume - geometry ratios, that are used for the standard ASTM D36 test. Correlations would, of course, be necessary. The second, is to further develop the flow index concept of Krchma (18) that has good possibilities of characterizing asphalt rubber flow or consistency properties. This technique promises relatively rapid and economical measurements.

13.1.1.2 With regards to capillary viscometry, there appears to be two areas that can be considered. These are increased bore size to the number 800 tube and revised loading techniques and shear rate corrections. Additional reduction in vacuum may be necessary to produce acceptable flow times and shear rates and to prevent vacuum separation of asphalt from rubber particles. A modest experiment could be conducted to compare viscosity and variability of the larger bore tube and the tube - vacuum combination that was used for this study. Loading techniques that have been considered (but not used in this study in order to correlate data obtained in this study with other viscosity measurements) include vibration of the tube during loading and packing with high pressure syringes.

13.1.1.3 There is no question that viscosity is an important property from both production and application as well as from research points of view. Viscosity is affected by asphalt properties, rubber type and particle size, proportions of asphalt and rubber components and the extent of physico-chemical reaction between rubber and asphalt. In addition, sprayability and consequent job quality and aggregate adhesion are severely affected by viscosity. Hence, it is strongly recommended that additional work be conducted to develop reliable and sensitive methods of measuring and evaluating mass viscosity of asphalt-rubber systems at elevated temperatures [in the region of field applications such as 350-400 F (177-204 C)]. Measuring systems and techniques should be developed that have both laboratory and field capability or that can be reliably correlated. The principle feature that has to be considered is to provide for specimen size adequate to accommodate rubber particles without particle to particle interference, and without sidewall or end effects. It would be worthwhile to consider both coaxial cylinder [as described by Jiminez (19)] and vane or rotational viscometers. The possible use of the laboratory mixer as a viscometer will be discussed below. It would seem advisable to devote a separate study to consider several possibilities of viscosity measurement of asphalt-rubber at elevated temperatures to establish a standard method for specification, production, and application quality control purposes.

13.1.1.4 Flow characteristics at lower temperatures appear to be measurable with the Schweyer Rheometer, although some modifications of the systems should be considered. The first suggested modification is to increase the frame stiffness of the instrument. At low temperatures, relatively large loads are necessary to force the specimen through the capillary. Extension of the frame during this loading is not constant and hence a simple linear correction cannot be applied. Stiffening of the frame would reduce variability of output and is a straightforward operation. A second suggested modification of capillary sizes available is intended to reduce the effect of particle interference on measurements.

13.1.1.4.1 Asphalt-rubber mixtures contain agglomerations of several particles held together with thin asphalt films. These clusters in turn, can produce interference similar to that of simple large particles. Measurements made with different capillary sizes showed, as should be expected, an effect due to capillary size. It should also be noted that asphalt (without rubber) showed the same size effect and the question is open as to the cause of the phenomena. Without additional study, it is difficult to estimate the tube size that would be required to eliminate, or at least to stabilize, the effect of particle interference. The largest tube size available for the instrument used for this study is 9.47 mm. (approximately 3/8 inch). This capillary size does not provide for the effect of clusters or agglomerations of particles. It is recommended that larger size tubes be developed, along with appropriate constants, that can be accommodated by the existing instrument. The chamber of the instrument can accept tubes with outside diameters of approximately 3/4 inch. It may also be possible to machine the chamber to accept even larger tubes and to provide sleeves to hold the original tubes.

13.1.1.5 Force-ductility appears to show good possibilities for measurement of low temperature properties of asphalt-rubber and should receive additional development. Force-ductility was the only simple test used during this study that was sensitive to both mixing temperature and holding (or reaction) time. In addition, the test provides data and information on tensile strength and can, as will be subsequently discussed, provide creep compliance or stiffness modulus information compatible with theoretical elastic or viscoelastic models.

13.1.1.5.1 Recommendations for modification of the test method and apparatus include changes of load read-out and specimen geometry. Load cells with improved precision over the force ring used for this study are recommended along with compatible recording equipment. These load cells have been installed as part of another project. Standard ductility specimens, as were used in this study, have a constantly decreasing cross section down to a minimum area of 1 $cm.^2$ Preliminary tests show the feasibility of modifying specimen geometry to provide a constant cross section of 1 $cm.^2$ for a length of approximately 6 cm.

13.1.1.5.2 The test envisioned is neither constant stress nor constant strain that would be ideal but requires expensive equipment, preferably electrohydraulic closed loop devices with process controllers. The test would be a constant deformation rate as provided by the ductility drive mechanism. Load versus time is provided by the load cell and elongation versus time is provided by recording head movement and the increase of gage length of the specimen. Strain can be calculated on the basis of engineering strain (ε') or true strain (ε) for large deformations. These strains are calculated as follows:

$$\varepsilon' = \frac{\Delta L}{L}$$

$$\varepsilon = \ln (1+\varepsilon')$$

where ΔL = change of length

L = original gage length

13.1.5.3 Stress calculations can be based on original cross sectional area or on reduced areas measured manually by micrometer. A plot of strain versus time and stress versus time can be used to generate the logarithmic function curve of compliance or stiffness versus time. Stiffness at short times (traffic loads) and at long times (thermal effects) would not be measured directly as part of the test but could be extrapolated, with sufficient precision and accuracy, for initial use in theoretical examinations of field behavior of asphalt rubber membranes and interlayers.

13.2 Asphalt - Rubber Reactions

13.2.1 Green and Tolonen (13) have discussed physico - chemical aspects of the asphalt rubber reaction in some detail. Discussions presented here are background for recommendations for testing and development programs and to report observations made which the scanning electron microscope (SEM).

13.2.2 It is generally agreed that rubber particles of the type used for this study swell to two to three times their original volume when fully reacted in hot asphalt. It is further postulated that swelling is the result of rubber particles imbibing or reacting with certain components of the base asphalt. Heat is necessary to liquify asphalt and contributes to the rate of reaction. Laboratory testing has been conducted that shows practically complete breakdown (dissolving) of rubber particles if the asphalt rubber mixture is subjected to sufficient reaction time and temperature. Rubber particles heated in air do not show a tendency to swell or melt at times of up to one hour at 400F (204C), hence the conclusion of a physio-chemical reaction between rubber and asphalt appears to be justified.

13.2.3 It has also been pointed out that there are different types of rubber (synthetic types and natural) used for different tires (truck tires versus conventional automobile tires and radial tires versus bias tires). In fact, the rubber types may be different for tread rubber than for sidewall material (21).

13.2.4 Tire rubber compounding involves the addition of extender oils to impart certain properties and characteristics to rubber to better function for an intended purpose. In addition, rubber compounding often uses reclaimed tire rubber that has been treated with petroleum products (chemically unsaturated resin oil obtained in the refining of gasoline (22)). These extender oils and reclaiming agents can be characterized by the same techniques used to determine the fractional composition of asphalt. In fact, some components of asphalt maltenes are very similar to extender oils and reclaiming agents when analyzed by the acid precipitation method described by Rostler (14).

13.2.5 It follows, then, if asphalt and rubber react chemically, that chemical composition or proportions of components of either asphalt or rubber can affect properties and probably performance of asphalt-rubber mixtures. This study showed that mixing temperature and reaction time have measurable effects on certain properties of these mixtures. Only minimal control can be exercised over reclaimed tires (sorting by type, separation of treads and sidewalls, selection of grinding method, and particle size). It is generally accepted that asphalts from the same specification grade can vary chemically due to different crude sources and with different refining methods.

13.2.6 It is recommended that studies be initiated to consider the effects of asphalt and rubber properties on characteristics of asphalt-rubber mixtures. Information on these procedures and on mixtures using a single asphalt-rubber combination generated from this study can serve as datum to develop subsequent studies. Presently, research is being conducted that will consider several chemical and physical aspects of asphalt-rubber mixtures. This study should be complemented by others that vary certain components of asphalt (such as the Rostler parameters) and use rubber of known composition. These rubbers can be aged materials prepared from stocks of known composition or perhaps special lots of reclaimed tires can be processed through a conventional reclaiming plant for this study.

13.3 Test Specimen Preparation

13.3.1 As was discussed previously, laboratory specimens for this study were prepared by heating asphalt in an open beaker, introducing rubber and then mixing with a high speed propeller. This technique was established by Jimenez (19) in a previous study and, for consistency, used in this project.

13.3.2 Measurements made after subjecting the asphalt (without rubber) to the same mixing showed that about the same amount of hardening takes place as occurs in the rolling thin film circulating oven exposure. Laboratory mixing obviously removes some of the more volatile components of the asphalt. One aspect of this loss is that these fractions are not available to react with the rubber.

13.3.3 Field preparation of the mixture, on the other hand, is accomplished in a closed tank that is nearly full and hence, in an atmosphere that has a greatly reduced oxygen content compared to laboratory preparation. Furthermore, the closed field system probably has much less loss of volatiles than does the laboratory method.

13.3.4 This brings up the question of similarity of properties of laboratory versus field mixed materials. Field mixed materials were not feasible for use in this study because of the obvious logistics difficulties as well as the lack of control of mixing temperature and holding times for field mixtures. The companion study that is presently underway is considering two sub studies to provide insight into the question.

- Field mixed materials were sampled and will be subjected to the same tests that were performed during this study.

- A mixing device has been developed by Arizona Department of Transportation that shows promise of retaining volatile components of asphalt during the mixing operation.

13.3.5 An asphalt-rubber field experimental project is being conducted by Arizona Department of Transportation (Buckeye - Liberty Project S-371-924). This project includes 9 test sections on a two-lane, heavily trafficed highway. This project, along with considering several structural applications, had included several combinations of asphalt and rubber types along with the combinations of material used for this project.

13.3.5.1 All field produced asphalt rubbers were sampled and frozen for future testing as part of an ongoing study.

13.3.5.2 All tests made during this study will be conducted on the field prepared materials and comparisons will be made between properties and variability of field and laboratory mixed materials. It is conceeded that some confounding may occur because neither the asphalt nor the rubber used for the field study was from the same lot as was used for the laboratory study. If necessary, however, laboratory mixtures can be prepared from component materials (asphalt, rubber, and additives) samples obtained from the field project.

13.3.6 The ongoing project will use a laboratory mixing device recently developed by the Arizona Department of Transportation referred to as the "Arizona Torque Fork", (ATF). This device has several features that show promise for both research purposes and for routine laboratory preparations of asphalt rubber mixtures.

13.3.6.1 First, the device uses a controlled electrical heating system that is considerably safer than the open flame that was employed as part of this study. Elimination of the open flame is of importance if high volatility additives such as kerosene and certain extender oils are to be used in laboratory preparations.

13.3.6.2 The mixing chamber of the device is covered (but not sealed) and should significantly reduce the loss of volatile asphalt components during long term mixing.

13.3.6.3 One of the most important features of the mixer is the ability to simultaneously record temperature, viscosity, and mixing time. Time is recorded on the strip chart and temperature is sensed by a resistance thermometer and recorded on the same strip chart. Torque of the mixer motor is indirectly measured by current flow required to maintain constant speed and is also recorded on the same strip chart. Viscosity can be determined from torque by calibration with standard materials of known viscosity.

13.3.6.4 Instantaneous measurement of viscosity during mixing provides several research opportunities. First, the effect of heat and time on base asphalt can be determined much more conveniently with this device than with conventional capillary viscometers. Secondly, if viscosity is an indicator of the extent of reaction between particular asphalt and rubber combinations, the device can easily monitor the rate and extent of these reactions for a large number of material combinations.

13.3.7 Two experiments utilizing the ATF are recommended:

- A comparison of field mixed asphalt rubber properties with those prepared in the laboratory. This will be an effort to standardize laboratory preparation methods and techniques.

- A study of the rate of reaction of various combinations of asphalt type, rubber type, and additives.

13.3.7.1 The first study would be a combined field and laboratory effort. Typical field projects using two commercial asphalt-rubber producers would be selected for study. Base materials (asphalt, rubber, and additives) would be extensively sampled for later laboratory study. Field production would be monitored (by field viscosity) throughout the production and application of the material. Laboratory mixtures would be prepared with the open beaker and ATF and be constantly monitored for viscosity as a function of mixing time and temperature. If correlations exist between field and laboratory viscosities, standard laboratory preparation procedures could be developed. If desirable, force-ductility properties could be used as a property of the prepared mixture for comparison of field and laboratory prepared mixtures. Incidently, test variability (repeatibility) information would also be generated during the program.

13.3.7.2 The second study would be a laboratory effort designed to measure the interaction of asphalt type, rubber type, and additives on viscosity and force-ductility parameters. Asphalts would be characterized by the usual specification properties and by the Rostler parameters. Rubber would be characterized by size, origin (natural or devulcanized content), and processing method. The experiment would monitor viscosity during mixing with the ATF for each combination of asphalt and rubber and would measure force ductility parameters of the final product.

13.4 Correlations of Field Performance and Material Properties to Establish Specification Criteria and Limits.

13.4.1 In order to develop specifications two areas must be considered:

- Material components, properties, and characteristics necessary for a specific function or application.

- Production and application procedures that provide the desired quality of material to a particular service environment.

These areas must be considered for either a prescription or performance specification and must be supported with standardized sampling and test methods.

Specifications for conventional asphalt materials and applications have developed with years of scientific and applied study and experience. Observations relating field performance to materials properties and application methods has resulted in the development of specifications that use a combination of several properties that are used to evaluate the material and hence, to predict performance.

13.4.2 Asphalt-rubber, on the other hand, is a new material that has some characteristics in common with conventional materials but is also quite different in many respects. Furthermore, the material is being used for some structural purposes that are in themselves, a departure from conventional paving techniques (for example, stress absorbing membrane interlayer (SAMI) and membranes for soil encapsulation). Field experience has shown that the material is capable of extraordinary performance and has also shown that, in a few isolated instances, that substandard performance and failure can occur. Without doubt, a complete specification could be developed with time, that would be adequate for most construction purposes. It seems reasonable, however, to recommend a systematic experimental program that can more efficiently and effectively develop a specification system than would a conventional trial-and-error approach.

13.4.3 This study has shown that conventional asphalt testing and technology will require modification, but can probably be used for asphalt-rubber. Additional field-laboratory studies, in somewhat the same format as the Buckeye-Liberty Project, should be designed to further consider both material components and the effect of production techniques on performance. Components and techniques could be selected in such a manner that accelerated results would be available in a reasonable time frame.

13.5 Scanning Electron Microscope (SEM) Study of Asphalt-Rubber.

13.5.1 A limited study of asphalt-rubber and of the rubber was conducted to provide some insight on the nature of the material. A secondary objective was to consider the possibility of reacted particles of rubber connecting or being welded together to form a continuous network of rubber particles.

13.5.2 Review of unreacted rubber particles showed a gradation of sizes and shapes. Some particles are elongated, some are cubical and all have irregular fractured faces. Examination of the asphalt rubber suggests a system of rubber aggregate particles with an asphalt binder.

13.5.3 An asphalt rubber mixture was extracted with cold (room temperature) trichloroethylene with repeated washings until the solvent was clear and light straw colored. Subjective estimates were that 95 percent or more of the binder was removed but that either some binder remained because rubber particles were knitted together to form a coherent rubber structure. Viewing of SEM photomicrographs of this material shows some weldment that is probably the asphalt binder and not rubber to rubber adhesion.

13.5.4 Additional cold washing with trichloroethylene were repeated until individual particles rather than the coherent net were produced. Viewing of photomicrographs of these particles showed no evidence of rubber to rubber weldment. Since the possibility exists that the solvent may have destroyed rubber to rubber weldment, particles of rubber were soaked in the solvent for 24 hours at room temperature, dried and examined with the SEM. Large magnifications were used to examine fractures within particles and contact surfaces between particles. It was reasoned that rounding or other evidence of chemical action of the solvent would be apparent, at least subjectively. All edges were sharp with no apparent rounding due to rubber solution by the solvent.

13.5.5 Based on these limited observations, there is no conclusive evidence that rubber to rubber weldment occurs to produce a continuous rubber network.

13.5.6 This study, while probably not definitive for establishing the presence or absence of a connected rubber network, did show the value of scanning electron microscopy as a tool in asphalt rubber research and evaluation. It is strongly recommended that the method be employed for all future work for both analysis and documentation of materials used.

14.0 Acknowledgements

14.1 Engineers Testing Laboratories, Inc. wishes to express their appreciation to the Research Division of Arizona Department of Transportation and to the U.S. Department of Transportation, Federal Highway Administration for funding and administration of the study. They wish to also express their appreciation for direction and helpful input from the Project Research Advisory Panel.

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Figure 1
Mixing System

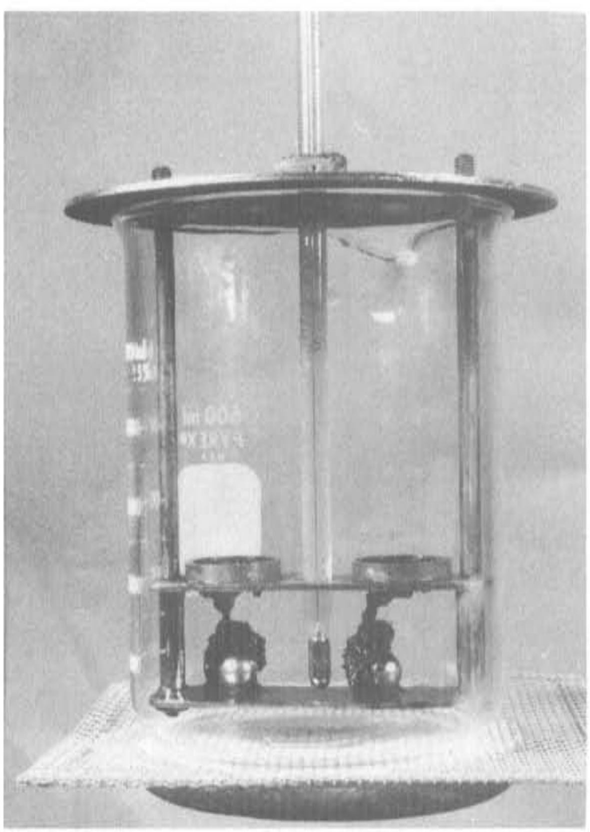

Figure 2
Ring and Ball Softening Point

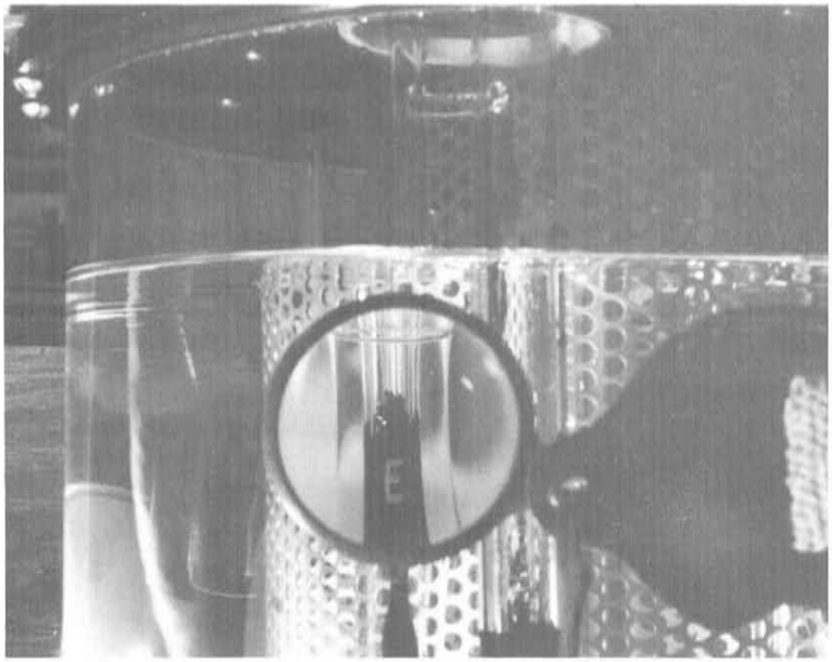

Figure 3
Absolute Viscosity

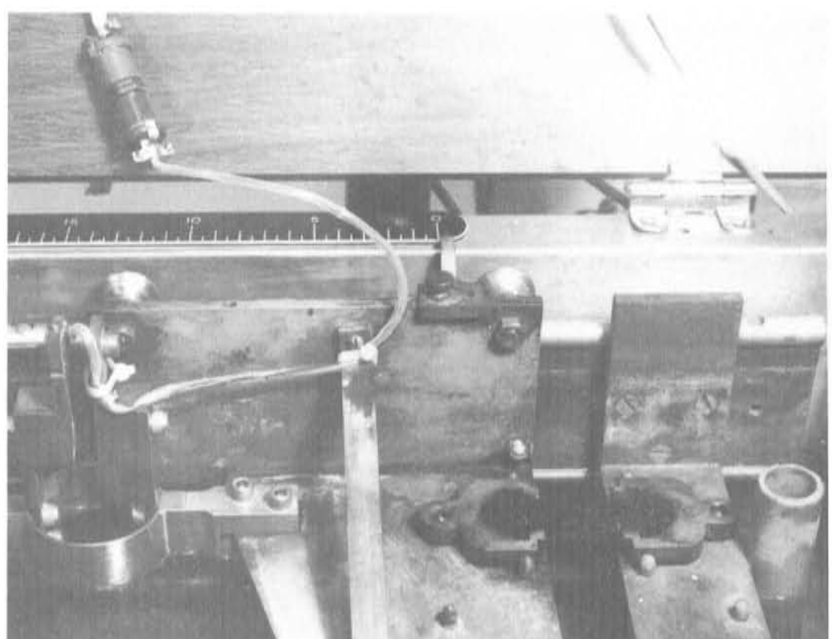

Figure 4
Force Ring and Specimen

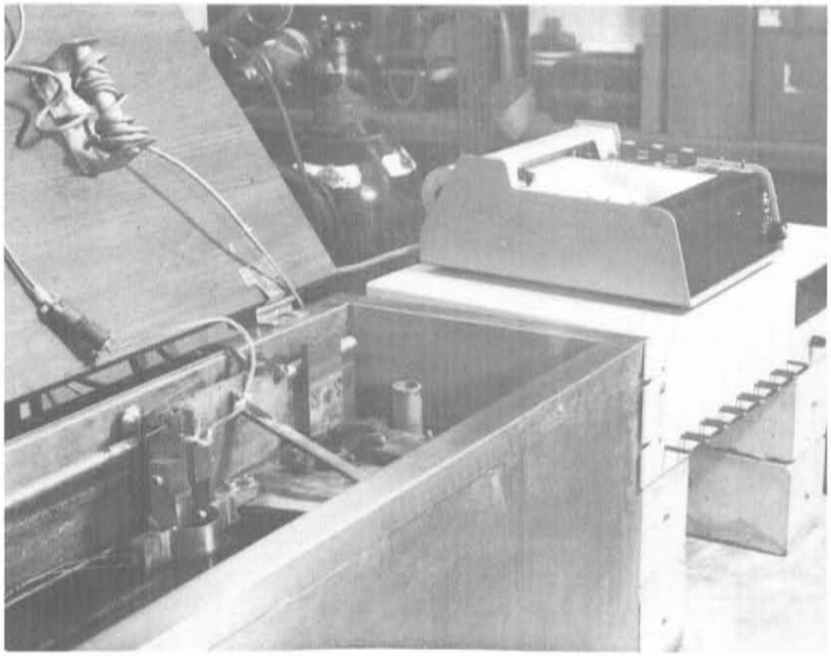

Figure 5
Force-Ductility Set-up

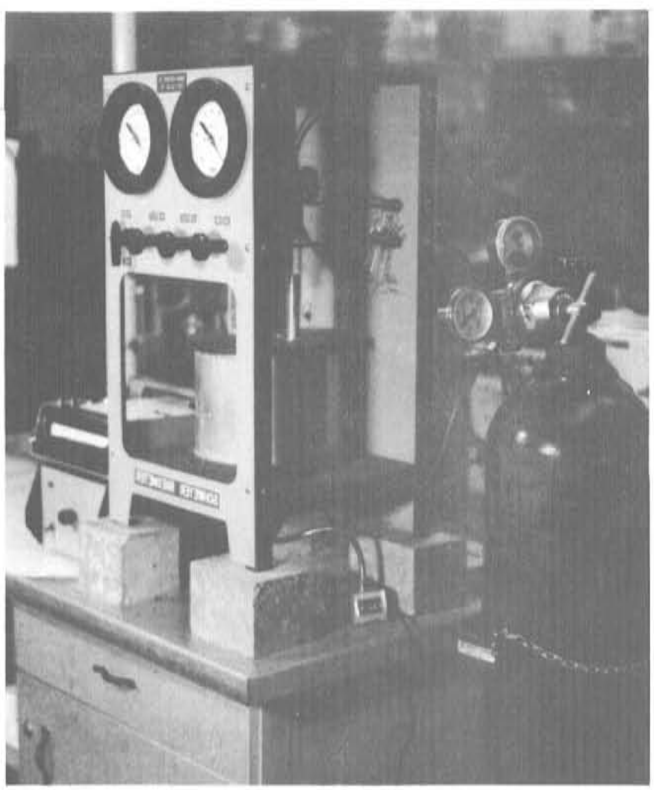

Figure 6
Schweyer Rheometer

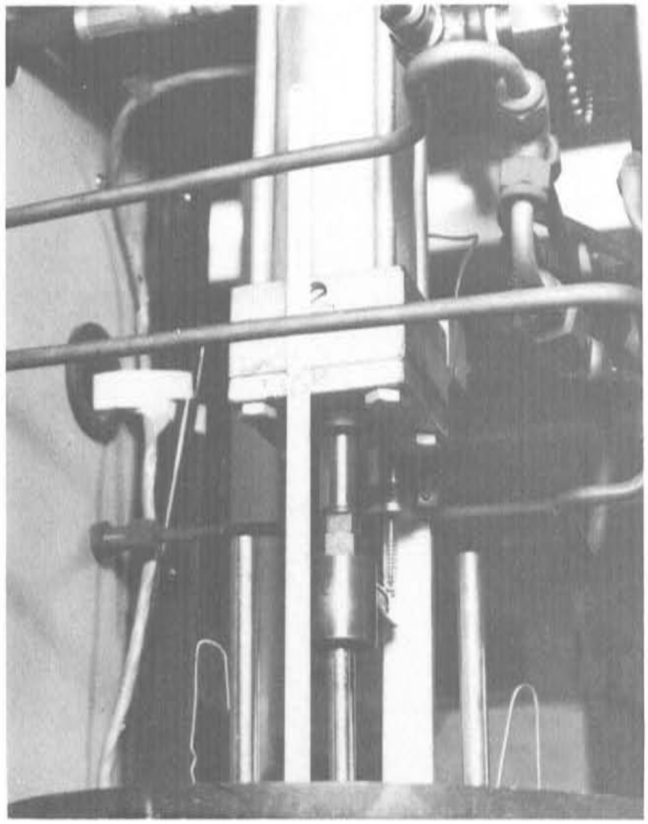

Figure 7
Rheometer Load Piston

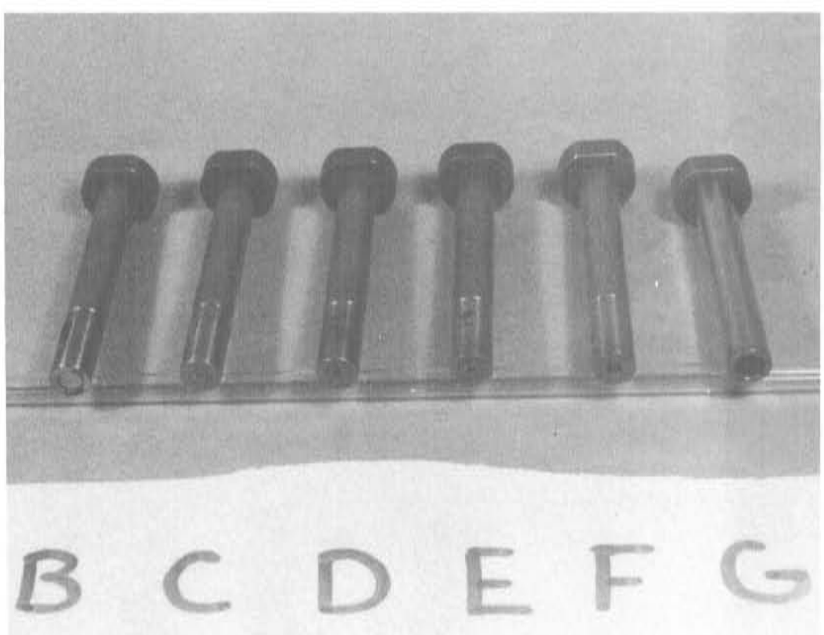

Figure 8
Rheometer Tubes

Table A-1
SOFTENING POINT
All Data

DATA MATRIX

TIME		TEMPERATURE 350		375		400		Totals
0.5		161 163	162.0 2.0	162 161	161.5 1.0	158 155	156.5 3.0	1888
		148 151	149.4 3.0	155 154	154.5 1.0	160 160	160.0 0	
1.0		155 156	155.5 1.0	160 159	159.5 1.0	162 160	161.0 2.0	1926
		159 158	158.5 1.0	163 164	163.5 1.0	164 166	165.0 2.0	
2.0		152 154	153.0 2.0	162 163	162.5 1.0	161 164	162.5 3.0	1878
		159 161	160.0 2.0	156 155	155.5 1.0	146 145	145.5 1.0	
Totals		1877		1914		1901		

Left Ball Right Ball	162 163	162.0 2.0	Mean Range

Range: 145.5 - 165.0

CELL MEANS AND RANGE
(Based on Replicate Means)

TIME	TEMPERATURE 350	375	400
0.5	155.75 12.5	158.00 7.0	158.25 3.5
1.0	157.00 3.0	161.50 4.0	163.00 4.0
2.0	156.50 7.0	159.00 7.0	154.00 17.0

Mean Range	155.75 12.5

TABLE A-2
ANOVA
SOFTENING POINT
All Data

FOSTER & BURR q:

Matrix Σx_i 5692

Matrix Σx_i^2 900,892

ν 3.00

q 0.269 $\quad q_{0.99}$ 0.287 $\quad q_{0.999}$ 0.363

$$Y_{ijk} = \mu + T_i + H_j + (TH)_{ij} + \epsilon_{(ij)k}$$

TOTALS

Temperature (T_i)

$T_{1..}$ = 1877
$T_{2..}$ = 1914
$T_{3..}$ = 1901
5692

Time (H_j)

$T_{.1.}$ = 1888
$T_{.2.}$ = 1926
$T_{.3.}$ = 1878
5692

Temperature x Time $(TH)_{ij}$

$T_{11.}$ = 623
$T_{12.}$ = 628
$T_{13.}$ = 626

$T_{21.}$ = 632
$T_{22.}$ = 646
$T_{23.}$ = 636

$T_{31.}$ = 633
$T_{32.}$ = 652
$T_{33.}$ = 616
5692

$T_{...}$ = 5692

$\Sigma\Sigma\Sigma Y_{ijk}$ = ________

TABLE A-3
ANOVA
SOFTENING POINT
All Data

(0) $CT = (T...)^2 \div n = (5692)^2 \div (36) = 8.999684444E + 05$

(1) $SS\ (Temp.) = \frac{1}{(12)} [(1877)^2 + \dots + (1901)^2] - CT = 58.72$

(2) $SS\ (Time) = \frac{1}{(12)} [(1888)^2 + \dots + (1878)^2] - CT = 106.89$

(3) $SS\ (H \times T) = \frac{1}{(4)} [(623)^2 + \dots + (616)^2] - CT - SS\ (T) - SS\ (H)$

$= 84.44$

(4) $SS\ (Total) = \Sigma\Sigma\Sigma x_{ijk}^2 - CT = 923.56$

(5) $SS\ (Error) = SS\ (Total) - SS\ (T) - SS\ (H) - SS\ (T \times H)$

$= (4) - (1) - (2) - (3) = 673.50$

ANALYSIS OF VARIANCE

	Source	df	SS	MS	F	F.05	F.01
(1)	T_i	2	58.72	29.36	1.18	3.35	
(2)	H_j	2	106.89	53.44	2.14	3.35	
(3)	$(TH)_{ij}$	4	84.44	21.11	0.85	2.73	
(5)	Error	27	673.50	24.94			
(4)	Total	35	923.56				

TABLE A-4

SOFTENING POINT
Mean of Runs

DATA MATRIX

		TEMPERATURE 350	375	400	Totals
TIME	0.5	162.0	161.5	156.5	944.0
		149.5	154.5	160.0	
	1	155.5	159.5	161.0	963.0
		158.5	163.5	165.0	
	2	153.0	162.5	162.5	939.0
		160.0	155.5	145.5	
Totals		938.5	957.0	950.5	2846.0

TABLE A-5
ANOVA
SOFTENING POINT
Mean of Runs

FOSTER & BURR q:

Matrix Σx_i = 2846.0

Matrix Σx_i^2 = 450,432.0

ν = 1.00

q = 0.278 $\quad q_{0.99}$ = 0.576 $\quad q_{0.999}$ = 0.750

$Y_{ijk} = \mu + T_i + H_j + (TH)_{ij} + \epsilon_{(ij)k}$

TOTALS

Temperature (T_i)

$T_{1..}$ = 938.5
$T_{2..}$ = 957.0
$T_{3..}$ = 950.5
2846.0

Time (H_j)

$T_{.1.}$ = 944.0
$T_{.2.}$ = 963.0
$T_{.3.}$ = 939.0
2846.0

Temperature x Time $(TH)_{ij}$

$T_{11.}$ = 311.5
$T_{12.}$ = 314.0
$T_{13.}$ = 313.0

$T_{21.}$ = 316.0
$T_{22.}$ = 323.0
$T_{23.}$ = 318.0

$T_{31.}$ = 316.5
$T_{32.}$ = 326.0
$T_{33.}$ = 308.0

2846.0

$T_{...}$ = 2846.0

$\Sigma\Sigma\Sigma Y_{ijk}$ = ______

TABLE A-6
ANOVA
SOFTENING POINT
Mean of Runs

(0) $CT = (T_{...})^2 \div n = (2846)^2 \div (18) =$ 4.499842222E + 05

(1) $SS\ (Temp.) = \frac{1}{(6)}\,[(938.5)^2 + \dots + (950.5)^2] - CT =$ 29.36

(2) $SS\ (Time) = \frac{1}{(6)}\,[(944.0)^2 + \dots + (939.0)^2] - CT =$ 53.44

(3) $SS\ (H \times T) = \frac{1}{(2)}\,[(311.5)^2 + \dots + (308.0)^2] - CT - SS\ (T) - SS\ (H)$

$=$ 42.22

(4) $SS\ (Total) = \Sigma\Sigma\Sigma x_{ijk}^2 - CT =$ 447.78

(5) SS (Error) = SS (Total) - SS (T) - SS (H) - SS (T x H)

= (4) - (1) - (2) - (3) = 322.75

ANALYSIS OF VARIANCE

	Source	df	SS	MS	F	F.05	F.01
(1)	T_i	2	29.36	14.68	0.41	4.26	
(2)	H_j	2	53.44	26.72	0.75	4.26	
(3)	$(TH)_{ij}$	4	42.22	10.56	0.29	3.63	
(5)	Error	9	322.75	35.86			
(4)	Total	17	447.78				

SOFTENING POINT
All Data
Cell Means

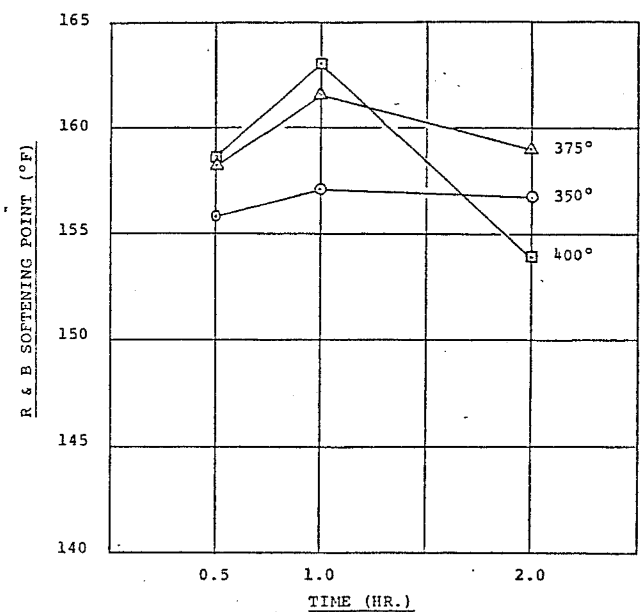

FIGURE A-1
R & B SOFT. VS TIME

SOFTENING POINT
All Data
Cell Means

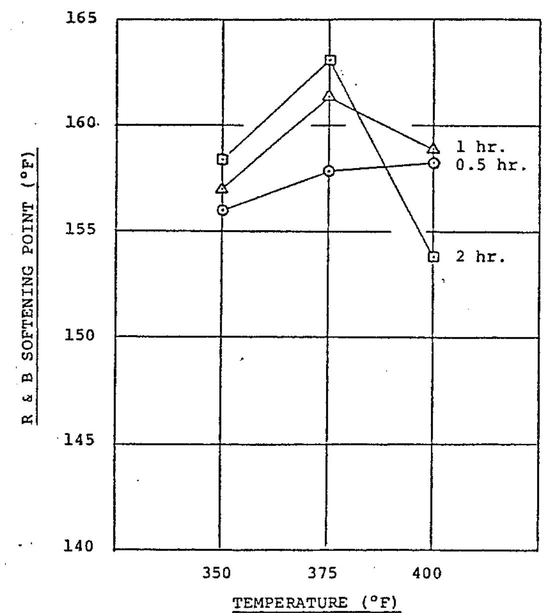

FIGURE A-2
R & B SOFT. VS TEMP.

TABLE B-1

ABSOLUTE VISCOSITY (140F)
All Data

DATA MATRIX

TIME		TEMPERATURE 350		375		400		Totals
0.5		91,257	85,829	138,695	115,924	118,100	89,400	1,055,036
		80,402		93,152		60,700		
		45,934	36,755	54,305	45,614	192,062	153,996	
		27,577		36,922		115,930		
1.0		67,955	53,782	80,275	91,858	321,130	318,555	1,875,258
		39,609		103,441		315,980		
		119,580	95,025	210,284	202,713	202,898	175,697	
		70,469		195,142		148,495		
2.0		133,217	123,054	228,988	229,300	477,016	459,450	2,335,303
		112,890		229,612		441,883		
		228,928	199,298	127,319	107,607	44,438	48,945	
		169,667		87,894		53,451		
Totals		1,187,485		1,586,029		2,492,083		5,265,597

Upper	91,257	85,829	Mean
Lower	80,402		

Range: 477,016 - 27,577

CELL MEANS

TIME	TEMPERATURE 350	375	400
0.5	61,292	80,768	121,698
1.0	74,403	147,286	247,126
2.0	161,176	168,453	254,197

TABLE B-2
ANOVA
ABSOLUTE VISCOSITY (140F)
All Data

FOSTER & BURR q:

Matrix Σx_i 5.265597000E + 06

Matrix Σx_i^2 1.177345470E + 12

ν 3.00

q 0.483 $q_{0.99}$ 0.287 $q_{0.999}$ 0.363

$$Y_{ijk} = \mu + T_i + H_j + (TH)_{ij} + \varepsilon_{(ij)k}$$

TOTALS

Temperature (T_i)

$T_{1..}$ = ______
$T_{2..}$ = ______
$T_{3..}$ = ______
______

Time (H_j)

$T_{.1.}$ = ______
$T_{.2.}$ = ______
$T_{.3.}$ = ______
______

Temperature x Time $(TH)_{ij}$

$T_{11.}$ = ______
$T_{12.}$ = ______
$T_{13.}$ = ______

$T_{21.}$ = ______
$T_{22.}$ = ______
$T_{23.}$ = ______

$T_{31.}$ = ______
$T_{32.}$ = ______
$T_{33.}$ = ______
______

$T_{...}$ = ______

$\Sigma\Sigma\Sigma Y_{ijk}$ = ______

TABLE B-3

LOG_{10} ABSOLUTE VISCOSITY (140F)

All Data

TRANSFORMED DATA MATRIX (LOG_{10})

TIME		TEMPERATURE 350	375	400	Totals
0.5		4.960	5.142	5.072	
		4.905	4.969	4.783	58.583
		4.662	4.735	5.283	
		4.441	4.567	5.064	
1.0		4.832	4.905	5.507	
		4.598	5.015	5.500	61.375
		5.078	5.323	5.307	
		4.848	5.290	5.172	
2.0		5.125	5.360	5.679	
		5.053	5.361	5.645	62.238
		5.360	5.105	4.648	
		5.230	4.944	4.728	
Totals		59.092	60.716	62.388	182.196

CELL MEANS

TIME	TEMPERATURE 350	375	400	
0.5	4.742	4.853	5.051	4.882
1.0	4.839	5.133	5.372	5.116
2.0	5.192	5.193	5.175	5.187
	4.924	5.060	5.199	5.061

TABLE B-4
ANOVA
ABSOLUTE VISCOSITY (140F)
All Data (Log_{10} Transform)

FOSTER & BURR q:

Matrix Σx_i 182.196

Matrix Σx_i^2 925.410

ν 3.0

q 0.277 $q_{0.99}$ 0.287 $q_{0.999}$ 0.363

$$Y_{ijk} = \mu + T_i + H_j + (TH)_{ij} + \epsilon_{(ij)k}$$

TOTALS

Temperature (T_i)

$T_{1..}$ = 59.092
$T_{2..}$ = 60.716
$T_{3..}$ = 62.388
182.196

Time (H_j)

$T_{.1.}$ = 58.513
$T_{.2.}$ = 61.375
$T_{.3.}$ = 62.238
182.196

Temperature x Time $(TH)_{ij}$

$T_{11.}$ = 18.968
$T_{12.}$ = 19.356
$T_{13.}$ = 20.768

$T_{21.}$ = 19.413
$T_{22.}$ = 20.533
$T_{23.}$ = 20.770

$T_{31.}$ = 20.202
$T_{32.}$ = 21.486
$T_{33.}$ = 20.700
182.196

$T_{...}$ = 182.196

$\Sigma\Sigma\Sigma Y_{ijk}$ = ______

TABLE B-5
ANOVA
LOG_{10} ABSOLUTE VISCOSITY
All Data

(0) $CT = (T...)^2 \div n = (182.196)^2 \div (36) = \underline{922.094}$

(1) $SS\ (Temp.) = \frac{1}{(12)} [(59.092)^2 + \cdots + (62.388)^2] - CT = \underline{0.453}$

(2) $SS\ (Time) = \frac{1}{(12)} [(58.583)^2 + \ldots + (62.238)^2] - CT = \underline{0.608}$

(3) $SS\ (H \times T) = \frac{1}{(4)} [(18.968)^2 + \ldots + (20.700)^2] - CT - SS\ (T) - SS\ (H)$

$= \underline{0.313}$

(4) $SS\ (Total) = \Sigma\Sigma\Sigma x_{ijk}^2 - CT = \underline{3.316}$

(5) $SS\ (Error) = SS\ (Total) - SS\ (T) - SS\ (H) - SS\ (T \times H)$

$= (4) - (1) - (2) - (3) = \underline{1.942}$

	ANALYSIS OF VARIANCE						
	Source	df	SS	MS	F	F.05	F.01
(1)	T_i	2	0.453	0.226	3.146	3.35	5.40
(2)	H_j	2	0.608	0.304	4.228	3.35	5.49
(3)	$(TH)_{ij}$	4	0.313	0.078	1.086	2.73	4.11
(5)	Error	27	1.942	0.072			
(4)	Total	35	3.316				

TABLE B-6
LOG_{10} ABSOLUTE VISCOSITY
All Data
Newman-Keuls

Cell Means: Table B-3

Ms (error): Table B-5

$$s_{\bar{y}ij} = \sqrt{\frac{Ms\ (error)}{n}} = \sqrt{\frac{0.072}{4}} = 0.1342$$

$R_k = q_{.05}(k, df)\ s_{\bar{y}ij}$; df = 27 [from Ms (error)]

ξ	(k, df)	R_k
9	4.764	0.639
8	4.643	0.623
7	4.503	0.604
6	4.338	0.582
5	4.134	0.555
4	3.873	0.520
3	3.509	0.471
2	2.904	0.390

(Appendix 6, Ref.(13))

TABLE B-7
LOG_{10} ABSOLUTE VISCOSITY
All Data
Newman-Keuls, Continued

RANK	1	2	3	4	5	6	7	8	9
(i, j)	3,2	2,3	1,3	3,3	2,2	3,1	2,1	1,2	1,1
Mean	5.372	5.193	5.192	5.175	5.133	5.051	4.853	4.839	4.742

RANK	LEAST SIGNIF. RANGE	K	K-1	K-2	K-3	K-4	K-5	K-6	K-7	K-8
		9	8	7	6	5	4	3	2	-
1	0.64	0.630	0.533	0.519	0.321	0.239	0.197	0.180	0.179	0
2	0.62	0.451	0.354	0.340	0.142	0.060	0.018	0.001	0	
3	0.60	0.450	0.353	0.339	0.141	0.059	0.017	0		
4	0.58	0.433	0.336	0.322	0.124	0.042	0			
5	0.56	0.391	0.294	0.280	0.082	0				
6	0.52	0.309	0.212	0.198	0					
7	0.47	0.111	0.014	0						
8	0.39	0.097	0							

No Significant Differences

LOG_{10} ABSOLUTE VISCOSITY
All Data, Cell Means

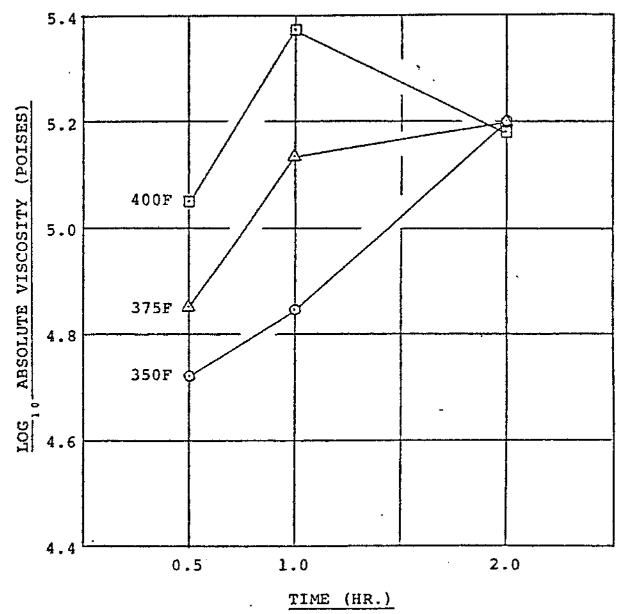

FIGURE B-1
LOG_{10} ABSOLUTE VISCOSITY
vs
TIME

LOG$_{10}$ ABSOLUTE VISCOSITY

All Data, Cell Means

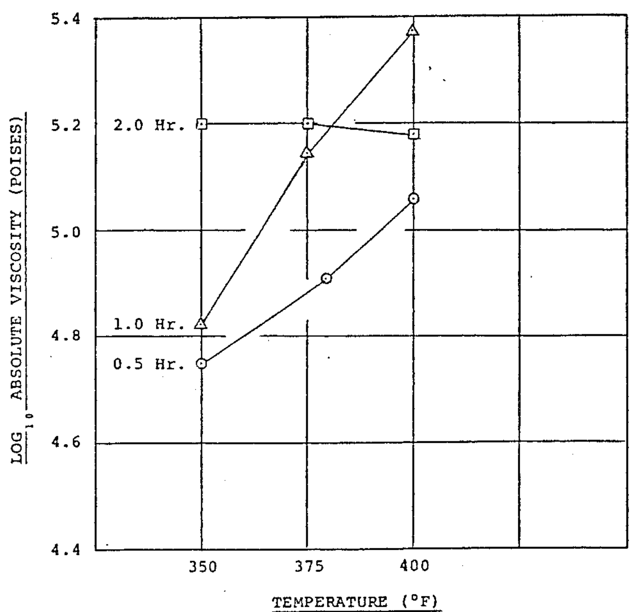

FIGURE B-2
LOG$_{10}$ ABSOLUTE VISCOSITY vs TEMPERATURE

TABLE B-8

LOG$_{10}$ ABSOLUTE VISCOSITY

Mean of Runs

DATA MATRIX: (Entries calculated from raw data matrix, Table B-1)

TIME	TEMPERATURE 350	375	400	Totals
0.5	4.934	5.064	4.951	29.361
	4.565	4.659	5.188	
1.0	4.731	4.963	5.503	30.727
	4.978	5.307	5.245	
2.0	5.090	5.360	5.662	31.134
	5.300	5.032	4.690	
Totals	29.598	30.385	31.239	91.222

CELL MEANS

TIME	TEMPERATURE 350	375	400
0.5	4.750	4.862	5.070
1.0	4.855	5.135	5.374
2.0	5.195	5.196	5.176

TABLE B-9
ANOVA
LOG_{10} ABSOLUTE VISCOSITY
Mean of Runs

FOSTER & BURR q:

Matrix Σx_i = 91.222

Matrix Σx_i^2 = 463.816

ν = 1.0

q = 0.339 $q_{0.99}$ = 0.576 $q_{0.999}$ = 0.750

$Y_{ijk} = \mu + T_i + H_j + (TH)_{ij} + \epsilon_{(ij)k}$

TOTALS

Temperature (T_i)

$T_{1..}$ = 29.598
$T_{2..}$ = 30.385
$T_{3..}$ = 31.239
91.222

Time (H_j)

$T_{.1.}$ = 29.361
$T_{.2.}$ = 30.727
$T_{.3.}$ = 31.134
91.222

Temperature x Time $(TH)_{ij}$

$T_{11.}$ = 9.499
$T_{12.}$ = 9.709
$T_{13.}$ = 10.390

$T_{21.}$ = 9.723
$T_{22.}$ = 10.270
$T_{23.}$ = 10.392

$T_{31.}$ = 10.139
$T_{32.}$ = 10.748
$T_{33.}$ = 10.352
91.222

$T_{...}$ = 91.222

$\Sigma\Sigma\Sigma Y_{ijk}$ = ________

TABLE B-10
ANOVA
LOG_{10} ABSOLUTE VISCOSITY
Mean of Runs

(0) $CT = (T_{...})^2 \div n = (91.222)^2 \div (18) =$ 462.303

(1) $SS\ (Temp.) = \frac{1}{(6)} [(29.598)^2 + \dots + (31.239)^2] - CT =$ 0.245

(2) $SS\ (Time) = \frac{1}{(6)} [(29.361)^2 + \dots + (31.134)^2] - CT =$ 0.308

(3) $SS\ (H \times T) = \frac{1}{(2)} [(9.499)^2 + \dots + (10.352)^2] - CT - SS\ (T) - SS\ (H)$

$=$ 0.132

(4) $SS\ (Total) = \Sigma\Sigma\Sigma x_{ijk}^2 - CT =$ 1.533

(5) SS (Error) = SS (Total) − SS (T) − SS (H) − SS (T x H)

= (4) − (1) − (2) − (3) = 0.849

ANALYSIS OF VARIANCE

	Source	df	SS	MS	F	F.05	F.01
(1)	T_i	2	0.245	0.122	1.297	4.26	
(2)	H_j	2	0.308	0.154	1.631	4.26	
(3)	$(TH)_{ij}$	4	0.132	0.033	0.34	3.63	
(5)	Error	9	0.849	0.094			
(4)	Total	17	1.533				

TABLE B-11

LOG_{10} ABSOLUTE VISCOSITY

Mean of Runs

Newman-Keuls

Cell Means: Table B-5; MS(error): Table B-10

$S_{\bar{y}ij} = \sqrt{0.094/2} = 0.2168$

ξ	(K, 9)	R_k	ξ	(K, 9)	R_k
9	5.595	1.21	5	4.756	1.03
8	5.432	1.18	4	4.415	0.96
7	5.244	1.14	3	3.949	0.86
6	5.024	1.09	2	3.199	0.69

RANK	1	2	3	4	5	6	7	8	9
(i,j)	3,2	2,3	1,3	3,3	2,2	3,1	2,1	1,2	1,1
Mean	5.372	5.196	5.195	5.176	5.135	5.070	4.862	4.855	4.750

RANK	LEAST SIGNIF. RANGE	K	K-1	K-2	K-3	K-4	K-5	K-6	K-7	K-8
		9	8	7	6	5	4	3	2	1
1	$1^{\underline{21}}$	0.624	0.519	0.512	0.304	0.239	0.198	0.179	0.178	0
2	$1^{\underline{18}}$	0.446	0.341	0.334	0.126	0.061	0.020	0.001	0	
3	$1^{\underline{14}}$	0.445	0.340	0.333	0.125	0.060	0.019	0		
4	$1^{\underline{09}}$	0.426	0.321	0.314	0.106	0.041	0			
5	$1^{\underline{03}}$	0.385	0.280	0.273	0.065	0				
6	$0^{\underline{96}}$	0.320	0.215	0.208	0					
7	$0^{\underline{86}}$	0.112	0.007	0						
8	$0^{\underline{69}}$	0.105	0							

No Significant Differences

TABLE C-1
DUCTILITY @ 39.3F (Cm.)
All Data

DATA MATRIX: (*Missing Data by ave.)

TIME		TEMPERATURE 350	375	400	Totals
	0.5	13.5	17.0	18.0	
		13.5	16.5	19.5	
		14.0	16.5	16.0	299.0
		12.0	19.5	20.0	
		16.0	17.0	18.5	
		14.0	18.0	19.5	
	1.0	16.5	17.0	14.5	
		17.0	18.0	15.5	
		15.0	15.5	18.0	298.0
		14.5	17.5	17.0	
		16.5	16.5	17.0	
		*15.5	18.0	18.5	
	2.0	14.5	17.0	18.5	
		16.5	18.0	15.0	
		16.5	15.5	16.0	319.0
		16.0	20.5	22.0	
		14.0	20.0	24.0	
		16.0	19.0	20.0	
Totals		271.5	317.0	327.5	916.0

CELL MEANS

TIME	TEMPERATURE 350	375	400	
0.5	13.83	17.42	18.58	16.61
1.0	15.83	17.08	16.75	16.55
2.0	15.58	18.33	19.25	17.72
	15.08	17.61	18.19	16.96

TABLE C-2
ANOVA
DUCTILITY @ 39.2F (Cm.)
All Data

FOSTER & BURR q:

Matrix Σx_i 916.0

Matrix Σx_i^2 15,806.00

ν 5.0

q 0.256 $q_{0.99}$ 0.217 $q_{0.999}$ 0.261

$$Y_{ijk} = \mu + T_i + H_j + (TH)_{ij} + \epsilon_{(ij)k}$$

TOTALS

Temperature (T_i)

$T_{1..}$ = ______
$T_{2..}$ = ______
$T_{3..}$ = ______
______

Time (H_j)

$T_{.1.}$ = ______
$T_{.2.}$ = ______
$T_{.3.}$ = ______
______

Temperature x Time $(TH)_{ij}$

$T_{11.}$ = ______
$T_{12.}$ = ______
$T_{13.}$ = ______

$T_{21.}$ = ______
$T_{22.}$ = ______
$T_{23.}$ = ______

$T_{31.}$ = ______
$T_{32.}$ = ______
$T_{33.}$ = ______
______

$T_{...}$ = ______

$\Sigma\Sigma\Sigma Y_{ijk}$ = ______

TABLE C-3

LOG_{10} DUCTILITY @ 39.2F (Cm.)

All Data

DATA MATRIX:

TIME		350	375	400	Totals
		TEMPERATURE			
TIME	0.5	1.1303	1.2304	1.2553	
		1.1303	1.2175	1.2900	
		1.1461	1.2175	1.2041	21.8848
		1.0792	1.2900	1.3010	
		1.2041	1.2304	1.2672	
		1.1461	1.2553	1.2900	
	1.0	1.2175	1.2304	1.1614	
		1.2304	1.2553	1.1903	
		1.1761	1.1903	1.2553	21.9200
		1.1614	1.2430	1.2304	
		1.2175	1.2175	1.2304	
		1.1903	1.2553	1.2672	
	2.0	1.1614	1.2304	1.2672	
		1.2175	1.2553	1.1761	
		1.2175	1.1903	1.2041	22.3893
		1.2041	1.3118	1.3424	
		1.1461	1.3010	1.3802	
		1.2041	1.2788	1.3010	
Totals		21.1800	22.4005	22.6136	66.1941

CELL MEANS

TIME		350	375	400	
		TEMPERATURE			
TIME	0.5	1.1394	1.2402	1.2679	1.2158
	1.0	1.1989	1.2320	1.2225	1.2178
	2.0	1.1918	1.2613	1.2785	1.2439
		1.1767	1.2445	1.2563	1.2258

Table C-4
ANOVA
LOG_{10} DUCTILITY @ 39.2F (Cm.)
All Data

FOSTER & BURR q:

Matrix Σx_i 66.1941

Matrix Σx_i^2 81.3146

ν 5.0

q 0.207 $q_{0.99}$ 0.217 $q_{0.999}$ 0.261

$$Y_{ijk} = \mu + T_i + H_j + (TH)_{ij} + \varepsilon_{(ij)k}$$

TOTALS

Temperature (T_i)

$T_{1..}$ = 21.1800
$T_{2..}$ = 22.4005
$T_{3..}$ = 22.6136
66.1941

Time (H_j)

$T_{.1.}$ = 21.8848
$T_{.2.}$ = 21.9200
$T_{.3.}$ = 22.3893
66.1941

Temperature x Time $(TH)_{ij}$

$T_{11.}$ = 6.8361
$T_{12.}$ = 7.1932
$T_{13.}$ = 7.1507

$T_{21.}$ = 7.4411
$T_{22.}$ = 7.3918
$T_{23.}$ = 7.5676

$T_{31.}$ = 7.6076
$T_{32.}$ = 7.3350
$T_{33.}$ = 7.6710
66.1941

$T_{...}$ = 66.1941

$\Sigma\Sigma\Sigma Y_{ijk}$ = ______

TABLE C-5

ANOVA

LOG_{10} DUCTILITY @ 39.2F (Cm.)

All Data

(0) $CT = (T...)^2 \div n = (66.1941)^2 \div (54) = $ 8.114183102E + 01

(1) $SS\ (Temp.) = \frac{1}{(18)} [(21.1800)^2 + \cdots + (22.6136)^2] - CT = $ 0.0665

(2) $SS\ (Time) = \frac{1}{(18)} [(21.8848)^2 + \ldots + (22.3893)^2] - CT = $ 0.0088

(3) $SS\ (H \times T) = \frac{1}{(6)} [(6.8361)^2 + \ldots + (7.6710)^2] - CT - SS\ (T) - SS\ (H)$

$= $ 0.0172

(4) $SS\ (Total) = \Sigma\Sigma\Sigma x_{ijk}^2 - CT = $ 0.1728

(5) SS (Error) = SS (Total) - SS (T) - SS (H) - SS (T x H)

= (4) - (1) - (2) - (3) = 0.0802

ANALYSIS OF VARIANCE

	Source	df	SS	MS	F	F.05	F.01
(1)	T_i	2	0.0665	0.0332	18.6	3.21	5.13
(2)	H_j	2	0.0088	0.0044	2.47	3.21	5.13
(3)	$(TH)_{ij}$	4	0.0172	0.0043	2.42	2.59	3.79
(5)	Error	45	0.0802	0.0018			
(4)	Total	53					

TABLE C-6

LOG_{10} DUCTILITY @ 39.2F (Cm.)

All Data

Newman-Keuls

Cell Means: Table C-3

Ms(Error): Table C-5

$$s_{\bar{y}ij} = \sqrt{\frac{Ms(Error)}{6}} = \sqrt{\frac{0.018}{6}} = 0.0548$$

k	k, 45	R_k	(Apprx. 6, A & McL)
9	4.614	0.252	df = 45
8	4.501	0.246	
7	4.370	0.239	
6	4.215	0.231	
5	4.024	0.220	
4	3.778	0.207	
3	3.431	0.188	
2	2.851	0.156	

TABLE C-7

LOG_{10} DUCTILITY @ 39.2 (1 cm.)

All Data

Newman-Keuls, Continued

RANK	1	2	3	4	5	6	7	8	9
(i,j)	3,3	3,1	2,3	2,1	2,2	3,2	1,2	1,3	1,1
Mean	1.2785	1.2679	1.2613	1.2402	1.2320	1.2225	1.1989	1.1918	1.1394

Rank	LSR	k	k-1	k-2	k-3	k-4	k-5	k-6	k-7	k-8
		9	8	7	6	5	4	3	2	1
1	0.25	0.139	0.087	0.080	0.056	0.047	0.038	0.017	0.011	0
2	0.25	0.129	0.076	0.069	0.045	0.036	0.028	0.007	0	
3	0.24	0.122	0.070	0.062	0.039	0.029	0.021	0		
4	0.23	0.101	0.048	0.041	0.018	0.008	0			
5	0.22	0.093	0.040	0.033	0.010	0				
6	0.21	0.083	0.031	0.024	0					
7	0.19	0.060	0.007	0						
8	0.16	0.052	0							

No Significant Differences

TABLE C-8

LOG_{10} DUCTILITY @ 39.2 (1 cm.)

TEMPERATURE

NEWMAN - KEULS

One-Way Anova of Temp. (q = 0.396 < q crit.)

ANALYSIS OF VARIANCE						
Source	df	SS	MS	F	F.05	F.01
Treat.	2	0.0665	0.0332	16.09	3.19	5.07
Error	51	0.1053	0.0021			
Total	53	0.1718	-			

$S\bar{y} = \sqrt{\frac{0.0021}{18}} = 0.0108$

RANK	1	2	3
(i)	3	2	1
Mean	1.2563	1.2445	1.1767

Rank	LSR*	k	k-1
		3	2
1	0.0369	0.0796	0.0118
2	0.0307	0.0678	-

*Least Significant Range

D_{400} greater than D_{350}

D_{375} greater than D_{350}

LOG_{10} DUCTILITY @ 39.3F (Cm.)

All Data, Cell Means

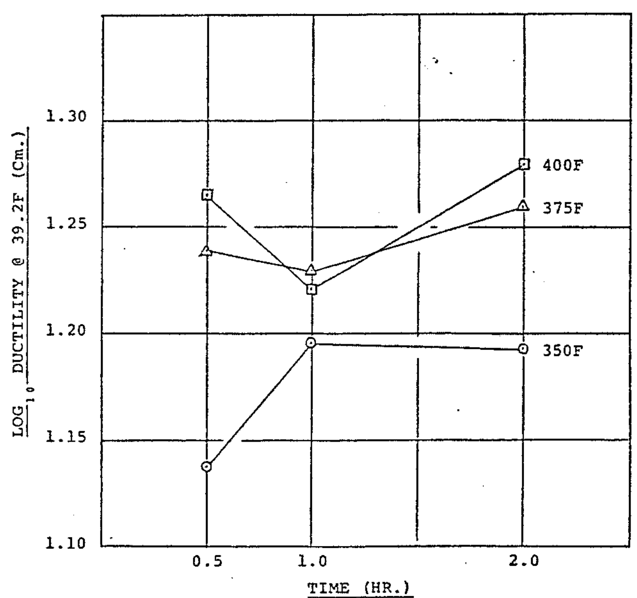

FIGURE C-1
LOG_{10} DUCTILITY @ 39.2F vs TIME

LOG_{10} DUCTILITY @ 39.2F (Cm.)

All Data, Cell Means

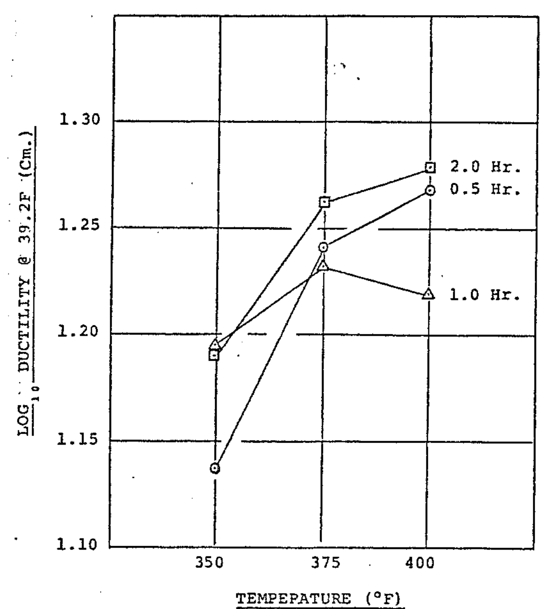

FIGURE C-2
LOG_{10} DUCTILITY @ 39.2F vs TEMPERATURE

TABLE C-9

DUCTILITY @ 39.2F (Cm.)

Means of Runs

DATA MATRIX: (Each entry is mean of three specimens.)

TIME		TEMPERATURE 350	375	400	Totals
	0.5	13.67	16.67	17.83	99.67
		14.00	18.17	19.33	
	1.0	16.17	16.83	16.00	99.33
		15.50	17.33	17.50	
	2.0	15.83	16.83	16.50	106.32
		15.33	19.83	22.00	
Totals		90.50	105.66	109.16	305.32

CELL MEANS

TIME	TEMPERATURE 350	375	400
0.5	13.84	17.42	18.58
1.0	15.84	17.08	16.75
2.0	15.58	18.33	19.25

Table C-10

ANOVA

DUCTILITY @ 39.2F (Cm.)

Means of Runs

FOSTER & BURR q:

Matrix Σx_i 305.32

Matrix Σx_i^2 5248.337

ν 1.0

q 0.457 $q_{0.99}$ 0.576 $q_{0.999}$ 0.750

$Y_{ijk} = \mu + T_i + H_j + (TH)_{ij} + \epsilon_{(ij)k}$

TOTALS

Temperature (T_i)

$T_{1..}$ = 90.50
$T_{2..}$ = 105.66
$T_{3..}$ = 109.16
305.32

Time (H_j)

$T_{.1.}$ = 99.67
$T_{.2.}$ = 99.33
$T_{.3.}$ = 106.32
305.32

Temperature x Time $(TH)_{ij}$

$T_{11.}$ = 27.67
$T_{12.}$ = 31.67
$T_{13.}$ = 31.16

$T_{21.}$ = 34.84
$T_{22.}$ = 34.16
$T_{23.}$ = 36.66

$T_{31.}$ = 37.16
$T_{32.}$ = 33.50
$T_{33.}$ = 38.50
305.32

$T_{...}$ = 305.32

$\Sigma\Sigma\Sigma Y_{ijk}$ = ______

TABLE C-11
ANOVA
DUCTILITY @ 39.2F
Mean of Runs

(0) CT = $(T...)^2 \div n = (305.32)^2 \div (\ 18\ )$ = 5.178905689E + 03

(1) SS (Temp.) = $\frac{1}{(6)}$ [(90.50)² + ... + (109.16)²] - CT = 32.79

(2) SS (Time) = $\frac{1}{(6)}$ [(99.67)² + ... + (106.32)²] - CT = 5.18

(3) SS (T x H) = $\frac{1}{(2)}$ [(27.67)² + ... + (38.50)²] - CT - SS (T) - SS (H)

= 7.93

(4) SS (Total) = $\Sigma\Sigma\Sigma x_{ijk}^2$ - CT = 69.43

(5) SS (Error) = SS (Total) - SS (T) - SS (H) - SS (T x H)

= (4) - (1) - (2) - (3) = 23.53

ANALYSIS OF VARIANCE

	Source	df	SS	MS	F	F.05	F.01
(1)	T_i	2	32.79	16.40	6.27	4.26	8.02
(2)	H_j	2	5.18	2.59	0.99	4.26	8.02
(3)	$(TH)_{ij}$	4	7.93	1.98	0.76	3.63	6.42
(5)	Error	9	23.53	2.61			
(4)	Total	17	69.43				

TABLE C-12
DUCTILITY @ 39.2F (Cm.)
Means of Runs
Newman-Keuls

Cell Means: Table C-9; Ms(Error): Table C-11

$S_{\bar{y}ij} = \sqrt{2.61/2} = 1.14$

ξ	(k, df)	R_{k-}	ξ	(k, df)	R_{k-}
9	5.595	6.39	5	4.756	5.43
8	5.432	6.21	4	4.415	5.04
7	5.244	5.99	3	3.949	4.51
6	5.024	5.74	2	3.199	3.65

RANK	1	2	3	4	5	6	7	8	9
(i,j)	3,3	3,1	2,3	2,1	2,2	3,2	1,2	1,3	1,1
Mean	19.25	18.58	18.33	17.42	17.08	16.75	15.84	15.58	13.84

Rank	LSR	k	k-1	k-2	k-3	k-4	k-5	k-6	k-7	k-8
		9	8	7	6	5	4	3	2	1
1	6.39	5.41	3.67	3.41	2.50	2.17	1.83	0.92	0.67	0
2	6.21	4.74	3.00	2.74	1.83	1.50	1.16	0.25	0	
3	5.99	4.49	2.75	2.49	1.58	1.25	0.91	0		
4	5.74	3.58	1.84	1.58	0.67	0.34	0			
5	5.43	3.24	1.50	1.24	0.33	0				
6	5.04	2.91	1.17	0.91	0					
7	4.51	2.00	0.26	0						
8	3.65	1.74	0							

TABLE D-1

DUCTILITY AT 77F

Data Matrix

Time		Temperature 350	Temperature 375	Temperature 400
Time	0.5	8.5 10 18	15 13 13	17 18.5
	1.0	14 15 15.5	18 14 16	16 21
	2.0	15 17 18	16.5 17 16	18.5 20

Cell Means

Time		Temperature 350	Temperature 375	Temperature 400
Time	0.5	12.2	13.7	17.8
	1.0	14.8	16.0	18.5
	2.0	16.7	16.5	19.2

TABLE D-2

DUCTILITY AT 77F

ANALYSIS OF VARIANCE						
Source	df	SS	MS	F	F.05	F.01
Treat.	8	105.82	13.23	2.36	2.64	4.00
Error	15	83.92	5.59			
Total	23	189.74	-			

TABLE E-1

MAX. DUCTILITY LOAD, 39.2F, (N)

All Data

DATA MATRIX

TIME		TEMPERATURE 350	375	400	Totals
0.5		53.44	47.10	42.71	
		52.95	46.12	48.56	
		51.49	48.56	44.17	780.92
		34.90	40.76	46.12	
		28.56	43.19	46.12	
		26.61	39.78	39.78	
1.0		38.32	55.39	55.27	
		43.19	56.36	54.41	
		42.71	51.00	51.00	907.61
		51.00	59.29	49.05	
		51.00	53.39	46.12	
		38.85	62.21	49.05	
2.0		51.00	53.44	56.36	
		46.61	40.76	48.07	
		59.29	51.47	53.44	776.54
		53.92	30.03	21.26	
		51.00	32.95	23.69	
		52.95	28.56	21.74	
Totals		827.79	840.36	796.92	2465.07

CELL MEANS

TIME	TEMPERATURE 350	375	400
0.5	41.33	44.25	44.58
1.0	42.51	56.27	50.82
2.0	52.46	39.54	37.43

ANOVA

MAX. DUCTILITY LOAD 39.2F, (N)

All Data

FOSTER & BURR q:

Matrix Σx_i 2477.22

Matrix Σx_i^2 118,821.694

ν 5.0

q 0.280 $q_{0.99}$ 0.217 $q_{0.999}$ 0.261

$$Y_{ijk} = \mu + T_i + H_j + (TH)_{ij} + \varepsilon_{(ij)k}$$

TOTALS

Temperature (T_i)

$T_{1..}$ = ______

$T_{2..}$ = ______

$T_{3..}$ = ______

Time (H_j)

$T_{.1.}$ = ______

$T_{.2.}$ = ______

$T_{.3.}$ = ______

Temperature x Time $(TH)_{ij}$

$T_{11.}$ = ______

$T_{12.}$ = ______

$T_{13.}$ = ______

$T_{21.}$ = ______

$T_{22.}$ = ______

$T_{23.}$ = ______

$T_{31.}$ = ______

$T_{32.}$ = ______

$T_{33.}$ = ______

$T_{...}$ = ______

$\Sigma\Sigma\Sigma Y_{ijk}$ = ______

TABLE E- 2

MAX. DUCTILITY LOAD, 39.2F (N)

$\sqrt{B} - \sqrt{B-x}$ TRANSFORM

All Data

DATA MATRIX

B = 62.21 = Max. Value
x = Measured Value

TIME		TEMPERATURE 350	375	400	Totals
0.5		4.926	4.000	3.471	
		4.844	3.876	4.193	
		4.613	4.193	3.640	65.260
		2.661	3.256	3.876	
		2.086	3.526	3.876	
		1.921	3.151	3.151	
1.0		3.000	5.276	5.253	
		3.526	5.469	5.094	
		3.471	4.539	4.539	83.678
		4.539	6.179	4.260	
		4.539	4.917	3.876	
		3.054	7.887	4.260	
2.0		4.539	4.926	5.469	
		3.938	3.256	4.127	
		6.179	4.610	4.926	67.835
		5.008	2.215	1.488	
		4.539	2.478	1.681	
		4.844	2.086	1.526	
Totals		72.227	75.840	68.706	216.773

CELL MEANS

TIME	TEMPERATURE 350	375	400
0.5	3.509	3.667	3.701
1.0	3.688	5.711	4.547
2.0	4.841	3.262	3.203

TABLE E-3

ANOVA (MAXIMUM DUCTILITY LOAD)

$\sqrt{B} - \sqrt{B-x}$ All Data

FOSTER & BURR q:

Matrix Σx_i 216.773

Matrix Σx_i^2 954.471485

ν 5.0

q 0.206 $q_{0.99}$ 0.217 $q_{0.999}$ 0.261

$$Y_{ijk} = \mu + T_i + H_j + (TH)_{ij} + \epsilon_{(ij)k}$$

TOTALS

Temperature (T_i)

$T_{1..}$ = 72.227
$T_{2..}$ = 75.840
$T_{3..}$ = 68.706
216.773

Time (H_j)

$T_{.1.}$ = 65.260
$T_{.2.}$ = 83.678
$T_{.3.}$ = 67.835
216.773

Temperature x Time $(TH)_{ij}$

$T_{11.}$ = 21.051
$T_{12.}$ = 22.129
$T_{13.}$ = 29.047

$T_{21.}$ = 22.002
$T_{22.}$ = 34.267
$T_{23.}$ = 19.571

$T_{31.}$ = 22.207
$T_{32.}$ = 27.282
$T_{33.}$ = 19.217

216.773

$T_{...}$ = 216.773

$\Sigma\Sigma\Sigma Y_{ijk}$ = ________

TABLE E-4

ANOVA (MAXIMUM DUCTILITY LOAD)

$\sqrt{B}$ $\sqrt{B-x}$ All Data

(0) CT = $(T...)^2 \div n = (216.773)^2 \div (\ 54\ ) =$ 8.701950654E + 02

(1) SS (Temp.) = $\frac{1}{(18)}$ $[(72.227)^2 + \cdots + (68.706)^2] - CT =$ 1.414

(2) SS (Time) = $\frac{1}{(18)}$ $[(65.260)^2 + \cdots + (67.835)^2] - CT =$ 11.053

(3) SS (H x T) = $\frac{1}{(6)}$ $[(21.051)^2 + \cdots + (19.217)^2] - CT - SS(T) - SS(H)$

= 21.448

(4) SS (Total) = $\Sigma\Sigma\Sigma x_{ijk}^2 - CT =$ 84.276

(5) SS (Error) = SS (Total) - SS (T) - SS (H) - SS (T x H)

= (4) - (1) - (2) - (3) = 50.362

	ANALYSIS OF VARIANCE						
	Source	df	SS	MS	F	F.05	F.01
(1)	T_i	2	1.414	0.707	0.63	3.21	5.13
(2)	H_j	2	11.053	5.526	4.94	3.21	5.13
(3)	$(TH)_{ij}$	4	21.448	5.362	4.79	2.59	3.79
(5)	Error	45	50.362	1.119			
(4)	Total	84	84.276				

MAX. DUCTILITY LOAD 39.2F (N)

$\sqrt{B}$ - $\sqrt{B-x}$ Transform (All Data)

B = 62.21 = Max., All Data; x = Max. Load

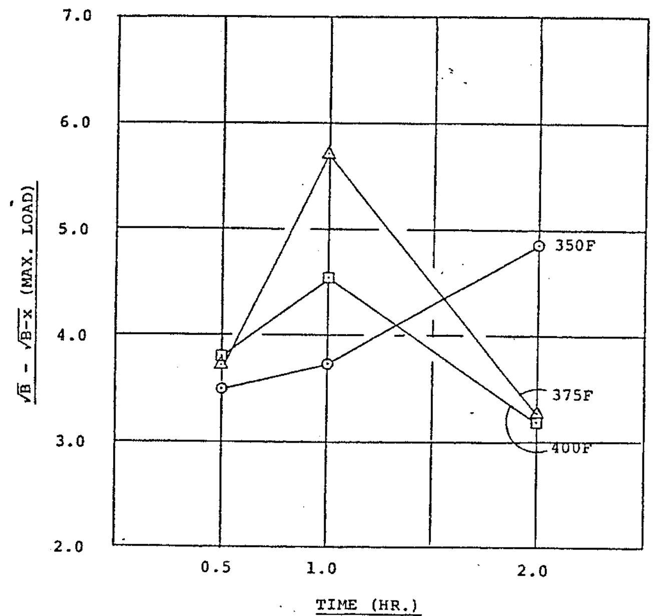

FIGURE E-1

MAX. DUCTILITY LOAD, 39.2F
$\sqrt{B}$ - $\sqrt{B-x}$ TRANSFORMATION
VS TIME

MAX. DUCTILITY LOAD, 39.2F (N)

$\sqrt{B} - \sqrt{B-x}$ Transform (All Data)

B = 62.21 = Max., All Data; x = Max. Load

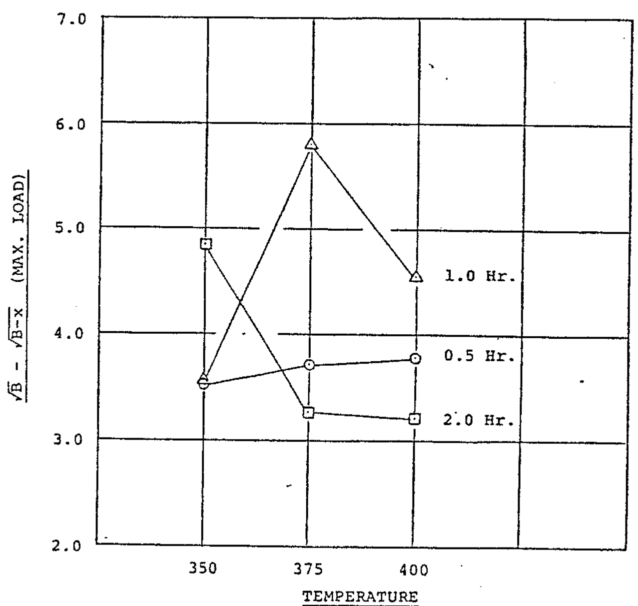

FIGURE E-2

MAX. DUCTILITY LOAD, 39.2F

$\sqrt{B} - \sqrt{B-x}$ Transformation

VS TIME

TABLE E-5

MAX. DUCTILITY LOAD, 39.2F (N)

$\sqrt{B} - \sqrt{B-x}$ Transform (All Data)

Newman-Keuls

Cell Means, E-1; Ms(Error), E-4

$S_{\bar{y}ij} = \sqrt{1.119/6} = 0.4319$

ξ	(k, df)	R_k	ξ	(k, df)	R_k
9	5.595	2.416	5	4.756	2.054
8	5.432	2.346	4	4.415	1.907
7	5.244	2.265	3	3.949	1.705
6	5.024	2.170	2	3.199	1.382

RANK	1	2	3	4	5	6	7	8	9
(i,j)	2,2	1,3	3,2	3,1	1,2	2,1	1,1	2,3	3,3
Mean	5.711	4.841	4.547	3.701	3.688	3.667	3.509	3.262	3.203

RANK	LEAST SIGNIF. RANGE	k 9	k-1 8	k-2 7	k-3 6	k-4 5	k-5 4	k-6 3	k-7 2	k-8 1
1	2.42	2.508	2.449	2.202	2.044	2.023	2.010	1.164	0.870	0
2	2.35	1.638	1.579	1.332	1.174	1.153	1.140	0.294	0	
3	2.26	1.344	1.285	1.038	0.880	0.859	1.846	0		
4	2.17	0.498	0.439	0.192	0.034	0.013	0			
5	2.05	0.485	0.426	0.179	0.021	0				
6	1.91	0.464	0.405	0.158	0					
7	1.70	0.306	0.247	0						
8	1.39	0.059	0							

Cell 2,2 Greater Than All Others.

TABLE F-1

FORCE - DUCTILITY ENERGY (N-M)

All Data

DATA MATRIX: (*Mean, missing data)

TIME		TEMPERATURE 350	375	400	Totals
	0.5	5.34	6.26	5.56	
		5.46	5.49	6.73	
		5.61	5.79	4.85	95.22
		3.49	6.04	6.14	
		3.59	5.32	5.72	
		2.98	5.34	5.51	
	1.0	4.64	6.95	5.73	
		5.48	7.23	5.97	
		4.75	5.53	7.16	111.94
		5.42	7.82	5.89	
		6.52	6.69	5.33	
		5.97*	8.22	6.64	
	2.0	5.62	6.39	7.12	
		5.82	4.57	5.02	
		7.48	6.89	5.59	95.70
		6.11	4.27	3.14	
		5.38	4.76	3.84	
		6.80	3.86	3.04	
Totals		96.46	107.42	98.98	302.86

CELL MEANS:

TIME	TEMPERATURE 350	375	400
0.5	4.412	5.707	5.752
1.0	5.463	7.073	6.120
2.0	6.202	5.123	4.625

TABLE F-2

ANOVA

FORCE - DUCTILITY ENERGY (N-M)

All Data

FOSTER & BURR q:

Matrix Σx_i 302.86

Matrix Σx_i^2 1773.512400

ν 5.0

q 0.174 $q_{0.99}$ 0.217 $q_{0.999}$ 0.261

$$Y_{ijk} = \mu + T_i + H_j + (TH)_{ij} + \epsilon_{(ij)k}$$

TOTALS

Temperature (T_i)

$T_{1..}$ = 96.46

$T_{2..}$ = 107.42

$T_{3..}$ = 98.98

302.86

Time (H_j)

$T_{.1.}$ = 95.22

$T_{.2.}$ = 111.94

$T_{.3.}$ = 95.70

302.86

Temperature x Time $(TH)_{ij}$

$T_{11.}$ = 26.47

$T_{12.}$ = 32.78

$T_{13.}$ = 37.21

$T_{21.}$ = 34.24

$T_{22.}$ = 42.44

$T_{23.}$ = 30.74

$T_{31.}$ = 34.51

$T_{32.}$ = 36.72

$T_{33.}$ = 27.75

302.86

$T_{...}$ = 302.86

$\Sigma\Sigma\Sigma Y_{ijk}$ = ______

TABLE F-3
ANOVA
FORCE - DUCTILITY ENERGY (N-M)
All Data

(0) $CT = (T...)^2 \div n = (302.86)^2 \div (54) =$ 1698.595919

(1) $SS\ (Temp.) = \frac{1}{(18)} [(96.46)^2 + \cdots + (98.98)^2] - CT =$ 3.661

(2) $SS\ (Time) = \frac{1}{(18)} [(95.22)^2 + \ldots + (95.70)^2] - CT =$ 10.065

(3) $SS\ (H \times T) = \frac{1}{(6)} [(26.47)^2 + \ldots + (27.75)^2] - CT - SS\ (T) - SS\ (H)$

$=$ 18.946

(4) $SS\ (Total) = \Sigma\Sigma\Sigma x_{ijk}^2 - CT =$ 74.916

(5) $SS\ (Error) = SS\ (Total) - SS\ (T) - SS\ (H) - SS\ (T \times H)$

$= (4) - (1) - (2) - (3) =$ 42.244

ANALYSIS OF VARIANCE

	Source	df	SS	MS	F	F.05	F.01
(1)	T_i	2	3.661	1.831	1.95	3.21	5.13
(2)	H_j	2	10.065	5.033	5.36	3.21	5.13
(3)	$(TH)_{ij}$	4	18.946	4.737	5.05	2.59	3.79
(5)	Error	45	42.244	0.939			
(4)	Total	53	74.916				

TABLE F-4
FORCE - DUCTILITY ENERGY (N-M)
ALL DATA
Newman-Keuls

Cell Means, F-1; Ms(Error), F-3

$S_{\bar{y}ij} = \sqrt{4.737/6} = 0.8885$

ξ	(k, df)	R_{k-}	ξ	(k, df)	R_{k-}
9	5.595	4.971	5	4.756	4.226
8	5.432	4.827	4	4.415	3.923
7	5.244	4.659	3	3.949	3.509
6	5.024	4.464	2	3.199	2.842

RANK	1	2	3	4	5	6	7	8	9
(i,j)	2,2	1,3	3,2	3,1	2,1	1,2	2,3	3,3	1,1
Mean	7.073	6.202	6.120	5.752	5.707	5.463	5.123	4.625	4.412

RANK	LEAST SIGNIF. RANGE	k	k-1	k-2	k-3	k-4	k-5	k-6	k-7	k-8
		9	8	7	6	5	4	3	2	1
1	4.97	2.661	2.448	1.950	1.610	1.366	1.321	0.953	0.871	0
2	4.83	1.790	1.577	1.079	0.739	0.495	0.450	0.082	0	
3	4.66	1.708	1.495	0.997	0.657	0.413	0.368	0		
4	4.46	1.340	1.127	0.629	0.289	0.045	0			
5	4.23	1.295	1.082	0.584	0.244	0				
6	3.92	1.051	0.838	0.340	0					
7	3.51	0.711	0.498	0						
8	2.84	0.213	0							

No Significant Differences

FORCE - DUCTILITY ENERGY (N-M)
All Data, Cell Means

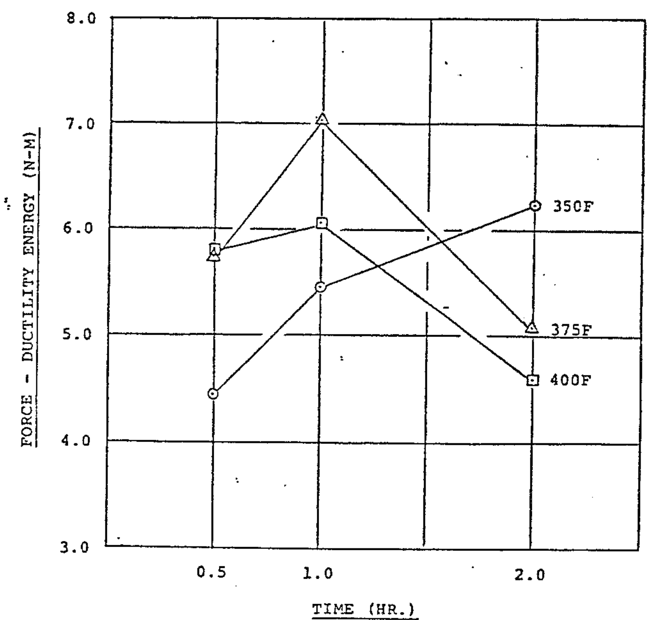

FIGURE F-1
FORCE - DUCTILITY ENERGY
VS
TIME

FORCE - DUCTILITY ENERGY (N-M)
All Data, Cell Means

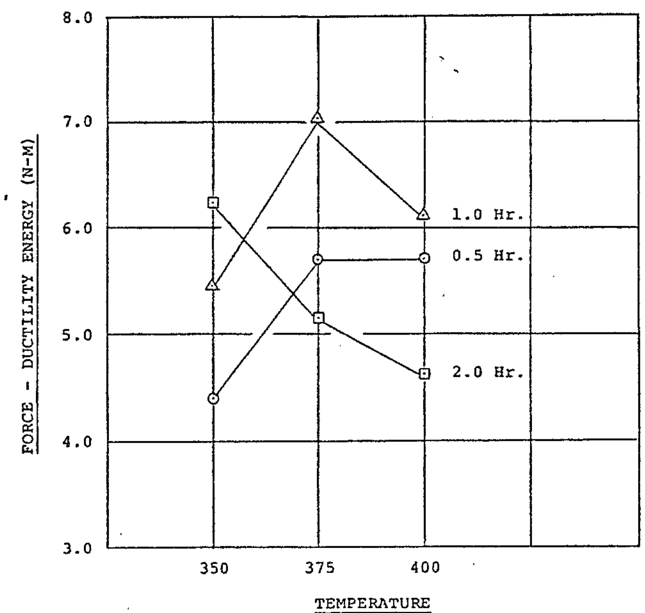

FIGURE F-2
FORCE - DUCTILITY ENERGY
VS
TEMPERATURE

TABLE F-5

FORCE-DUCTILITY ENERGY

HOLDING TIME

NEWMAN-KEULS

One-way Anova of Time (q = 0.365 Lq crit.)

ANALYSIS OF VARIANCE						
Source	df	SS	MS	F	F.05	F.01
Treat.	2	10.07	5.03	3.96	3.19	5.07
Error	51	64.85	1.27			
Total	53	74.92	-			

$S\bar{y} = \sqrt{\frac{1.27}{18}} = 0.2656$

RANK	1	2	3
(i)	2	1	3
Mean	6.22	5.32	5.29

Rank	LSR	k 3	k-1 2
1	0.908	0.93	0.90
2	0.755	0.03	0

No significant differences

FIGURE G-1
SCHWEYER RHEOMETER

IDENTIFICATION 22R

CHART NO. 54 TEMP. 4C TUBE 4G

(L), LVDT Chart Dsp., mm	4.2	9.0	13.4	17.0	30.0	
(T), Time Chart Dsp., mm	150.0	150.0	150.0	150.0	150.0	
(CS), Chart Speed, cm/min	1	1	1	1	1	
(P), Pressure, KPa	15	20	25	30	40	
τ Pa	1.1E04	1.4E04	1.8E04	2.2E04	2.9E04	
V mm/sec	3.72E-05	7.98E-05	1.19E-04	1.51E-04	2.66E-04	
$\dot{\gamma}$ sec^{-1}	3.07E-05	6.58E-05	9.80E-05	1.24E-04	2.19E-04	
η_a Pa-sec	3.5E08	2.2E08	1.8E08	1.8E08	1.3E08	

Regression ($\eta_a = a\dot{\gamma}^b$): a = 1.961E06 b = -4.956E-01 R^2 = 0.982

Rheologic Constants : $\eta_{.05}$ = 8.9E06 $\eta_{1.0}$ = 2.0E06 c = 5.1E-01

Plotting Points:

$\dot{\gamma}$	1.0E-05	1.0E-04	1.0E-03
η_a	5.9E08	1.9E08	6.1E07

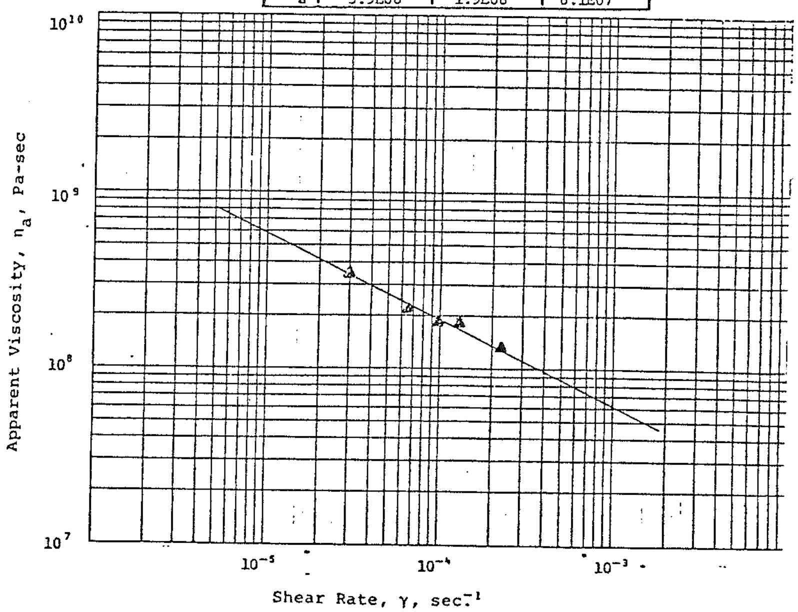

Table G-1

Schweyer Rheometer
Asphalt Cements

MATERIAL	TEMP.	TUBE	a	b	c	R^2	$\eta_{.05}$	$\eta_{1.0}$
375 1 Hr.	60F	E	6.0E06	-1.5E02	0.98	0.04	6.3E06	6.0E06
375 1 Hr.	60F	F	2.1E08	6.2E-01	1.60	0.26	3.4E07	2.1E08
375 1 Hr.	30F	E	1.0E08	-3.4E-01	0.66	0.31	2.8E08	1.0E08
375 1 Hr.	30F	F	3.5E08	-1.4E-01	0.86	0.21	5.4E08	3.5E08
375 1 Hr.	30F	G	1.6E07	-4.9E-01	0.51	0.91	6.9E07	1.6E07
Stock AR-1000	39.2F	D	8.5E09	1.0E00	2.00	0.86	4.1E08	8.5E09
Stock AR-1000	39.2F	E	7.1E06	-4.0E-01	0.60	0.02	2.4E07	7.1E06
Stock AR-1000	39.2F	F	6.7E07	8.2E-02	1.10	0.06	5.2E07	6.7E07
AR-1000 Aged @ 375 1 Hr.	30F	D	9.9E06	-5.6E-01	0.44	0.08	5.2E07	9.9E06
AR-1000 Aged @ 375 1 Hr.	30F	E	1.7E07	-5.1E-01	0.49	0.34	7.9E07	1.7E07
AR-1000 Aged @ 375 1 Hr.	30F	F	6.5E08	-1.3E-01	0.87	0.90	9.5E08	6.5E08
AR-1000 Aged @ 375 1 Hr.	60F	C	1.7E07	4.5E-01	1.50	0.29	4.3E06	1.7E07
AR-1000 Aged @ 375 1 Hr.	60F	D	3.1E06	4.0E-02	1.00	0.17	2.8E06	3.1E06
AR-1000 Aged @ 375 1 Hr.	60F	E	7.9EE05	-2.3E-01	0.77	0.52	1.6E06	7.9E05
AR-1000 Aged @ 375 1 Hr.	39.2F	D	3.6E08	1.4E-01	1.10	0.22	2.3E08	3.6E08
AR-1000 Aged @ 375 1 Hr.	39.2F	E	1.7E09	3.8E-01	1.40	0.75	5.4E08	1.7E09
AR-1000 Aged @ 375 1 Hr.	39.2F	F	6.5E08	1.7E-01	1.20	0.77	3.9E08	6.5E08
Stock AR-1000	60F	C	1.0E06	1.3E-01	1.10	0.81	6.9E05	1.0E06
Stock AR-1000	60F	D	5.7E05	-3.5E-02	0.97	0.14	6.3E05	5.7E05
Stock AR-1000	60F	B	2.8E06	7.7E-01	1.8	0.72	2.8E05	2.8E06
Stock AR-1000	30F	D	9.1E09	6.7E-01	1.7	0.82	1.2E09	9.1E09
Stock AR-1000	30F	E	5.7E10	7.4E-01	1.7	0.62	6.3E09	5.7E10
Stock AR-1000	30F	F	4.0E11	9.4E-01	1.9	0.94	2.4E10	4.0E11

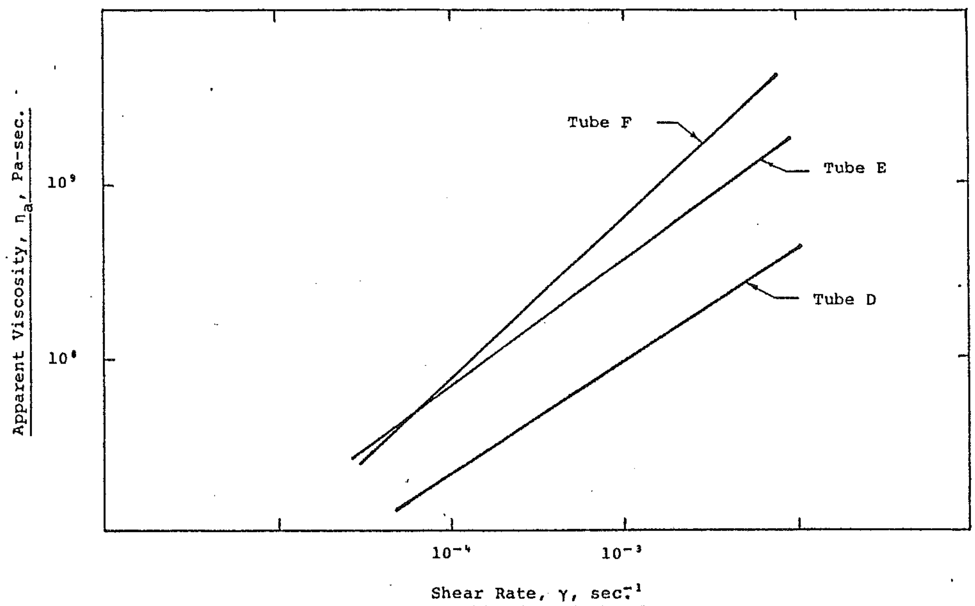

Figure G-2 - AR-1000 Unaged
Test Temperature, 30F

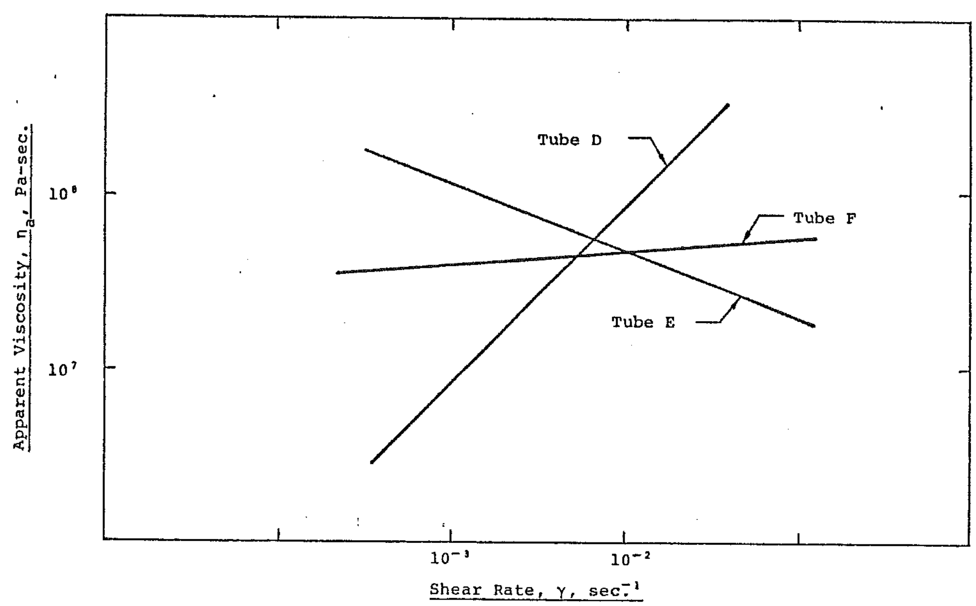

Figure G-1 - AR-1000 Unaged
Test Temperature, 39.2F

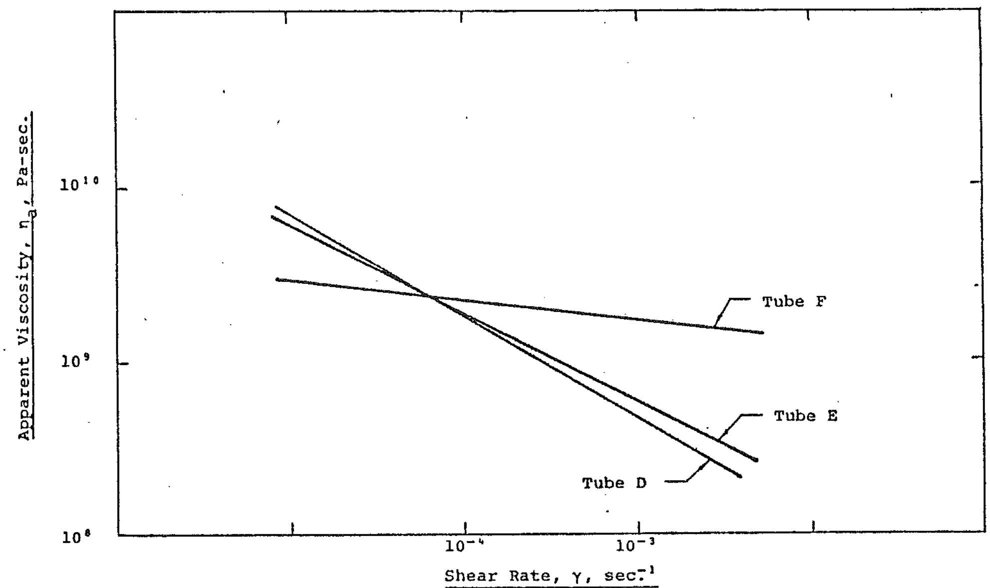

Figure G-4 - AR-1000 Aged 1 Hr. @ 375F
Test Temperature, 30F

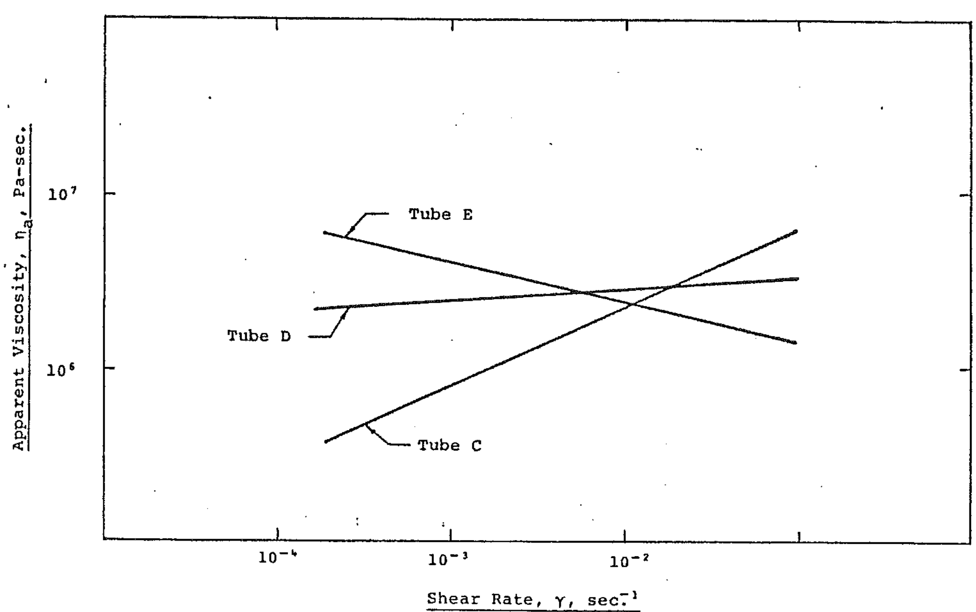

Figure G-3 - AR-1000 Aged 1 Hr. @ 375F
Test Temperature, 60F

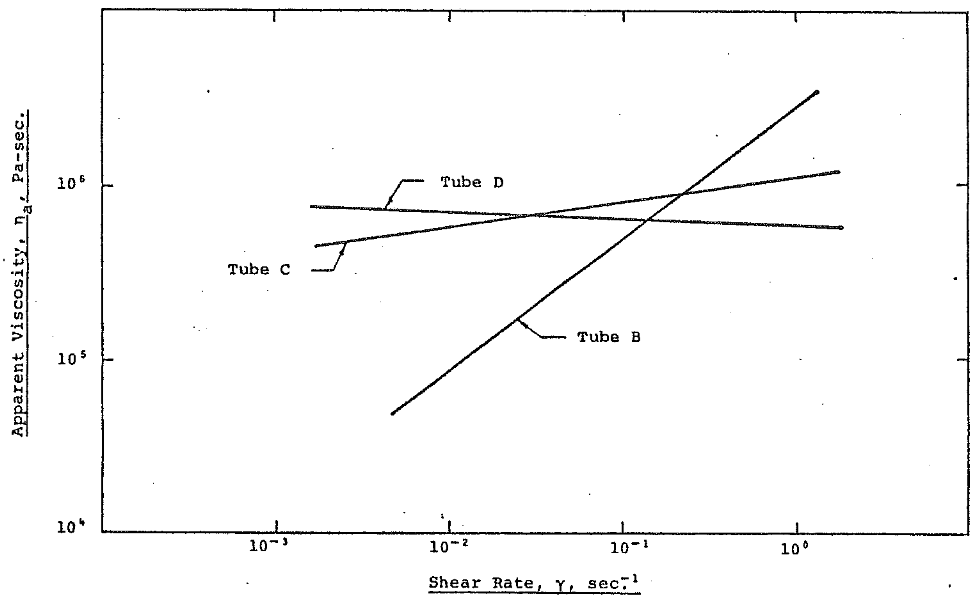

Figure G-6 - AR-1000 - Unaged
Test Temperature, 60F

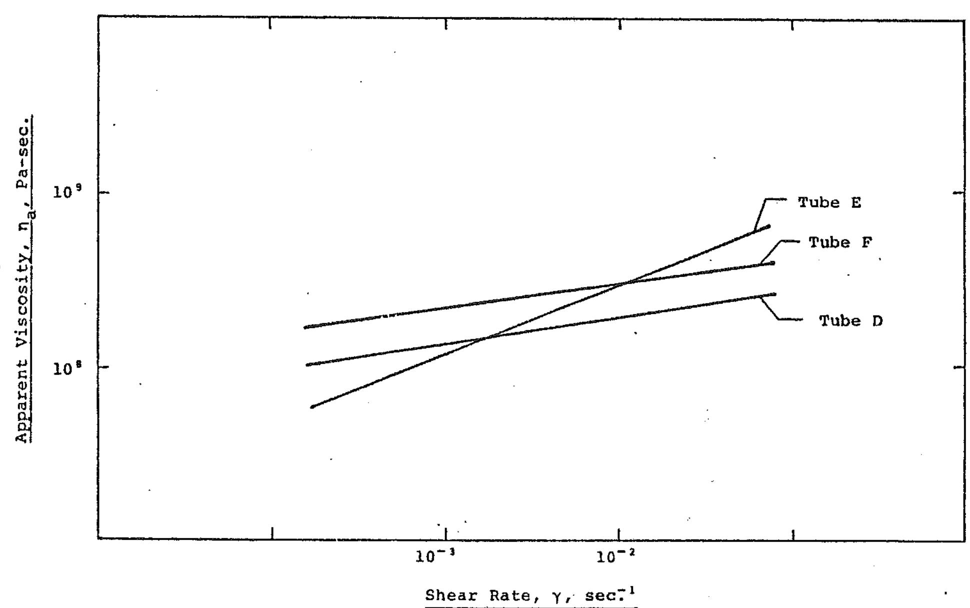

Figure G-5 - AR-1000 Aged 1 Hr. @ 375F
Test Temperature, 39.2F

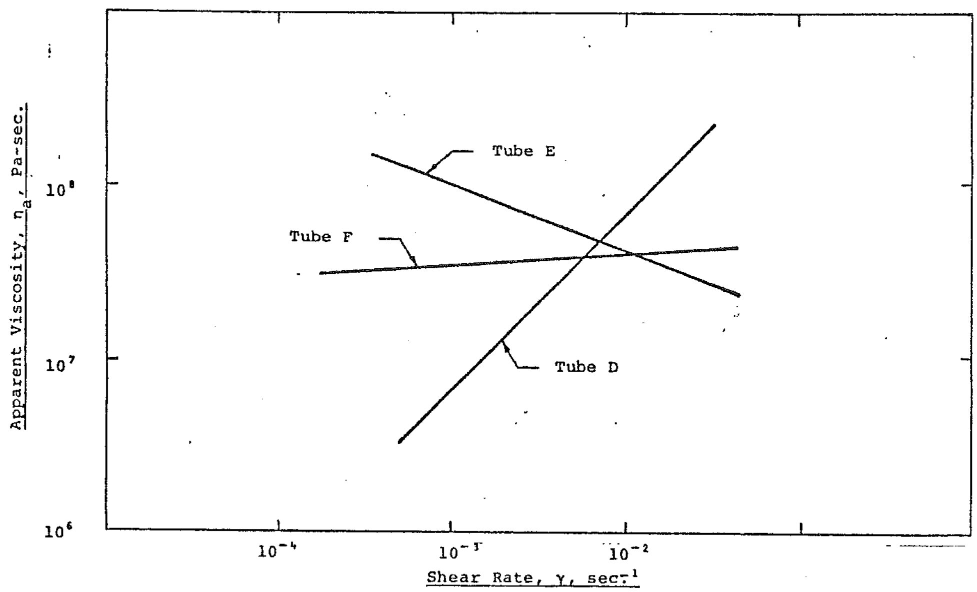

Figure G-8 - AR-1000 Unaged
Test Temperature = 39.2F

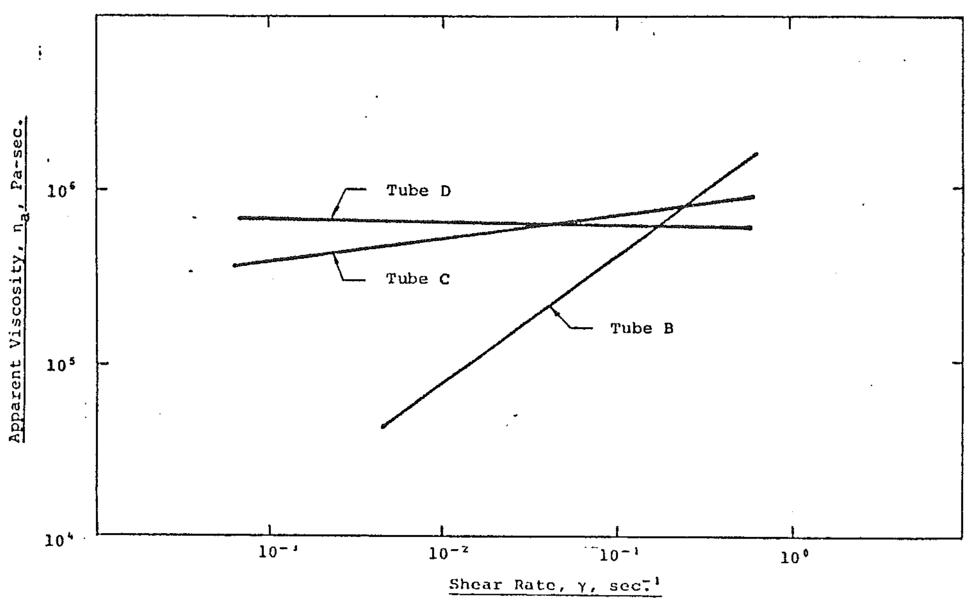

Figure G-7 - AR-1000 - Unaged
Test Temperature = 60F

TABLE G-2

ANOVA

Schweyer

no. 05

Holding Time (H_ε)	Tube (C_j)	Temperature (T_i): 350	375	400	TOTALS	
0.5	E	3.869 E+07	0.3134 E+07	4.714 E+07		
		10.86 E+07	13.75 E+07	17.81 E+07		
	F	3.982 E+07	39.95 E+07	91.40 E+07		
		6.072 E+07	94.05 E+07	9.312 E+07		
	G	1.239 E+07	18.25 E+07	10.06 E+07		
		3.335 E+07	0.8452 E+07	1.254 E+07		
1.0	E	24.15 E+07	57.47 E+07	18.58 E+07		
		86.01 E+07	17.11 E+07	43.93 E+07		
	F	15.49 E+07	124.0 E+07	56.70 E+07		
		79.17 E+07	6.474 E+07	66.76 E+07		
	G	4.219 E+07	5.389 E+07	11.26 E+07		
		1.145 E+07	4.875 E+07	14.67 E+07		
2.0	E	15.78 E+07	10.45 E+07	135.5 E+07		
		28.03 E+07	6.674 E+07	7.126 E+07		
	F	230.6 E+07	13.68 E+07	15.84 E+07		
		6.439 E+07	4.288 E+07	18.42 E+07		
	G	2.748 E+07	2.513 E+07	4.000 E+07		
		1.092 E+07	0.8348 E+07	0.8877 E+07		
TOTALS						

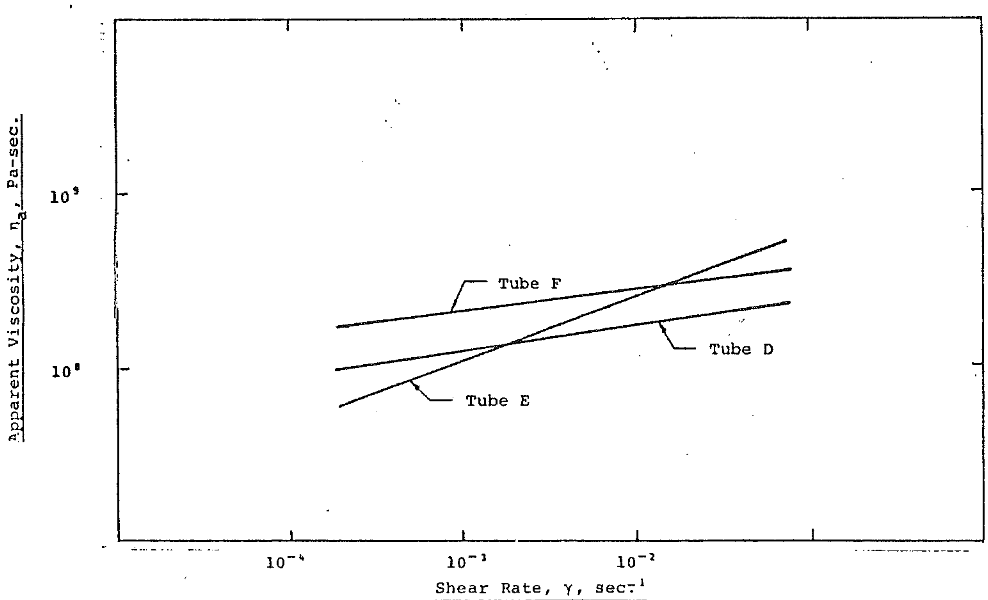

Figure G-9 - AR-1000 Aged at 375F for 1 Hour
Test Temperature = 39.2F

TABLE G-3

ANOVA
Schweyer
ηo. 05

FOSTER & BURR q:

Matrix Σx_i ______ Matrix Σx_i^2 ______

ν ______ q 0.287 $q_{.99}$ 0.198 $q_{.999}$ 0.265

$$Y_{ijkl} = \mu + T_i + C_j + H_k + (TC)_{ij} + (TH)_{ik} + (CH)_{jk} + (TCH)_{ijk} + \epsilon_{(ijk)l}$$

TOTALS

Temperature (T_i)	Tube (C_j)	Time (H_j)
$T_{1\cdots}$ =	$T_{\cdot 1\cdot\cdot}$ =	$T_{\cdot\cdot 1\cdot}$ =
$T_{2\cdots}$ =	$T_{\cdot 2\cdot\cdot}$ =	$T_{\cdot\cdot 2\cdot}$ =
$T_{3\cdots}$ =	$T_{\cdot 3\cdot\cdot}$ =	$T_{\cdot\cdot 3\cdot}$ =

Temperature x Tube $(TC)_{ij}$

$T_{11\cdot\cdot}$ =	$T_{21\cdot\cdot}$ =	$T_{31\cdot\cdot}$ =
$T_{12\cdot\cdot}$ =	$T_{22\cdot\cdot}$ =	$T_{32\cdot\cdot}$ =
$T_{13\cdot\cdot}$ =	$T_{23\cdot\cdot}$ =	$T_{33\cdot\cdot}$ =

Temperature x Time $(TH)_{ik}$

$T_{1\cdot 1\cdot}$ =	$T_{2\cdot 1\cdot}$ =	$T_{3\cdot 1\cdot}$ =
$T_{1\cdot 2\cdot}$ =	$T_{2\cdot 2\cdot}$ =	$T_{3\cdot 2\cdot}$ =
$T_{1\cdot 3\cdot}$ =	$T_{2\cdot 3\cdot}$ =	$T_{3\cdot 3\cdot}$ =

Tube x Time $(CH)_{jk}$

$T_{\cdot 11\cdot}$ =	$T_{\cdot 12\cdot}$ =	$T_{\cdot 13\cdot}$ =
$T_{\cdot 21\cdot}$ =	$T_{\cdot 22\cdot}$ =	$T_{\cdot 23\cdot}$ =
$T_{\cdot 31\cdot}$ =	$T_{\cdot 32\cdot}$ =	$T_{\cdot 33\cdot}$ =

TABLE G-3 (Cont.)

TOTALS - Continued

Temperature x Tube x Time $(TCH)_{ijk}$

$T_{111\cdot}$ = 147.29 E+06	$T_{112\cdot}$ = 1101.6 E+06	$T_{113\cdot}$ = 438.10 E+06
$T_{121\cdot}$ = 100.54 E+06	$T_{122\cdot}$ = 946.60 E+06	$T_{123\cdot}$ = 2370.4 E+06
$T_{131\cdot}$ = 45.740 E+06	$T_{132\cdot}$ = 53.640 E+06	$T_{133\cdot}$ = 38.40 E+06
$T_{211\cdot}$ = 140.63 E+06	$T_{212\cdot}$ = 745.80 E+06	$T_{213\cdot}$ = 171.24 E+06
$T_{221\cdot}$ = 1340.0 E+06	$T_{222\cdot}$ = 1304.7 E+06	$T_{223\cdot}$ = 179.68 E+06
$T_{231\cdot}$ = 190.95 E+06	$T_{232\cdot}$ = 102.64 E+06	$T_{233\cdot}$ = 33.478 E+06
$T_{311\cdot}$ = 225.24 E+06	$T_{312\cdot}$ = 625.10 E+06	$T_{313\cdot}$ = 1426.3 E+06
$T_{321\cdot}$ = 1007.1 E+06	$T_{322\cdot}$ = 1234.6 E+06	$T_{323\cdot}$ = 342.60 E+06
$T_{331\cdot}$ = 113.14 E+06	$T_{332\cdot}$ = 259.30 E+06	$T_{333\cdot}$ = 48.877 E+06

TABLE G-4

ANOVA

Schweyer

Log_{10} (no. 05 ÷ 1.00 E+07)

Holding Time (H_ε) / Tube (C_j) / Temperature (T_i)

Holding Time	Tube	350	375	400	TOTALS	
0.5	E	0.5876	-0.5039	0.6734		
		1.0358	1.1383	1.2507		
	F	0.6001	1.6015	1.9609		
		0.7833	1.9734	0.9690		
	G	0.0931	1.2613	1.0026		
		0.5231	-0.0730	0.0983		
1.0	E	1.3829	1.7594	1.2690		
		1.9345	1.2333	1.6428		
	F	1.9101	2.0934	1.7536		
		1.8986	0.8112	1.8245		
	G	0.6252	0.7315	1.0515		
		0.0588	0.6880	1.1664		
2.0	E	1.1981	1.0191	2.1319		
		1.4476	0.8244	0.8528		
	F	2.3629	1.1361	1.1988		
		0.8088	0.6323	1.2653		
	G	0.4390	0.4002	0.6021		
		0.0382	-0.0784	-0.0517		
TOTALS						

TABLE G-5

ANOVA

Schweyer

Log_{10} (no. 05 ÷ 1.00 E+07)

FOSTER & BURR q:

Matrix Σx_i 5.503 770 000 E-01   Matrix Σx_i^2 7.945 572 551 E+01

ν 1.000   q 0.106 $q_{.99}$ 0.198 $q_{.999}$ 0.265

$$Y_{ijkl} = \mu + T_i + C_j + H_k + (TC)_{ij} + (TH)_{ik} + (CH)_{jk} + (TCH)_{ijk} + \varepsilon_{(ijk)l}$$

TOTALS

Temperature (T_i)		Tube (C_j)		Time (H_j)	
$T_{1...}$ =	17.7277	$T_{.1..}$ =	20.8777	$T_{..1.}$ =	14.9755
$T_{2...}$ =	16.6481	$T_{.2..}$ =	25.5838	$T_{..2.}$ =	23.8347
$T_{3...}$ =	20.6619	$T_{.3..}$ =	8.5762	$T_{..3.}$ =	16.2275
	55.0377		55.0377		53.0377

Temperature x Tube $(TC)_{ij}$

$T_{11..}$ =	7.5865	$T_{21..}$ =	5.4706	$T_{31..}$ =	7.8206
$T_{12..}$ =	8.3638	$T_{22..}$ =	8.2479	$T_{32..}$ =	8.9721
$T_{13..}$ =	1.7774	$T_{23..}$ =	2.9296	$T_{33..}$ =	3.8692
	17.7277		16.6481		20.6619

Temperature x Time $(TH)_{ik}$

$T_{1.1.}$ =	3.6230	$T_{2.1.}$ =	5.3976	$T_{3.1.}$ =	5.9549
$T_{1.2.}$ =	7.8101	$T_{2.2.}$ =	7.3168	$T_{3.2.}$ =	8.7078
$T_{1.3.}$ =	6.2946	$T_{2.3.}$ =	3.9337	$T_{3.3.}$ =	5.9992
	17.7277		16.6481		20.6619

Tube x Time $(CH)_{jk}$

$T_{.11.}$ =	4.1819	$T_{.12.}$ =	9.2219	$T_{.13.}$ =	7.4739
$T_{.21.}$ =	7.8882	$T_{.22.}$ =	10.2914	$T_{.23.}$ =	7.4042
$T_{.31.}$ =	2.9054	$T_{.32.}$ =	4.3214	$T_{.33.}$ =	1.3494
	14.9755		23.8347		16.2275

TABLE G-5 (Cont.)

<u>TOTALS</u> - Continued

Temperature x Tube x Time $(TCH)_{ijk}$

$T_{111\cdot}$	=	1.6234	$T_{112\cdot}$	=	3.3174	$T_{113\cdot}$	=	2.6457
$T_{121\cdot}$	=	1.3834	$T_{122\cdot}$	=	3.8087	$T_{123\cdot}$	=	3.1717
$T_{131\cdot}$	=	0.6162	$T_{132\cdot}$	=	0.6840	$T_{133\cdot}$	=	0.4772
		3.6230			7.8101			6.2946
$T_{211\cdot}$	=	0.6344	$T_{212\cdot}$	=	2.9927	$T_{213\cdot}$	=	1.8435
$T_{221\cdot}$	=	3.5749	$T_{222\cdot}$	=	2.9046	$T_{223\cdot}$	=	1.7684
$T_{231\cdot}$	=	1.1883	$T_{232\cdot}$	=	1.4195	$T_{233\cdot}$	=	0.3218
		5.3976			7.3168			3.9337
$T_{311\cdot}$	=	1.9241	$T_{312\cdot}$	=	2.9118	$T_{313\cdot}$	=	2.9847
$T_{321\cdot}$	=	2.9299	$T_{322\cdot}$	=	3.5781	$T_{323\cdot}$	=	2.4641
$T_{331\cdot}$	=	1.1009	$T_{332\cdot}$	=	2.2179	$T_{333\cdot}$	=	0.5504
		5.9549			8.7078			5.9992

TABLE G-6

ANOVA
Schweyer
Log_{10} (no. 05 ÷ 1.00 E+07)

(0) $CT = (T....)^2 \div n = (\ 55.0377\ )^2 \div 54 =$ 5.609 534 113 E+01

(1) $SS(T_i) = \frac{1}{18}[(\ 17.7277\ )^2 + \ldots + (\ 20.6619\ )^2] - \overset{(0)}{CT} =$ 4.793 640 400 E-01

(2) $SS(C_j) = \frac{1}{18}[(\ 20.8777\ )^2 + \ldots + (\ 8.5762\ )^2] - \overset{(0)}{CT} =$ 8.569 124 760 E+00

(3) $SS(H_k) = \frac{1}{18}[(\ 14.9755\ )^2 + \ldots + (\ 16.2275\ )^2] - \overset{(0)}{CT} =$ 2.554 118 930 E+00

(4) $SS(TC)_{ij} = \frac{1}{6}[(\ 7.5865\ )^2 + \ldots + (\ 3.8692\ )^2] - \overset{(0)}{CT} - \overset{(1)}{SS(T_i)} - \overset{(2)}{SS(C_j)} =$ 4.955 417 200 E-01

(5) $SS(TH)_{ik} = \frac{1}{6}[(\ 3.6230\ )^2 + \ldots + (\ 5.9992\ )^2] - \overset{(0)}{CT} - \overset{(1)}{SS(T_i)} - \overset{(3)}{SS(H_k)} =$ 7.322 491 200 E-01

(6) $SS(CH)_{jk} = \frac{1}{6}[(\ 4.1819\ )^2 + \ldots + (\ 1.3494\ )^2] - \overset{(0)}{CT} - \overset{(2)}{SS(C_j)} - \overset{(3)}{SS(H_k)} =$ 1.162 486 310 E+00

(7) $SS(TCH)_{ijk} = \frac{1}{2}[(\ 1.6234\ )^2 + \ldots + (\ 0.5504\ )^2]$

$- \overset{(0)}{CT} - \overset{(1)}{SS(T_i)} - \overset{(2)}{SS(C_j)} - \overset{(3)}{SS(H_k)}$

$- \overset{(4)}{SS(TC_{ij})} - \overset{(5)}{SS(TH_{ik})} - \overset{(6)}{SS(CH_{jk})}$

= 1.816 263 090 E+00

(8) $SS(Total) = \Sigma\Sigma\Sigma\Sigma x^2_{ijkl} - \overset{(0)}{CT} =$ 2.336 038 438 E+01

(9) $SS(Error) = SS(Total) - SS(T_i) - SS(C_j) - SS(H_k)$

$- SS(TC)_{ij} - SS(TH)_{ik} - SS(CH)_{jk} - SS(TCH)_{ijk}$

= (8) - (1) - (2) - (3) - (4) - (5) - (6) - (7)

= 7.551 236 410 E+00

TABLE G-7

ANOVA
Schweyer
$\log_{10}$ (ηo. 05 ÷ 1.00 E+07)

	SOURCE	df	SS	MS	F	F.05	F.01
(1)	T_i	2	0.4794	0.2397	0.86	3.35	5.49
(2)	C_j	2	8.5691	4.2846	15.32	3.35	5.49
(3)	H_k	2	2.5541	1.2771	4.57	3.35	5.49
(4)	$(TC)_{ij}$	4	0.4955	0.1239	0.44	2.73	4.11
(5)	$(TH)_{ik}$	4	0.7322	0.1831	0.65	2.73	4.11
(6)	$(CH)_{jk}$	4	1.1625	0.2906	1.04	2.73	4.11
(7)	$(TCH)_{ijk}$	8	1.8163	0.2270	0.81	2.31	3.26
(9)	Error	27	7.5512	0.2797			
(8)	Total	53	23.3604				

TABLE G-8

ANOVA
Schweyer
Shear Susceptibility Index, C

Holding Time (H_ε)
Tube (C_j)
Temperature (T_i)

Holding Time	Tube	350	375	400	TOTALS	
0.5	E	1.003	0.468	0.516		
		0.911	0.890	1.087		
	F	0.668	0.996	1.042		
		0.715	0.723	0.801		
	G	0.450	0.393	0.674		
		0.618	0.391	0.410		
1.0	E	0.846	1.014	0.730		
		1.071	0.890	0.939		
	F	0.755	1.021	0.896		
		0.969	0.682	1.030		
	G	0.699	0.450	0.797		
		0.299	0.500	0.722		
2.0	E	1.108	0.665	1.485		
		1.105	0.987	0.974		
	F	1.036	0.699	0.834		
		0.739	0.759	1.013		
	G	0.567	0.456	0.570		
		0.374	0.506	0.508		
TOTALS						

TABLE G-9

ANOVA
Schweyer
Shear Susceptibility Index, C

FOSTER & BURR q:

Matrix Σx_i 41.419 Matrix Σx_i^2 3.506 341 700 E+01

ν 1.00 q 0.092 $q_{.99}$ 0.198 $q_{.999}$ 0.265

$$Y_{ijkl} = \mu + T_i + C_j + H_k + (TC)_{ij} + (TH)_{ik} + (CH)_{jk} + (TCH)_{ijk} + \varepsilon_{(ijk)l}$$

TOTALS

Temperature (T_i)		Tube (C_j)		Time (H_j)	
$T_{1...}$ =	13.933	$T_{.1..}$ =	16.657	$T_{..1.}$ =	12.756
$T_{2...}$ =	12.409	$T_{.2..}$ =	15.378	$T_{..2.}$ =	14.278
$T_{3...}$ =	15.077	$T_{.3..}$ =	9.384	$T_{..3.}$ =	14.385
	41.419		41.419		41.419

Temperature x Tube $(TC)_{ij}$

$T_{11..}$ =	6.044	$T_{21..}$ =	4.833	$T_{31..}$ =	5.780
$T_{12..}$ =	4.882	$T_{22..}$ =	4.880	$T_{32..}$ =	5.616
$T_{13..}$ =	3.007	$T_{23..}$ =	2.696	$T_{33..}$ =	3.681
	13.933		12.409		15.077

Temperature x Time $(TH)_{ik}$

$T_{1.1.}$ =	4.365	$T_{2.1.}$ =	3.861	$T_{3.1.}$ =	4.530
$T_{1.2.}$ =	4.639	$T_{2.2.}$ =	4.476	$T_{3.2.}$ =	5.163
$T_{1.3.}$ =	4.929	$T_{2.3.}$ =	4.072	$T_{3.3.}$ =	5.384
	13.933		12.409		15.077

Tube x Time $(CH)_{jk}$

$T_{.11.}$ =	4.875	$T_{.12.}$ =	5.458	$T_{.13.}$ =	6.324
$T_{.21.}$ =	4.945	$T_{.22.}$ =	5.353	$T_{.23.}$ =	5.080
$T_{.31.}$ =	2.936	$T_{.32.}$ =	3.467	$T_{.33.}$ =	2.981
	12.756		14.278		14.385

TABLE G-9 (Cont.)

TOTALS - Continued

Temperature x Tube x Time $(TCH)_{ijk}$

$T_{111.}$ =	1.914	$T_{112.}$ =	1.917	$T_{113.}$ =	2.213
$T_{121.}$ =	1.383	$T_{122.}$ =	1.724	$T_{123.}$ =	1.775
$T_{131.}$ =	1.068	$T_{132.}$ =	0.998	$T_{133.}$ =	0.941
	4.365		4.639		4.929
$T_{211.}$ =	1.358	$T_{212.}$ =	1.823	$T_{213.}$ =	1.652
$T_{221.}$ =	1.719	$T_{222.}$ =	1.703	$T_{223.}$ =	1.458
$T_{231.}$ =	0.784	$T_{232.}$ =	0.950	$T_{233.}$ =	0.962
	3.861		4.476		4.072
$T_{311.}$ =	1.603	$T_{312.}$ =	1.718	$T_{313.}$ =	2.459
$T_{321.}$ =	1.843	$T_{322.}$ =	1.926	$T_{323.}$ =	1.847
$T_{331.}$ =	1.084	$T_{332.}$ =	1.519	$T_{333.}$ =	1.078
	4.530		5.163		5.384

TABLE G-10

ANOVA
Schweyer
Shear Susceptibility Index, C

(0) $CT = (T....)^2 \div n = (\ 41.419\ )^2 \div 54 = \underline{3.176\ 914\ 002\ E+01}$

(1) $SS(T_i) = \frac{1}{18}[(\ 13.933\ )^2 + \ldots + (\ 15.077\ )^2] - \overset{(0)}{CT} =$ $\underline{1.990\ 654\ 800\ E-01}$

(2) $SS(C_j) = \frac{1}{18}[(\ 16.657\ )^2 + \ldots + (\ 9.384\ )^2] - \overset{(0)}{CT} =$ $\underline{1.657\ 192\ 700\ E+00}$

(3) $SS(H_k) = \frac{1}{18}[(\ 12.756\ )^2 + \ldots + (\ 14.385\ )^2] - \overset{(0)}{CT} =$ $\underline{9.225\ 137\ 000\ E-02}$

(4) $SS(TC)_{ij} = \frac{1}{6}[(\ 6.044\ )^2 + \ldots + (\ 3.681\ )^2] - \overset{(0)}{CT} - \overset{(1)}{SS(T_i)} - \overset{(2)}{SS(C_j)} = \underline{8.064\ 030\ 000\ E-02}$

(5) $SS(TH)_{ik} = \frac{1}{6}[(\ 4.365\ )^2 + \ldots + (\ 5.384\ )^2] - \overset{(0)}{CT} - \overset{(1)}{SS(T_i)} - \overset{(3)}{SS(H_k)} = \underline{3.230\ 863\ 000\ E-02}$

(6) $SS(CH)_{jk} = \frac{1}{6}[(\ 4.875\ )^2 + \ldots + (\ 2.981\ )^2] - \overset{(0)}{CT} - \overset{(2)}{SS(C_j)} - \overset{(3)}{SS(H_k)} = \underline{1.282\ 400\ 800\ E-01}$

(7) $SS(TCH)_{ijk} = \frac{1}{2}[(\ 1.914\ )^2 + \ldots + (\ 1.078\ )^2]$
$- \overset{(0)}{CT} - \overset{(1)}{SS(T_i)} - \overset{(2)}{SS(C_j)} - \overset{(3)}{SS(H_k)}$
$- \overset{(4)}{SS(TC_{ij})} - \overset{(5)}{SS(TH_{ik})} - \overset{(6)}{SS(CH_{jk})}$
$= \underline{1.947\ 779\ 200\ E-01}$

(8) $SS(Total) = \Sigma\Sigma\Sigma\Sigma x^2_{ijkl} - \overset{(0)}{CT} = \underline{3.294\ 276\ 980\ E+00}$

(9) $SS(Error) = SS(Total) - SS(T_i) - SS(C_j) - SS(H_k)$
$- SS(TC)_{ij} - SS(TH)_{ik} - SS(CH)_{jk} - SS(TCH)_{ijk}$
$= (8) - (1) - (2) - (3) - (4) - (5) - (6) - (7)$
$= \underline{8.918\ 005\ 000\ E-01}$

TABLE G-11

ANOVA
Schweyer
Shear Susceptibility Index, C

	SOURCE	df	SS	MS	F	F.05	F.01
(1)	T_i	2	0.1991	0.0995	3.01	3.35	5.49
(2)	C_j	2	1.6752	0.8376	25.36	3.35	5.49
(3)	H_k	2	0.0923	0.0461	1.40	3.35	5.49
(4)	$(TC)_{ij}$	4	0.0806	0.0202	0.61	2.73	4.11
(5)	$(TH)_{ik}$	4	0.0323	0.0081	0.24	2.73	4.11
(6)	$(CH)_{jk}$	4	0.1282	0.0321	0.97	2.73	4.11
(7)	$(TCH)_{ijk}$	8	0.1948	0.0243	99.74	2.31	3.26
(9)	Error	27	0.8918	0.0330	-	-	-
(8)	Total	53	3.2943	-	-	-	-

TABLE G-12

ANOVA

Schweyer

a Parameter

Holding Time (H_ε)
Tube (C_j)
Temperature (T_i)

Holding Time	Tube	350	375	400	TOTALS
0.5	E	39.01 E+06	0.03851 E+06	11.05 E+06	
		83.10 E+06	39.53 E+06	231.1 E+06	
	F	14.75 E+06	2.964 E+06	1036.0 E+06	
		25.82 E+06	48.93 E+06	51.31 E+06	
	G	2.382 E+06	41.05 E+06	37.91 E+06	
		10.63 E+06	1.442 E+06	2.145 E+06	
1.0	E	152.5 E+06	59.85 E+06	88.70 E+06	
		1064.0 E+06	1322.0 E+06	423.5 E+06	
	F	74.35 E+06	10.37 E+06	415.0 E+06	
		722.0 E+06	96.54 E+06	724.5 E+06	
	G	17.12 E+06	24.95 E+06	61.34 E+06	
		1.403 E+06	10.91 E+06	63.71 E+06	
2.0	E	21.79 E+06	38.35 E+06	5796.0 E+06	
		383.3 E+06	55.51 E+06	96.20 E+06	
	F	256.6 E+06	4.927 E+06	11.02 E+06	
		29.48 E+06	64.14 E+06	65.84 E+06	
	G	7.654 E+06	20.81 E+06	191.7 E+06	
		1.676 E+06	1.900 E+06	2.031 E+06	
TOTALS					

TABLE G-13

ANOVA

Schweyer

a Parameter

FOSTER & BURR q:

Matrix Σx_i ______ Matrix Σx_i^2 ______

ν ______ q 0.782 $q_{.99}$ 0.198 $q_{.999}$ 0.265

$$Y_{ijkl} = \mu + T_i + C_j + H_k + (TC)_{ij} + (TH)_{ik} + (CH)_{jk} + (TCH)_{ijk} + \epsilon_{(ijk)l}$$

TOTALS

Temperature (T_i)	Tube (C_j)	Time (H_j)
$T_{1...}$ = ______	$T_{.1..}$ = ______	$T_{..1.}$ = ______
$T_{2...}$ = ______	$T_{.2..}$ = ______	$T_{..2.}$ = ______
$T_{3...}$ = ______	$T_{.3..}$ = ______	$T_{..3.}$ = ______

Temperature x Tube $(TC)_{ij}$

$T_{11..}$ = ______	$T_{21..}$ = ______	$T_{31..}$ = ______
$T_{12..}$ = ______	$T_{22..}$ = ______	$T_{32..}$ = ______
$T_{13..}$ = ______	$T_{23..}$ = ______	$T_{33..}$ = ______

Temperature x Time $(TH)_{ik}$

$T_{1.1.}$ = ______	$T_{2.1.}$ = ______	$T_{3.1.}$ = ______
$T_{1.2.}$ = ______	$T_{2.2.}$ = ______	$T_{3.2.}$ = ______
$T_{1.3.}$ = ______	$T_{2.3.}$ = ______	$T_{3.3.}$ = ______

Tube x Time $(CH)_{jk}$

$T_{.11.}$ = ______	$T_{.12.}$ = ______	$T_{.13.}$ = ______
$T_{.21.}$ = ______	$T_{.22.}$ = ______	$T_{.23.}$ = ______
$T_{.31.}$ = ______	$T_{.32.}$ = ______	$T_{.33.}$ = ______

TABLE G-13 (Cont.)

TOTALS - Continued

Temperature x Tube x Time $(TCH)_{ijk}$

$T_{111\cdot}$ = 122.1 E+06	$T_{112\cdot}$ = 1217 E+06	$T_{113\cdot}$ = 405.1 E+06
$T_{121\cdot}$ = 40.57 E+06	$T_{122\cdot}$ = 796.4 E+06	$T_{123\cdot}$ = 286.1 E+06
$T_{131\cdot}$ = 13.01 E+06	$T_{132\cdot}$ = 18.52 E+06	$T_{133\cdot}$ = 9.330 E+06
$T_{211\cdot}$ = 39.57 E+06	$T_{212\cdot}$ = 1382 E+06	$T_{213\cdot}$ = 93.86 E+06
$T_{221\cdot}$ = 51.89 E+06	$T_{222\cdot}$ = 106.9 E+06	$T_{223\cdot}$ = 69.07 E+06
$T_{231\cdot}$ = 42.49 E+06	$T_{232\cdot}$ = 35.86 E+06	$T_{233\cdot}$ = 22.71 E+06
$T_{311\cdot}$ = 242.2 E+06	$T_{312\cdot}$ = 506.2 E+06	$T_{313\cdot}$ = 5892 E+06
$T_{321\cdot}$ = 1087 E+06	$T_{322\cdot}$ = 1140 E+06	$T_{323\cdot}$ = 76.86 E+06
$T_{331\cdot}$ = 40.06 E+06	$T_{332\cdot}$ = 125.1 E+06	$T_{333\cdot}$ = 193.7 E+06

TABLE G-14

ANOVA

Schweyer
Log_{10} a

Holding Time (H_ε)
Tube (C_j)
Temperature (T_i)

Holding Time	Tube	350	375	400	TOTALS	
0.5	E	7.5912	4.5856	7.0434		
		7.9196	7.5969	8.3638		
	F	7.1688	6.4719	9.0154		
		7.4120	7.6896	7.7102		
	G	6.3769	7.6133	7.5788		
		7.0265	6.1596	6.3314		
1.0	E	8.1833	7.7771	7.9175		
		9.0269	9.1212	8.6269		
	F	7.8713	7.0158	8.6180		
		8.8585	7.9847	8.8600		
	G	7.2335	7.3971	7.7877		
		6.1471	7.0378	7.8042		
2.0	E	7.3383	7.5838	9.7631		
		8.5835	7.7444	7.9832		
	F	8.4093	6.6926	7.0422		
		7.4695	7.8071	7.8185		
	G	6.8839	7.3183	8.2826		
		6.2243	6.2788	6.3077		
TOTALS						

TABLE G-15
ANOVA
Schweyer
Log_{10} a

FOSTER & BURR q:

Matrix Σx_i 408.4546   Matrix Σx_i^2 3.135 966 880 E+03

ν 1.000   q 0.097 $q_{.99}$ 0.198 $q_{.999}$ 0.265

$Y_{ijkl} = \mu + T_i + C_j + H_k + (TC)_{ij} + (TH)_{ik} + (CH)_{jk} + (TCH)_{ijk} + \epsilon_{(ijk)l}$

TOTALS

Temperature (T_i)		Tube (C_j)		Time (H_j)	
$T_{1...}$ =	135.7246	$T_{.1..}$ =	142.7497	$T_{..1.}$ =	129.6549
$T_{2...}$ =	129.8756	$T_{.2..}$ =	139.9154	$T_{..2.}$ =	143.2686
$T_{3...}$ =	143.8548	$T_{.3..}$ =	125.7895	$T_{..3.}$ =	135.5311
	408.4550		408.4546		408.4546

Temperature x Tube $(TC)_{ij}$

$T_{11..}$ =	43.1005	$T_{21..}$ =	50.6529	$T_{31..}$ =	48.9963
$T_{12..}$ =	45.4679	$T_{22..}$ =	49.2083	$T_{32..}$ =	45.2392
$T_{13..}$ =	41.0865	$T_{23..}$ =	43.4074	$T_{33..}$ =	41.2956
	129.6549		143.2686		135.5311

Temperature x Time $(TH)_{ik}$

$T_{1.1.}$ =	43.4950	$T_{2.1.}$ =	40.1169	$T_{3.1.}$ =	46.0430
$T_{1.2.}$ =	47.3206	$T_{2.2.}$ =	46.3337	$T_{3.2.}$ =	49.6143
$T_{1.3.}$ =	44.9088	$T_{2.3.}$ =	43.4250	$T_{3.3.}$ =	47.1973
	135.7244		129.8756		142.8546

Tube x Time $(CH)_{jk}$

$T_{.11.}$ =	43.1005	$T_{.12.}$ =	50.6529	$T_{.13.}$ =	48.9963
$T_{.21.}$ =	45.4679	$T_{.22.}$ =	49.2083	$T_{.23.}$ =	45.2392
$T_{.31.}$ =	41.0865	$T_{.32.}$ =	43.4072	$T_{.33.}$ =	41.2956
	129.6549		143.2686		135.5311

TABLE G-15 (Cont.)

TOTALS - Continued

Temperature x Tube x Time $(TCH)_{ijk}$

$T_{111.}$ =	15.5108	$T_{112.}$ =	17.2102	$T_{113.}$ =	15.9218
$T_{121.}$ =	14.5807	$T_{122.}$ =	16.7298	$T_{123.}$ =	15.8738
$T_{131.}$ =	13.4035	$T_{132.}$ =	13.3806	$T_{133.}$ =	13.1082
	43.4950		47.3206		44.9088
					$T_{1...}$ = 135.724-
$T_{211.}$ =	12.1825	$T_{212.}$ =	16.8983	$T_{213.}$ =	15.3281
$T_{221.}$ =	14.1615	$T_{222.}$ =	15.0005	$T_{223.}$ =	14.4997
$T_{231.}$ =	13.7729	$T_{232.}$ =	14.4349	$T_{233.}$ =	13.5970
	40.1169		46.3337		43.4248
					$T_{2...}$ = 129.875-
$T_{311.}$ =	15.4072	$T_{312.}$ =	16.5444	$T_{313.}$ =	17.7463
$T_{321.}$ =	16.7256	$T_{322.}$ =	17.4781	$T_{323.}$ =	14.8607
$T_{331.}$ =	13.9102	$T_{332.}$ =	15.5920	$T_{333.}$ =	14.5903
	46.0430		49.6145		47.1973
					$T_{3...}$ = 142.854

TABLE G-16

ANOVA
Schweyer
Log_{10} a

(0) $CT = (T....)^2 \div n = (\ 408.4546\ )^2 \div 54 = \underline{3.089\ 540\ 006\ E{+}03}$

(1) $SS(T_i) = \frac{1}{18}\,[(\ 135.7246\ )^2 + \ldots + (\ 142.8548\ )^2] - \overset{(0)}{CT} =$
$\underline{4.700\ 583\ 000\ E{+}00}$

(2) $SS(C_j) = \frac{1}{18}\,[(\ 142.7497\ )^2 + \ldots + (\ 125.7895\ )^2] - \overset{(0)}{CT} =$
$\underline{9.170\ 790\ 000\ E{+}000}$

(3) $SS(H_k) = \frac{1}{18}\,[(\ 29.6549\ )^2 + \ldots + (\ 135.5311\ )^2] - \overset{(0)}{CT} =$
$\underline{5.180\ 211\ 000\ E{+}00}$

(4) $SS(TC)_{ij} = \frac{1}{6}\,[(\ 43.1005)^2 + \ldots + (\ 41.2956\ )^2] - \overset{(0)}{CT} - \overset{(1)}{SS(T_i)} -$
$\overset{(2)}{SS(C_j)} = \underline{2.756\ 549\ 000\ E{+}00}$

(5) $SS(TH)_{ik} = \frac{1}{6}\,[(\ 43.4950\ )^2 + \ldots + (\ 47.1973\ )^2] - \overset{(0)}{CT} - \overset{(1)}{SS(T_i)} -$
$\overset{(3)}{SS(H_k)} = \underline{3.931\ 520\ 000\ E{-}01}$

(6) $SS(CH)_{jk} = \frac{1}{6}\,[(\ 43.1005\ )^2 + \ldots + (\ 41.2956\ )^2] - \overset{(0)}{CT} -$
$\overset{(2)}{SS(C_j)} - \overset{(3)}{SS(H_k)} = \underline{2.277\ 021\ 000\ E{+}00}$

(7) $SS(TCH)_{ijk} = \frac{1}{2}\,[(\ 15.5108\ )^2 + \ldots + (\ 14.5903\ )^2]$
$- \overset{(0)}{CT} - \overset{(1)}{SS(T_i)} - \overset{(2)}{SS(C_j)} - \overset{(3)}{SS(H_k)}$
$- \overset{(4)}{SS(TC_{ij})} - \overset{(5)}{SS(TH_{ik})} - \overset{(6)}{SS(CH_{jk})}$
$= \underline{3.310\ 444\ 000\ E{+}00}$

(8) $SS(Total) = \Sigma\Sigma\Sigma\Sigma x^2_{ijkl} - \overset{(0)}{CT} = \underline{4.642\ 687\ 400\ E{+}01}$

(9) $SS(Error) = SS(Total) - SS(T_i) - SS(C_j) - SS(H_k)$
$- SS(TC)_{ij} - SS(TH)_{ik} - SS(CH)_{jk} - SS(TCH)_{ijk}$
$= (8) - (1) - (2) - (3) - (4) - (5) - (6) - (7)$
$= \underline{1.863\ 802\ 400\ E{+}01}$

TABLE G-17

ANOVA
Schweyer
Log_{10} a

	SOURCE	df	SS	MS	F	F.05	F.01
(1)	T_i	2	4.701	2.350	3.40	3.35	5.49
(2)	C_j	2	9.171	4.585	6.64	3.35	5.49
(3)	H_k	2	5.180	2.590	3.75	3.35	5.49
(4)	$(TC)_{ij}$	4	2.757	0.689	1.00	2.73	4.11
(5)	$(TH)_{ik}$	4	0.393	0.098	0.14	2.73	4.11
(6)	$(CH)_{jk}$	4	2.277	0.569	0.82	2.73	4.11
(7)	$(TCH)_{ijk}$	8	3.310	0.414	4.80	2.31	3.26
(9)	Error	27	18.638	0.690	-		
(8)	Total	53	46.427	-			

TABLE G-18

ANOVA

Schweyer

R^2 ($\eta_a = a\,\dot{\gamma}^b$)

Holding Time (H_ε) / Tube (C_j) / Temperature (T_i)

Holding Time	Tube	350	375	400	TOTALS	
0.5	E	1.467 E-04	0.755	0.834		
		392 E-04	0.184	0.0577		
	F	0.894	4.20 E-04	0.018		
		0.513	0.593	0.280		
	G	1.000	0.961	0.567		
		0.907	0.971	0.926		
1.0	E	0.228	2.00 E-04	0.255		
		0.0104	0.223	4.03 E-04		
	F	0.795	3.20 E-04	0.0869		
		0.00311	0.845	50.6 E-04		
	G	0.644	0.959	0.863		
		0.973	0.795	0.646		
2.0	E	0.213	0.684	0.288		
		0.00262	0.0142	57.2 E-04		
	F	0.00606	0.273	0.671		
		0.994	0.804	31.3 E-04		
	G	0.863	0.875	0.671		
		0.944	0.972	0.986		
TOTALS						

TABLE G-19

ANOVA

Schweyer

R^2

FOSTER & BURR q:

Matrix Σx_i 26.103 Matrix Σx_i^2 2.078 522 622 E+01

ν 1.000 q 0.092 $q_{.99}$ 0.198 $q_{.999}$ 0.265

$$Y_{ijkl} = \mu + T_i + C_j + H_k + (TC)_{ij} + (TH)_{ik} + (CH)_{jk} + (TCH)_{ijk} + \varepsilon_{(ijk)l}$$

TOTALS

Temperature (T_i)		Tube (C_j)		Time (H_j)	
$T_{1\cdots}$ =	9.030	$T_{\cdot 1 \cdot\cdot}$ =	3.795	$T_{\cdot\cdot 1 \cdot}$ =	9.500
$T_{2\cdots}$ =	9.909	$T_{\cdot 2 \cdot\cdot}$ =	6.785	$T_{\cdot\cdot 2 \cdot}$ =	7.333
$T_{3\cdots}$ =	7.164	$T_{\cdot 3 \cdot\cdot}$ =	15.523	$T_{\cdot\cdot 3 \cdot}$ =	9.270
	26.103		26.103		26.103

Temperature x Tube $(TC)_{ij}$

$T_{11\cdot\cdot}$ =	0.493	$T_{21\cdot\cdot}$ =	1.860	$T_{31\cdot\cdot}$ =	1.441
$T_{12\cdot\cdot}$ =	3.205	$T_{22\cdot\cdot}$ =	2.516	$T_{32\cdot\cdot}$ =	1.064
$T_{13\cdot\cdot}$ =	5.331	$T_{23\cdot\cdot}$ =	5.532	$T_{33\cdot\cdot}$ =	4.659
	9.029		9.908		7.164

Temperature x Time $(TH)_{ik}$

$T_{1\cdot 1\cdot}$ =	3.353	$T_{2\cdot 1\cdot}$ =	3.464	$T_{3\cdot 1\cdot}$ =	2.683
$T_{1\cdot 2\cdot}$ =	2.654	$T_{2\cdot 2\cdot}$ =	2.823	$T_{3\cdot 2\cdot}$ =	1.856
$T_{1\cdot 3\cdot}$ =	3.023	$T_{2\cdot 3\cdot}$ =	3.622	$T_{3\cdot 3\cdot}$ =	2.625
	9.029		9.909		7.164

Tube x Time $(CH)_{jk}$

$T_{\cdot 11\cdot}$ =	1.870	$T_{\cdot 12\cdot}$ =	0.717	$T_{\cdot 13\cdot}$ =	1.208
$T_{\cdot 21\cdot}$ =	2.298	$T_{\cdot 22\cdot}$ =	1.735	$T_{\cdot 23\cdot}$ =	2.751
$T_{\cdot 31\cdot}$ =	5.332	$T_{\cdot 32\cdot}$ =	4.880	$T_{\cdot 33\cdot}$ =	5.311
	9.500		7.332		9.270

TABLE G-19 (Cont.)

TOTALS - Continued

Temperature x Tube x Time $(TCH)_{ijk}$

$T_{111\cdot}$ =	0.039	$T_{112\cdot}$ =	0.238	$T_{113\cdot}$ =	0.216
$T_{121\cdot}$ =	1.407	$T_{122\cdot}$ =	0.798	$T_{123\cdot}$ =	1.000
$T_{131\cdot}$ =	1.907	$T_{132\cdot}$ =	1.617	$T_{133\cdot}$ =	1.807
	3.353		2.653		3.023
$T_{211\cdot}$ =	0.939	$T_{212\cdot}$ =	0.223	$T_{213\cdot}$ =	0.698
$T_{221\cdot}$ =	0.593	$T_{222\cdot}$ =	0.845	$T_{223\cdot}$ =	1.077
$T_{231\cdot}$ =	1.932	$T_{232\cdot}$ =	1.754	$T_{233\cdot}$ =	1.847
	3.464		2.822		3.622
$T_{311\cdot}$ =	0.892	$T_{312\cdot}$ =	0.255	$T_{313\cdot}$ =	0.294
$T_{321\cdot}$ =	0.298	$T_{322\cdot}$ =	0.092	$T_{323\cdot}$ =	0.674
$T_{331\cdot}$ =	1.493	$T_{332\cdot}$ =	1.509	$T_{333\cdot}$ =	1.657
	2.683		1,856		2.625

TABLE G-20

ANOVA
Schweyer
R^2

(0) $CT = (T....)^2 \div n = (\ 26.103\ )^2 \div 54 =$ 1.261 790 017 E+01

(1) $SS(T_i) = \frac{1}{18}[(\ 9.030\ )^2 + \ldots + (\ 7.164\ )^2] - \overset{(0)}{CT} =$ 2.183 263 300 E-01

(2) $SS(C_j) = \frac{1}{18}[(3.795\ )^2 + \ldots + (\ 15.523\ )^2] - \overset{(0)}{CT} =$ 4.126 643 110 E+00

(3) $SS(H_k) = \frac{1}{18}[(9.500\ )^2 + \ldots + (\ 9.270\ )^2] - \overset{(0)}{CT} =$ 1.549 776 100 E-01

(4) $SS(TC)_{ij} = \frac{1}{6}[(\ 0.493\ )^2 + \ldots + (\ 4.659\ )^2] - \overset{(0)}{CT} - \overset{(1)}{SS(T_i)} - \overset{(2)}{SS(C_j)} =$ 4.108 592 200 E-01

(5) $SS(TH)_{ik} = \frac{1}{6}[(\ 3.353\ )^2 + \ldots + (\ 2.625\ )^2] - \overset{(0)}{CT} - \overset{(1)}{SS(T_i)} - \overset{(3)}{SS(H_k)} =$ 1.649 806 000 E-02

(6) $SS(CH)_{jk} = \frac{1}{6}[(\ 1.870\ )^2 + \ldots + (\ 5.311\ )^2] - \overset{(0)}{CT} - \overset{(2)}{SS(C_j)} - \overset{(3)}{SS(H_k)} =$ 6.391 711 000 E-02

(7) $SS(TCH)_{ijk} = \frac{1}{2}[(0.039\ )^2 + \ldots + (1.657\ )^2]$
$- \overset{(0)}{CT} - \overset{(1)}{SS(T_i)} - \overset{(2)}{SS(C_j)} - \overset{(3)}{SS(H_k)}$
$- \overset{(4)}{SS(TC_{ij})} - \overset{(5)}{SS(TH_{ik})} - \overset{(6)}{SS(CH_{jk})}$
= 3.174 358 900 E-01

(8) $SS(Total) = \Sigma\Sigma\Sigma\Sigma x^2_{ijkl} - \overset{(0)}{CT} =$ 8.167 326 050 E+00

(9) $SS(Error) = SS(Total) - SS(T_i) - SS(C_j) - SS(H_k)$
$- SS(TC)_{ij} - SS(TH)_{ik} - SS(CH)_{jk} - SS(TCH)_{ijk}$
= (8) - (1) - (2) - (3) - (4) - (5) - (6) - (7)
= 2.858 668 720 E+00

TABLE G-21

ANOVA
Schweyer
R^2

	SOURCE	df	SS	MS	F	F.05	F.01
(1)	T_i	2	0.218	0.109	1.03	3.35	5.49
(2)	C_j	2	4.127	2.063	19.49	3.35	5.49
(3)	H_k	2	0.155	0.077	0.73	3.35	5.49
(4)	$(TC)_{ij}$	4	0.411	0.103	0.97	2.73	4.11
(5)	$(TH)_{ik}$	4	0.016	0.004	0.04	2.73	4.11
(6)	$(CH)_{jk}$	4	0.064	0.016	0.15	2.73	4.11
(7)	$(TCH)_{ijk}$	8	0.317	0.040	0.38	2.31	3.26
(9)	Error	27	2.859	0.106			
(8)	Total	53					

TABLE G-22

TUBE E
Schweyer
Log_{10} a

Time \ Temperature	350	375	400
0.5	7.5912	4.5856	7.0434
	7.9196	7.5969	8.3638
1.0	8.1833	7.7771	7.9175
	9.0269	9.1212	8.6269
2.0	7.3383	7.5838	9.7631
	8.5835	7.7444	7.9832

TABLE G-23

TUBE E

FOSTER & BURR q: Schweyer Log_{10} a

Matrix Σx_i 142.7497

Matrix Σx_i^2 1.151 957 216 E+03

ν 1.000

q 0.291 $q_{0.99}$ 0.576 $q_{0.999}$ 0.750

$Y_{ijk} = \mu + T_i + H_j + (TH)_{ij} + \epsilon_{(ij)k}$

TOTALS

Temperature (T_i)

$T_{1..}$ = 48.6428
$T_{2..}$ = 44.4090
$T_{3..}$ = 49.6979
142.7497

Time (H_j)

$T_{.1.}$ = 43.1005
$T_{.2.}$ = 50.6529
$T_{.3.}$ = 48.9963
142.7497

Temperature x Time $(TH)_{ij}$

$T_{11.}$ = 15.5108
$T_{12.}$ = 17.2102 48.6428
$T_{13.}$ = 15.9218

$T_{21.}$ = 12.1825
$T_{22.}$ = 16.8983 44.4090
$T_{23.}$ = 15.3282

$T_{31.}$ = 15.4072
$T_{32.}$ = 16.5444 49.6979
$T_{33.}$ = 17.7463

T... = 142.7497

$\Sigma\Sigma\Sigma Y_{ijk}$ = ______

TABLE G-24

TUBE E

Schweyer Log_{10} a

(0) CT = $(T...)^2 \div n = (142.7497)^2 \div (18) =$ 1.132 082 047 E+03

(1) SS (Temp.) = $\frac{1}{(6)}$ [(48.6428)^2 + ... + (49.6979)^2] − CT = 2.611 709 000 E+00

(2) SS (Time) = $\frac{1}{(6)}$ [(43.1005)^2 + ... + (48.9963)^2] − CT = 5.252 418 000 E+00

(3) SS (H x T) = $\frac{1}{(2)}$ [(15.5108)^2 + ... + (17.7463)^2] − CT − SS (T) − SS (H)

= 2.668 490 000 E+00

(4) SS (Total) = $\Sigma\Sigma\Sigma x_{ijk}^2$ − CT = 1.987 516 900 E+01

(5) SS (Error) = SS (Total) − SS (T) − SS (H) − SS (T x H)

= (4) − (1) − (2) − (3) = 9.342 552 000 E+00

ANALYSIS OF VARIANCE

	Source	df	SS	MS	F	F.05	F.01
(1)	T_i	2	2.6117	1.3059	1.26	4.26	8.02
(2)	H_j	2	5.2524	2.6262	2.53	4.26	8.02
(3)	$(TH)_{ij}$	4	2.6685	0.6671	0.64	3.63	6.42
(5)	Error	9	9.3426	1.0381			
(4)	Total	17	19.8752				

TABLE G-25

Tube F
Schweyer
Log_{10} a

Time \ Temperature	350	375	400
0.5	7.1688	6.4719	9.0154
	7.4120	7.6896	7.7102
1.0	7.8713	7.0158	8.6180
	8.8585	7.9847	8.8600
2.0	8.4093	6.6926	7.0422
	7.4695	7.8071	7.8185

TABLE G-26

Tube F

Schweyer Log_{10} a

FOSTER & BURR q:

Matrix Σx_i 139.9154

Matrix Σx_i^2 1.097 219 275 E+03

ν 1.000

q 0.152 $q_{0.99}$ 0.576 $q_{0.999}$ 0.750

$Y_{ijk} = \mu + T_i + H_j + (TH)_{ij} + \varepsilon_{(ij)k}$

TOTALS

Temperature (T_i)

$T_{1..}$ = 47.1894
$T_{2..}$ = 43.6617
$T_{3..}$ = 49.0643
139.9154

Time (H_j)

$T_{.1.}$ = 45.4679
$T_{.2.}$ = 49.2083
$T_{.3.}$ = 45.2392
139.9154

Temperature x Time $(TH)_{ij}$

$T_{11.}$ = 14.5808
$T_{12.}$ = 16.7298
$T_{13.}$ = 15.8788

$T_{21.}$ = 14.1615
$T_{22.}$ = 15.0005
$T_{23.}$ = 14.4997

$T_{31.}$ = 16.7256
$T_{32.}$ = 17.4780
$T_{33.}$ = 14.8607

T... = ________

$\Sigma\Sigma\Sigma Y_{ijk}$ = ________

TABLE G-27

ANOVA

Tube F

Schweyer Log_{10} a

(0) $CT = (T...)^2 \div n = (139.954)^2 \div (18) =$ 1.087 573 287 E+03

(1) $SS\ (Temp.) = \frac{1}{(6)} [(47.1894)^2 + \dots + (49.0643)^2] - CT =$ 2.508 222 000 E+0(

(2) $SS\ (Time) = \frac{1}{(6)} [(45.4679)^2 + \dots + (45.2392)^2] - CT =$ 1.655 369 000 E+0(

(3) $SS\ (H \times T) = \frac{1}{(2)} [(14.5808)^2 + \dots + (14.8607)^2] - CT - SS\ (T) - SS\ (H)$

$=$ 1.509 719 000 E+00

(4) $SS\ (Total) = \Sigma\Sigma\Sigma x_{ijk}^2 - CT =$ 9.645 988 000 E+00

(5) SS (Error) = SS (Total) - SS (T) - SS (H) - SS (T x H)

= (4) - (1) - (2) - (3) = 3.972 678 000 E+00

ANALYSIS OF VARIANCE

	Source	df	SS	MS	F	F.05	F.01
(1)	T_i	2	2.5082	1.2541	2.84	4.26	8.02
(2)	H_j	2	1.6554	0.8277	1.88	4.26	8.02
(3)	$(TH)_{ij}$	4	1.5097	0.3774	0.86	3.63	6.42
(5)	Error	9	3.9727	0.4414			
(4)	Total	17	9.6460				

TABLE G-28

TUBE G

Schweyer

Log_{10} a

Time \ Temperature	350	375	400
0.5	6.3769	7.6133	7.5788
	7.0265	6.1596	6.3314
1.0	7.2335	7.3971	7.7877
	6.1471	7.0378	7.8042
2.0	6.8839	7.3183	8.2826
	6.2243	6.2788	6.3077

TABLE G-29

Tube G

Schweyer Log_{10} a

FOSTER & BURR q:

Matrix Σx_i = 125.7895

Matrix Σx_i^2 = 8.868 756 633 E+02

ν = 1.000

q = 0.214 $q_{0.99}$ = 0.576 $q_{0.999}$ = 0.750

$Y_{ijk} = \mu + T_i + H_j + (TH)_{ij} + \epsilon_{(ij)k}$

TOTALS

Temperature (T_i)

$T_{1..}$ = 39.8922
$T_{2..}$ = 41.8049
$T_{3..}$ = 44.0924
125.7895

Time (H_j)

$T_{.1.}$ = 41.0865
$T_{.2.}$ = 43.4074
$T_{.3.}$ = 41.2956
125.7895

Temperature x Time $(TH)_{ij}$

$T_{11.}$ = 13.4034
$T_{12.}$ = 13.3806
$T_{13.}$ = 13.1082

$T_{21.}$ = 13.7729
$T_{22.}$ = 14.4349
$T_{23.}$ = 13.5971

$T_{31.}$ = 13.9102
$T_{32.}$ = 15.5919
$T_{33.}$ = 14.5903

T... = ________

$\Sigma\Sigma\Sigma Y_{ijk}$ = ________

TABLE G-30

Tube G

Schweyer Log_{10} a

(0) $CT = (T...)^2 \div n = (125.7895)^2 \div (18) =$ 8.790 554 617 E+02

(1) $SS\ (Temp.) = \frac{1}{(6)} [(39.8922)^2 + \cdots + (44.0924)^2] - CT =$ 1.474 042 100 E+

(2) $SS\ (Time) = \frac{1}{(6)} [(41.0865)^2 + \cdots + (41.2956)^2] - CT =$ 5.494 443 000 E-0

(3) $SS\ (H \times T) = \frac{1}{(2)} [(13.4034)^2 + \cdots + (14.5903)^2] - CT - SS\ (T) - SS\ (H)$

= 3.883 519 000 E-01

(4) $SS\ (Total) = \Sigma\Sigma\Sigma x_{ijk}^2 - CT =$ 7.820 201 600 E+00

(5) SS (Error) = SS (Total) - SS (T) - SS (H) - SS (T x H)

= (4) - (1) - (2) - (3) = 5.408 363 300 E+00

ANALYSIS OF VARIANCE

	Source	df	SS	MS	F	F.05	F.01
(1)	T_i	2	1.4740	0.7370	1.23	4.26	8.02
(2)	H_j	2	0.5494	0.2747	0.46	4.26	8.02
(3)	$(TH)_{ij}$	4	0.3884	0.0971	0.16	3.63	6.42
(5)	Error	9	5.4084	0.6009			
(4)	Total	17	7.8202				

TABLE G-31

TUBE E
Schweyer C

Time \ Temperature	350	375	400
0.5	1.003	0.468	0.516
	0.911	0.890	1.087
1.0	0.846	1.014	0.730
	1.071	0.809	0.988
2.0	1.108	0.665	1.485
	1.105	0.987	0.974

TABLE G-32

Tube E

Schweyer c

FOSTER & BURR q:

Matrix Σx_i 16.657

Matrix Σx_i^2 1.636 834 100 E+01

ν 1.000

q 0.210 $q_{0.99}$ 0.576 $q_{0.999}$ 0.750

$$Y_{ijk} = \mu + T_i + H_j + (TH)_{ij} + \epsilon_{(ij)k}$$

TOTALS

Temperature (T_i)

$T_{1..}$ = 6.044
$T_{2..}$ = 4.833
$T_{3..}$ = 5.780
16.657

Time (H_j)

$T_{.1.}$ = 4.875
$T_{.2.}$ = 5.458
$T_{.3.}$ = 6.324
16.657

Temperature x Time $(TH)_{ij}$

$T_{11.}$ = 1.914
$T_{12.}$ = 1.917
$T_{13.}$ = 2.213

$T_{21.}$ = 1.358
$T_{22.}$ = 1.823
$T_{23.}$ = 1.652

$T_{31.}$ = 1.603
$T_{32.}$ = 1.718
$T_{33.}$ = 2.459

$T_{...}$ =

$\Sigma\Sigma\Sigma Y_{ijk}$ =

TABLE G-33

ANOVA

Tube E

Schweyer c

(0) $CT = (T...)^2 \div n = (16.657)^2 \div (18) =$ 1.541 420 272 E+01

(1) $SS\ (Temp.) = \frac{1}{(6)} [(6.044)^2 + \cdots + (5.780)^2] - CT =$ 1.351 681 100 E

(2) $SS\ (Time) = \frac{1}{(6)} [(4.875)^2 + \cdots + (6.324)^2] - CT =$ 1.771 914 500 E-0

(3) $SS\ (H \times T) = \frac{1}{(2)} [(1.914)^2 + \cdots + (2.459)^2] - CT - SS\ (T) - SS\ (H)$

= 1.234 702 200 E-01

(4) $SS\ (Total) = \Sigma\Sigma\Sigma x_{ijk}^2 - CT =$ 9.541 382 800 E-01

(5) SS (Error) = SS (Total) - SS (T) - SS (H) - SS (T x H)

= (4) - (1) - (2) - (3) = 5.183 085 000 E-01

ANALYSIS OF VARIANCE

	Source	df	SS	MS	F	F.05	F.01
(1)	T_i	2	0.1352	0.0676	1.17	4.26	8.02
(2)	H_j	2	0.1772	0.0886	1.54	4.26	8.02
(3)	$(TH)_{ij}$	4	0.1235	0.0309	0.54	3.63	6.42
(5)	Error	9	0.5183	0.0576			
(4)	Total	17	0.9541				

TABLE G-34

TUBE F

Schweyer C

Time \ Temperature	350	375	400
0.5	0.668	0.996	1.042
	0.715	0.723	0.801
1.0	0.755	1.021	0.896
	0.969	0.682	1.030
2.0	1.036	0.699	0.834
	0.739	0.759	1.013

TABLE G-35

Tube F

Schweyer c

FOSTER & BURR q:

Matrix Σx_i 15.378

Matrix Σx_i^2 1.348 565 000 E+01

ν 1.000

q 0.175 $q_{0.99}$ 0.576 $q_{0.999}$ 0.750

$Y_{ijk} = \mu + T_i + H_j + (TH)_{ij} + \epsilon_{(ij)k}$

TOTALS

Temperature (T_i)

$T_{1..}$ = 4.882
$T_{2..}$ = 4.880
$T_{3..}$ = 5.616
15.378

Time (H_j)

$T_{.1.}$ = 4.945
$T_{.2.}$ = 5.353
$T_{.3.}$ = 5.080
15.378

Temperature x Time $(TH)_{ij}$

$T_{11.}$ = 1.383
$T_{12.}$ = 1.724
$T_{13.}$ = 1.775

$T_{21.}$ = 1.719
$T_{22.}$ = 1.703
$T_{23.}$ = 1.458

$T_{31.}$ = 1.843
$T_{32.}$ = 1.926
$T_{33.}$ = 1.847

T... =

$\Sigma\Sigma\Sigma Y_{ijk}$ =

TABLE G-36

ANOVA

Tube F

Schweyer c

(0) $CT = (T...)^2 \div n = (15.378)^2 \div (18) =$ 1.313 793 800 E+01

(1) $SS\ (Temp.) = \frac{1}{(6)} [(4.882)^2 + \dots + (5.616)^2] - CT =$ 6.002 533 000 E-(

(2) $SS\ (Time) = \frac{1}{(6)} [(4.945)^2 + \dots + (5.080)^2] - CT =$ 1.440 100 000 E-0:

(3) $SS\ (H \times T) = \frac{1}{(2)} [(1.383)^2 + \dots + (1.847)^2] - CT - SS\ (T) - SS\ (H)$

= 5.461 467 000 E-02

(4) $SS\ (Total) = \Sigma\Sigma\Sigma x_{ijk}^2 - CT =$ 3.477 120 000 E-01

(5) SS (Error) = SS (Total) - SS (T) - SS (H) - SS (T x H)

= (4) - (1) - (2) - (3) = 2.186 710 000 E-01

A N A L Y S I S O F V A R I A N C E

	Source	df	SS	MS	F	F.05	F.01
(1)	T_i	2	0.0600	0.0300	1.24	4.26	8.02
(2)	H_j	2	0.0144	0.0072	0.30	4.26	8.02
(3)	$(TH)_{ij}$	4	0.0546	0.0137	0.56	3.63	6.42
(5)	Error	9	0.2187	0.0243			
(4)	Total	17	0.3477				

TABLE G-37

TUBE G

Schweyer C

Time \ Temperature	350	375	400
0.5	0.450	0.393	0.674
	0.618	0.391	0.410
1.0	0.699	0.450	0.797
	0.299	0.500	0.722
2.0	0.567	0.456	0.570
	0.374	0.506	0.508

TABLE G-38

Tube G

Schweyer c

FOSTER & BURR q:

Matrix Σx_i 9.384

Matrix Σx_i^2 5.209 426 000 E+00

ν 1.000

q 0.341 $q_{0.99}$ 0.576 $q_{0.999}$ 0.750

$$Y_{ijk} = \mu + T_i + H_j + (TH)_{ij} + \epsilon_{(ij)k}$$

TOTALS

Temperature (T_i)

$T_{1..}$ = 3.007
$T_{2..}$ = 2.696
$T_{3..}$ = 3.681
9.384

Time (H_j)

$T_{.1.}$ = 2.936
$T_{.2.}$ = 3.467
$T_{.3.}$ = 2.981
9.384

Temperature x Time $(TH)_{ij}$

$T_{11.}$ = 1.068
$T_{12.}$ = 0.998
$T_{13.}$ = 0.941

$T_{21.}$ = 0.784
$T_{22.}$ = 0.950
$T_{23.}$ = 0.962

$T_{31.}$ = 1.084
$T_{32.}$ = 1.519
$T_{33.}$ = 1.078

$T_{...}$ = ______

$\Sigma\Sigma\Sigma Y_{ijk}$ = ______

TABLE G-39

ANOVA

Tube G

Schweyer c

(0) $CT = (T...)^2 \div n = (9.384)^2 \div (18) =$ 4.892 192 000 E+00

(1) $SS\ (Temp.) = \frac{1}{(6)} [(3.007)^2 + \cdots + (3.681)^2] - CT =$ 8.451 233 300 E02

(2) $SS\ (Time) = \frac{1}{(6)} [(2.936)^2 + \ldots + (2.981)^2] - CT =$ 2.889 900 000 E02

(3) $SS\ (H \times T) = \frac{1}{(2)} [(1.068)^2 + \ldots + (1.078)^2] - CT - SS\ (T) - SS\ (H)$

$=$ 4.900 166 700 E-02

(4) $SS\ (Total) = \Sigma\Sigma\Sigma x_{ijk}^2 - CT =$ 3.172 340 000 E-01

(5) $SS\ (Error) = SS\ (Total) - SS\ (T) - SS\ (H) - SS\ (T \times H)$

$= (4) - (1) - (2) - (3) =$ 1.548 210 000 E-012

A N A L Y S I S   O F   V A R I A N C E

	Source	df	SS	MS	F	F.05	F.01
(1)	T_i	2	0.0845	0.0423	2.46	4.26	8.02
(2)	H_j	2	0.0289	0.0144	0.84	4.26	8.02
(3)	$(TH)_{ij}$	4	0.0490	0.0123	0.71	3.63	6.42
(5)	Error	9	0.1548	0.0172			
(4)	Total	17	0.3172				

TABLE G-40

TUBE E

Schweyer R^2

Time \ Temperature	350	375	400
0.5	0.000147	0.755	0.834
	0.000392	0.184	0.0577
1.0	0.228	0.000200	0.255
	0.0104	0.223	0.000403
2.0	0.213	0.684	0.288
	0.00262	0.0142	0.000572

TABLE G-41

Tube E

Schweyer R^2

FOSTER & BURR q:

Matrix Σx_i 3.750 634 000 E+00

Matrix Σx_i^2 2.065 990 649 E+00

ν 1.000

q 0.248 $q_{0.99}$ 0.576 $q_{0.999}$ 0.750

$Y_{ijk} = \mu + T_i + H_j + (\ddot{TH})_{ij} + \epsilon_{(ij)k}$

TOTALS

Temperature (T_i)

$T_{1..}$ = 0.455

$T_{2..}$ = 1.860

$T_{3..}$ = 1.436

3.751

Time (H_j)

$T_{.1.}$ = 1.831

$T_{.2.}$ = 0.717

$T_{.3.}$ = 1.202

3.751

Temperature x Time $(TH)_{ij}$

$T_{11.}$ = 0.000539

$T_{12.}$ = 0.238

$T_{13.}$ = 0.216

$T_{21.}$ = 0.939

$T_{22.}$ = 0.223

$T_{23.}$ = 0.698

$T_{31.}$ = 0.892

$T_{32.}$ = 0.255

$T_{33.}$ = 0.289

T... =

$\Sigma\Sigma\Sigma Y_{ijk}$ =

TABLE G-42

ANOVA

Tube E

Schweyer R^2

(0) CT = $(T...)^2 \div n$ = $(3.750\ 634\ 000\ E{+}00)^2 \div (18)$ = 7.815 141 889 E-0

(1) SS (Temp.) = $\frac{1}{(6)}$ $[(0.455)^2 + \cdots + (1.436)^2]$ - CT = 1.732 736 444 E-0

(2) SS (Time) = $\frac{1}{(6)}$ $[(1.831)^2 + \cdots + (1.202)^2]$ - CT = 1.037 281 444 E-0

(3) SS (H x T) = $\frac{1}{(2)}$ $[(0.000\ 539)^2 + \cdots + (0.289)^2]$ - CT - SS (T) - SS (H) = 1.745 671 683 E-01

(4) SS (Total) = $\Sigma\Sigma\Sigma x_{ijk}^2$ - CT = 1.284 476 470 E+00

(5) SS (Error) = SS (Total) - SS (T) - SS (H) - SS (T x H)

= (4) - (1) - (2) - (3) = 8.329 085 137 E-01

ANALYSIS OF VARIANCE

	Source	df	SS	MS	F	F.05	F.01
(1)	T_i	2	0.1733	0.0866	0.94	4.26	8.02
(2)	H_j	2	0.1037	0.0519	0.56	4.26	8.02
(3)	$(TH)_{ij}$	4	0.1746	0.0436	0.47	3.63	6.42
(5)	Error	9	0.8329	0.0925			
(4)	Total	17	1.2845				

TABLE G-43

TUBE F
Schweyer R^2

Time / Temperature	350	375	400
0.5	0.894	0.000420	0.018
	0.513	0.593	0.280
1.0	0.795	0.000320	0.0369
	0.00311	0.845	0.00506
2.0	0.0606	0.273	0.671
	0.994	0.804	0.00313

TABLE G-44
Tube F
Schweyer R^2

FOSTER & BURR q:

Matrix Σx_i 6.839 540 000 E+00

Matrix Σx_i^2 5.009 319 321 E+00

ν 1.000

q 0.169 $q_{0.99}$ 0.576 $q_{0.999}$ 0.750

$$Y_{ijk} = \mu + T_i + H_j + (TH)_{ij} + \epsilon_{(ij)k}$$

TOTALS

Temperature (T_i)

$T_{1..}$ = 3.260
$T_{2..}$ = 2.516
$T_{3..}$ = 1.064
6.840

Time (H_j)

$T_{.1.}$ = 2.298
$T_{.2.}$ = 1.735
$T_{.3.}$ = 2.806
6.840

Temperature x Time $(TH)_{ij}$

$T_{11.}$ = 1.407
$T_{12.}$ = 0.798
$T_{13.}$ = 1.055

$T_{21.}$ = 0.593
$T_{22.}$ = 0.845
$T_{23.}$ = 1.077

$T_{31.}$ = 0.298
$T_{32.}$ = 0.092
$T_{33.}$ = 0.674

T... =

$\Sigma\Sigma\Sigma Y_{ijk}$ =

TABLE G-45

ANOVA

Tube F

Schweyer R^2

(0) $CT = (T...)^2 \div n = (6.839\ 540\ 000\ E{+}00)^2 \div (18) = 2.598\ 850\ 412\ E{+}0$

(1) $SS\ (Temp.) = (\frac{1}{6})\ [(3.260)^2 + \cdots + (1.064)^2] - CT = 4.161\ 415\ 880\ E{-}$

(2) $SS\ (Time) = (\frac{1}{6})\ [(2.298)^2 + \cdots + (2.806)^2] - CT = 9.526\ 042\ 100\ E{-}0$

(3) $SS\ (H \times T) = (\frac{1}{2})\ [(1.407)^2 + \cdots + (0.674)^2] - CT - SS\ (T) -$

$SS\ (H) = 1.430\ 600\ 790\ E{-}01$

(4) $SS\ (Total) = \Sigma\Sigma\Sigma x_{ijk}^2 - CT = 2.410\ 476\ 909\ E{+}00$

(5) $SS\ (Error) = SS\ (Total) - SS\ (T) - SS\ (H) - SS\ (T \times H)$

$= (4) - (1) - (2) - (3) = 1.756\ 006\ 821\ E{+}00$

	ANALYSIS OF VARIANCE						
	Source	df	SS	MS	F	F.05	F.01
(1)	T_i	2	0.4161	0.2081	1.07	4.26	8.02
(2)	H_j	2	0.0953	0.0476	0.24	4.26	8.02
(3)	$(TH)_{ij}$	4	0.1431	0.0358	0.18	3.63	6.42
(5)	Error	9	1.7560	0.1951			
(4)	Total	17	2.4105				

TABLE G-46

TUBE G

Schweyer R^2

Time \ Temperature	350	375	400
0.5	1.000	0.961	0.567
	0.907	0.971	0.926
1.0	0.644	0.959	0.863
	0.973	0.795	0.646
2.0	0.863	0.875	0.671
	0.944	0.972	0.986

TABLE G-47

Tube G

Schweyer R^2

FOSTER & BURR q:

Matrix Σx_i 15.523

Matrix Σx_i^2 1.371 198 300 E+01

ν 1.000

q 0.218 $q_{0.99}$ 0.576 $q_{0.999}$ 0.750

$Y_{ijk} = \mu + T_i + H_j + (TH)_{ij} + \varepsilon_{(ij)k}$

TOTALS

Temperature (T_i)

$T_{1..}$ = 5.331
$T_{2..}$ = 5.533
$T_{3..}$ = 4.659
15.523

Time (H_j)

$T_{.1.}$ = 5.332
$T_{.2.}$ = 4.880
$T_{.3.}$ = 5.311
15.523

Temperature x Time $(TH)_{ij}$

$T_{11.}$ = 1.907
$T_{12.}$ = 1.617
$T_{13.}$ = 1.807

$T_{21.}$ = 1.932
$T_{22.}$ = 1.754
$T_{23.}$ = 1.847

$T_{31.}$ = 1.493
$T_{32.}$ = 1.509
$T_{33.}$ = 1.657

T... = ______

$\Sigma\Sigma\Sigma Y_{ijk}$ = ______

TABLE G-48

ANOVA

Tube G

Schweyer R^2

(0) $CT = (T...)^2 \div n = (15.523)^2 \div (18) =$ 1.338 686 272 E+01

(1) $SS\ (Temp.) = \frac{1}{(6)} [(5.331)^2 + \dots + (4.659)^2] - CT =$ 6.979 245 000 E-02

(2) $SS\ (Time) = \frac{1}{(6)} [(5.332)^2 + \dots + (5.311)^2] - CT =$ 2.169 478 000 E-02

(3) $SS\ (H \times T) = \frac{1}{(2)} [(1.907)^2 + \dots + (1.657)^2] - CT - SS\ (T) - SS\ (H)$

= 1.610 755 000 E-02

(4) $SS\ (Total) = \Sigma\Sigma\Sigma x_{ijk}^2 - CT =$ 3.251 202 800 E-01

(5) SS (Error) = SS (Total) - SS (T) - SS (H) - SS (T x H)

= (4) - (1) - (2) - (3) = 2.175 255 000 E-01

ANALYSIS OF VARIANCE

	Source	df	SS	MS	F	F.05	F.01
(1)	T_i	2	0.0698	0.0349	1.44	4.26	8.02
(2)	H_j	2	0.0217	0.0108	0.45	4.26	8.02
(3)	$(TH)_{ij}$	4	0.0161	0.0040	0.17	3.63	6.42
(5)	Error	9	0.2175	0.0242			
(4)	Total	17	0.3251				

TABLE G-49

TUBE E

Schweyer

Log_{10} ($\eta_{0.05}$ ÷ 1.00 E+07)

Time \ Temperature	350	375	400
0.5	0.5876	0.5039	0.6734
	1.0358	1.1383	1.2507
1.0	1.3829	1.7594	1.2690
	1.9345	1.2333	1.6428
2.0	1.1981	1.0191	2.1319
	1.4476	0.8244	0.8528

TABLE G-50

Tube E

Schweyer

Log_{10} ($\eta_{0.05}$ ÷1.00 E+07)

FOSTER & BURR q:

Matrix Σx_i 20.8777

Matrix Σx_i^2 3.008 734 329 E+01

ν 1.000

q 0.318 $q_{0.99}$ 0.576 $q_{0.999}$ 0.750

$Y_{ijk} = \mu + T_i + H_j + \ddot{(TH)}_{ij} + \varepsilon_{(ij)k}$

TOTALS

Temperature (T_i)

$T_{1..}$ = 7.5865
$T_{2..}$ = 5.4706
$T_{3..}$ = 7.8206
20.8777

Time (H_j)

$T_{.1.}$ = 4.1819
$T_{.2.}$ = 9.2219
$T_{.3.}$ = 7.4739
20.8777

Temperature x Time $(TH)_{ij}$

$T_{11.}$ = 1.6234
$T_{12.}$ = 3.3174
$T_{13.}$ = 2.6457

$T_{21.}$ = 0.6344
$T_{22.}$ = 2.9927
$T_{23.}$ = 1.8435

$T_{31.}$ = 1.9241
$T_{32.}$ = 2.9118
$T_{33.}$ = 2.9847

T... = ______

$\Sigma\Sigma\Sigma Y_{ijk}$ = ______

TABLE G-50

ANOVA

Tube E

Schweyer

Log_{10} ($\eta_{0.05}$ ÷ 1.00 E + 07)

(0) $CT = (T...)^2 \div n = (20.8777)^2 \div (\ 18\ ) =$ 2.421 546 429 E+01

(1) $SS\ (Temp.) = \frac{1}{(6)}\ [(7.5865)^2 + \cdots + (7.8206)^2] - CT =$ 5.585 742 100 E-0

(2) $SS\ (Time) = \frac{1}{(6)}\ [(4.1819)^2 + \cdots + (7.4739)^2] - CT =$ 2.183 020 440 E+0

(3) $SS\ (H \times T) = \frac{1}{(2)}\ [(1.6234)^2 + \cdots + (2.9847)^2] - CT - SS\ (T) - SS\ (H)$

= 2.862 819 900 E-01

(4) $SS\ (Total) = \Sigma\Sigma\Sigma x_{ijk}^2 - CT =$ 5.871 879 000 E+00

(5) SS (Error) = SS (Total) - SS (T) - SS (H) - SS (T x H)

= (4) - (1) - (2) - (3) = 2.844 002 360 E+00

ANALYSIS OF VARIANCE

	Source	df	SS	MS	F	F.05	F.01
(1)	T_i	2	0.5586	0.2793	0.88	4.26	8.02
(2)	H_j	2	2.1830	1.0915	3.45	4.26	8.02
(3)	$(TH)_{ij}$	4	0.2863	0.0716	0.23	3.63	6.42
(5)	Error	9	2.8440	0.3160			
(4)	Total	17	5.8719				

TABLE G-51

TUBE F

Schweyer

Log_{10} (η0.05 ÷ 1.00 E+07)

Time \ Temperature	350	375	400
0.5	0.6001	1.6015	1.9609
	0.7833	1.9734	0.9690
1.0	1.9101	2.0934	1.7536
	1.8986	0.8112	1.8245
2.0	2.3629	1.1361	1.1988
	0.8088	0.6323	1.2653

TABLE G-52

Tube F

Schweyer

$\text{Log}_{10}\ (\eta_{0.05} \div 1.00\ E + 07)$

FOSTER & BURR q:

Matrix Σx_i = 25.5838

Matrix Σx_i^2 = 4.188 041 058 E+01

ν = 1.000

q = 0.320 $\quad q_{0.99}$ = 0.576 $\quad q_{0.999}$ = 0.750

$Y_{ijk} = \mu + T_i + H_j + (TH)_{ij} + \varepsilon_{(ij)k}$

TOTALS

Temperature (T_i)

$T_{1..}$ = 8.3638
$T_{2..}$ = 8.2479
$T_{3..}$ = 8.9721
25.5838

Time (H_j)

$T_{.1.}$ = 7.8882
$T_{.2.}$ = 10.2914
$T_{.3.}$ = 7.4042
25.5838

Temperature x Time $(TH)_{ij}$

$T_{11.}$ = 1.3834
$T_{12.}$ = 3.8087
$T_{13.}$ = 3.1717

$T_{21.}$ = 3.5749
$T_{22.}$ = 2.9046
$T_{23.}$ = 1.7684

$T_{31.}$ = 2.9299
$T_{32.}$ = 3.5781
$T_{33.}$ = 2.4641

$T_{...}$ = ______

$\Sigma\Sigma\Sigma Y_{ijk}$ = ______

TABLE G-53

ANOVA

Tube F

Schweyer

$\text{Log}_{10}\ (\eta_{0.05} \div 1.00\ E + 07)$

(0) $CT = (T_{...})^2 \div n = (25.5838)^2 \div (18) =$ 3.636 282 347 E+01

(1) $SS\ (\text{Temp.}) = \frac{1}{(6)} [(8.3638)^2 + \cdots + (8.9721)^2] - CT =$ 5.044 041 000 E-02

(2) $SS\ (\text{Time}) = \frac{1}{(6)} [(7.8882)^2 + \cdots + (7.4042)^2] - CT =$ 7.969 750 000 E-01

(3) $SS\ (H \times T) = \frac{1}{(2)} [(1.3834)^2 + \cdots + (2.4641)^2] - CT - SS\ (T) - SS\ (H)$

$=$ 1.930 973 270 E+00

(4) $SS\ (\text{Total}) = \Sigma\Sigma\Sigma x_{ijk}^2 - CT =$ 5.517 587 110 E+00

(5) $SS\ (\text{Error}) = SS\ (\text{Total}) - SS\ (T) - SS\ (H) - SS\ (T \times H)$

$= (4) - (1) - (2) - (3) =$ 2.739 198 430 E+00

ANALYSIS OF VARIANCE

	Source	df	SS	MS	F	F.05	F.01
(1)	T_i	2	0.0504	0.0252	0.08	4.26	8.02
(2)	H_j	2	0.7970	0.3985	1.31	4.26	8.02
(3)	$(TH)_{ij}$	4	1.9310	0.4827	1.59	3.63	6.42
(5)	Error	9	2.7392	0.3044			
(4)	Total	17	5.5176				

TABLE G-54

TUBE G

Schweyer

Log_{10} ($\eta 0.05 \div 1.00$ E+07)

Time / Temperature	350	375	400
0.5	0.0931	1.2613	1.0026
	0.5231	-0.0730	0.0983
1.0	0.6252	0.7315	1.0515
	0.0588	0.6880	1.1664
2.0	0.4390	0.4002	0.6021
	0.0382	-0.0784	-0.0517

TABLE G-55

Tube G

Schweyer

Log_{10} ($\eta_{0.05} \div 1.00$ E + 07)

FOSTER & BURR q:

Matrix Σx_i 8.5762

Matrix Σx_i^2 7.487 971 640 E+00

ν 1.000

q 0.273 $q_{0.99}$ 0.576 $q_{0.999}$ 0.750

$$Y_{ijk} = \mu + T_i + H_j + (TH)_{ij} + \varepsilon_{(ij)k}$$

TOTALS

Temperature (T_i)

$T_{1..}$ = 1.7774

$T_{2..}$ = 2.9296

$T_{3..}$ = 3.8692

8.5762

Time (H_j)

$T_{.1.}$ = 2.9054

$T_{.2.}$ = 4.3214

$T_{.3.}$ = 1.3494

8.5782

Temperature x Time $(TH)_{ij}$

$T_{11.}$ = 0.6162

$T_{12.}$ = 0.6840

$T_{13.}$ = 0.4772

$T_{21.}$ = 1.1883

$T_{22.}$ = 1.4195

$T_{23.}$ = 0.3218

$T_{31.}$ = 1.1009

$T_{32.}$ = 2.2179

$T_{33.}$ = 0.5504

T... = ______

$\Sigma\Sigma\Sigma Y_{ijk}$ = ______

TABLE G-56

ANOVA

Schweyer

$\text{Log}_{10}\ (n_{0.05} \div 1.00\ \text{E} + 07)$

(0) $CT = (T...)^2 \div n = (8.5762)^2 \div (\ 18\ ) =$ 4.086 178 136 E+00

(1) $SS\ (\text{Temp.}) = \frac{1}{(6)}\ [(1.7774)^2 + \dots + (3.8692)^2] - CT =$ 3.658 911240 E-0

(2) $SS\ (\text{Time}) = \frac{1}{(6)}\ [(2.9054)^2 + \dots + (1.3494)^2] - CT =$ 7.366 097 770 E-0

(3) $SS\ (H \times T) = \frac{1}{(2)}\ [(0.6162)^2 + \dots + (0.5504)^2] - CT - SS\ (T) - SS\ (H)$

$=$ 3.312 569 830 E-01

(4) $SS\ (\text{Total}) = \Sigma\Sigma\Sigma x_{ijk}^2 - CT =$ 3.401 793 504 E+00

(5) $SS\ (\text{Error}) = SS\ (\text{Total}) - SS\ (T) - SS\ (H) - SS\ (T \times H)$

$= (4) - (1) - (2) - (3) =$ 1.968 035 620 E+00

	A N A L Y S I S O F V A R I A N C E						
	Source	df	SS	MS	F	F.05	F.01
(1)	T_i	2	0.3659	0.1829	0.48	4.26	8.02
(2)	H_j	2	0.7366	0.3683	0.97	4.26	8.02
(3)	$(TH)_{ij}$	4	0.3313	0.0828	0.22	3.63	6.42
(5)	Error	9	3.4018	0.3780			
(4)	Total	17	1.9680				