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Final Report - Phase I

STRUCTURAL DESIGN OF ASPHALT PAVEMENTS (ARIZONA)

by

R. A. Jimenez

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Arizona Transportation and Traffic Institute
College of Engineering
The University of Arizona
Tucson, Arizona 85721

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16. Abstract

The report presents a new concept for the design of asphalt pavements in Arizona. The criteria for design are based on fatigue response of a selected 3-layered system to tensile stresses in the asphaltic surface course and to vertical compressive strains in the subgrade. These values of stresses and strains are calculated using elastic theory and Chevron's computer program. A new classification of highway loads is presented and environmental effects on stresses and strains are accounted for by the influence of regional temperature and rainfall on the moduli of the surface and subgrade. The new design procedure is illustrated using actual design data and the results obtained are compared to designs obtained by Arizona, Kansas, Texas and Shell procedures.

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Final Report - Phase I

STRUCTURAL DESIGN OF ASPHALT PAVEMENTS (ARIZONA)

INTRODUCTION

This report is concerned with the development of a pavement design procedure based on repeated stresses and strains. The procedure is intended for use by The Arizona Highway Department for asphalt pavements and as such it will contain a certain amount of empiricism unique to the state. Inasmuch as this is a preliminary (progress) report, much bibliographical material reviewed will not be discussed at this time; and, further, some design assumptions have been made with a minimum of written justification. The objective of the study is to yield a viable and tenable pavement design procedure. Presently, we believe that such a procedure must incorporate the use of elastic theory.

DESIGN PROCEDURES BASED ON ELASTIC THEORY

The review of American publications concerned with asphalt pavement design indicated three methods based directly on elastic theory. These procedures known as the Texas, Kansas, and Shell are described briefly in the following paragraph.

Texas

The details of this method were published in 1955 by the Highway Research Board (1)*. The design is based on prevention of overstress in the pavement system. The solution for stress was based on Boussinesq's (2) equations; however the shear stress at a point was reduced by a factor to account for differences in modulus of elasticity in a layered pavement system. Wheel loads of 24,000 and 10,000 lb. with 100 psi inflation were used to calculate stresses for different points at depths ranging from 1 inch to 28 inches. Mohr circles were drawn for stresses at each elevation and shear stress envelopes were drawn. Texas classified its soils on the basis of the soil's Mohr envelope for limiting shear strength. A comparison between the two envelopes of stress versus strength solved the pavement design problem of establishing the kind (strength) and depth of cover of soil required for each layer to prevent overstressing. Mathematical theory was used to establish the kind and amount of soils for other wheel loads. A somewhat complex procedure was used for preparing and testing soils for

*The numbers in parentheses correspond to the references listed at the end of the report.

classification (2). The design wheel load was obtained by averaging the 10 heaviest wheels anticipated on the roadway. Regional or environmental effects were accounted for by the method of specimen preparation and testing.

Kansas

The Kansas procedure utilized mathematical theory in that the design was based on limiting deflections of the pavement (3, 4). The thickness of pavement was proportional to the depth at which there was a specified vertical displacement under a standard load. This depth was calculated from a Boussinesq condition with the following equation:

$$Z = T = \sqrt{\left(\frac{3P}{2\pi ES}\right)^2 - a^2} \qquad \text{---- } 1$$

where: Z is the depth at which displacement S, occurs, in.

P is the load of a single wheel, lb.

S is the allowable deflections, in.

E is the soil's modulus of elasticity, psi, and

a is the radius of the circular loaded areas, in.

The basic equation was modified to consider adjustments in pavement thickness caused by (a) different traffic conditions, (b) variations in E of the subgrade soil caused by moisture, and (c) paving materials placed over the subgrade. The new equation took the form as follows.

$$T = \sqrt{\left(\frac{3Pmn}{2\pi ES}\right)^2 - a^2} \cdot \sqrt[3]{E/E_p} \qquad \text{---- } 2$$

where the new terms are:

m is a coefficient varying from 1/2 to 2 depending on traffic volume given in AADT

n is a coefficient varying from 1/2 to 1 depending on rainfall to adjust for the possibility that the subgrade will not be saturated

E_p is the modulus of elasticity of the pavement or surface course.

The term $\sqrt[3]{E/E_p}$ is an equivalency factor to convert depth of subgrade soil to thickness of pavement. In this approach, it was apparently assumed that all of the surface deflection resulted from compression of the subgrade only, that is, the pavement layer(s) was incompressible.

Kansas has set the allowable deflection, S, to be 0.10 inch and suggests a value for E_p equal to 15,000 psi. The value of 0.10 inch for S has been considered to be excessive since performance studies have indicated that the limiting pavement surface deflection should be in the neighborhood of 0.04 inch and also that approximately 30 percent of the surface deflection is due to compression of the pavement layers (5).

Shell

Whereas the Texas and Kansas methods of pavement design are based on the stress distribution in a homogeneous material, the Shell procedure is based on Burmister's equations for the stresses in a layered system. Although Burmister's equations were presented in 1943 (6), extensive solutions to practical and general conditions did not become available until the 1960's when electronic computers made this possible (7, 8 and 9).

Dormon and Metcalf (10) describe a development for flexible pavement design curves. The design procedure is based on the capability of the pavement system to resist repeated tensile strains at the underside of the bituminous surface and also repeated vertical compressive strains at the top of the subgrade. Traffic loadings are converted to an equivalent 9,000 lb. wheel with an 80 psi contact pressure. With this design wheel, the subgrade strain is computed at a point directly under the center of a circular load area of radius equal to 6 in.; the tensile strain of the surfacing is found assuming the load contact area to have a radius of 4.2 in.

Design charts have been developed for determining various thickness combinations of surface course and base course which will withstand the accumulated distress due to tensile strain at the surface and compressive strain at the subgrade. A chart was developed for each value of subgrade modulus (E_3) selected and for each it appears that E_1 and E_2 are held constant. Additionally, it was assumed that a linear logarithmic relationship existed between strain and number of load repetitions to cause failure.

The general basis or criteria of the Shell procedure will be used but modified in the development of the design procedure to be discussed.

CONCEPTS FOR PROPOSED DESIGN PROCEDURE

The reader is cautioned that several conditions to the design problem are assumed for the sake of practicality and/or because of ignorance at this time as to real effects of specific design factors. Now it is generally agreed that a pavement design procedure must include three basic factors, (a) the stresses that will be imposed on the pavement, (b) the strength or stress-resistance characteristics of the pavement layers, and (c) the effects of the environment on the stress distributing and resisting characteristics of the pavement. Basic to the design procedure must be knowledge as to the form and definition of failure of the pavement. Recognizing and acknowledging the efforts of others and assuming that certain minimum requirements will be specified for the manufactured layers (surface and base), the proposed design will have criteria of limiting numbers for (a) tensile stresses at the surface course and (b) vertical compressive strains at the subgrade. The tensile stress consideration is to control surface cracking and the compressive strain limitation is to guard against settling. It is expected that at the end of the design period, the pavement system will still be serviceable but in need of rehabilitation.

Wheel Loads and Effects

It is evident that the design procedure is greatly dependent on the number and effects of wheel loads. The effects of concern are those related to the tensile stress at the bottom of the surface course and also the strain at the top of the subgrade. It is believed necessary to account directly for every type of wheel load with reference to its effects on the design criteria. The use of wheel-load equivalency is not necessary nor perhaps justified (20) in a procedure relying on calculated stresses and strains. As a consequence, a classification of vehicles was set and defined as follows:

(a) Passenger - vehicle with one rear axle having two tires each with 1,400 lb. at 28 psi inflation pressure.

(b) 2P - a pick-up with one rear axle having two tires each with 1,600 lb. at 32 psi inflation pressure.

(c) 2S - a delivery truck with one rear axle having two tires each with 1,800 lb. at 45 psi inflation pressure.

(d) SA - a truck with one rear axle having 2 pairs of tires, each tire with 4,500 lb. at 80 psi inflation pressure and a center to center spacing of 13 in. This corresponds to the regular "single-axled" truck with duals and generally limited to an axle load of 18,000 lb.

(e) TA - a truck or bus with two SA configurations spaced 48 in. apart. This arrangement is similar to the "tandem-axled" truck except that it could be considered to be overloaded since it carries 36,000 lb. instead of 32,000 lb.

(f) Others - a truck or combination of SA and TA.

Figure 1 shows the standard highway vehicle classification and also the one described above.

It would seem that certain wheel axle units have been loaded too heavily, for example, the Passenger and TA, but for the time being these values will be kept for computational illustrations.

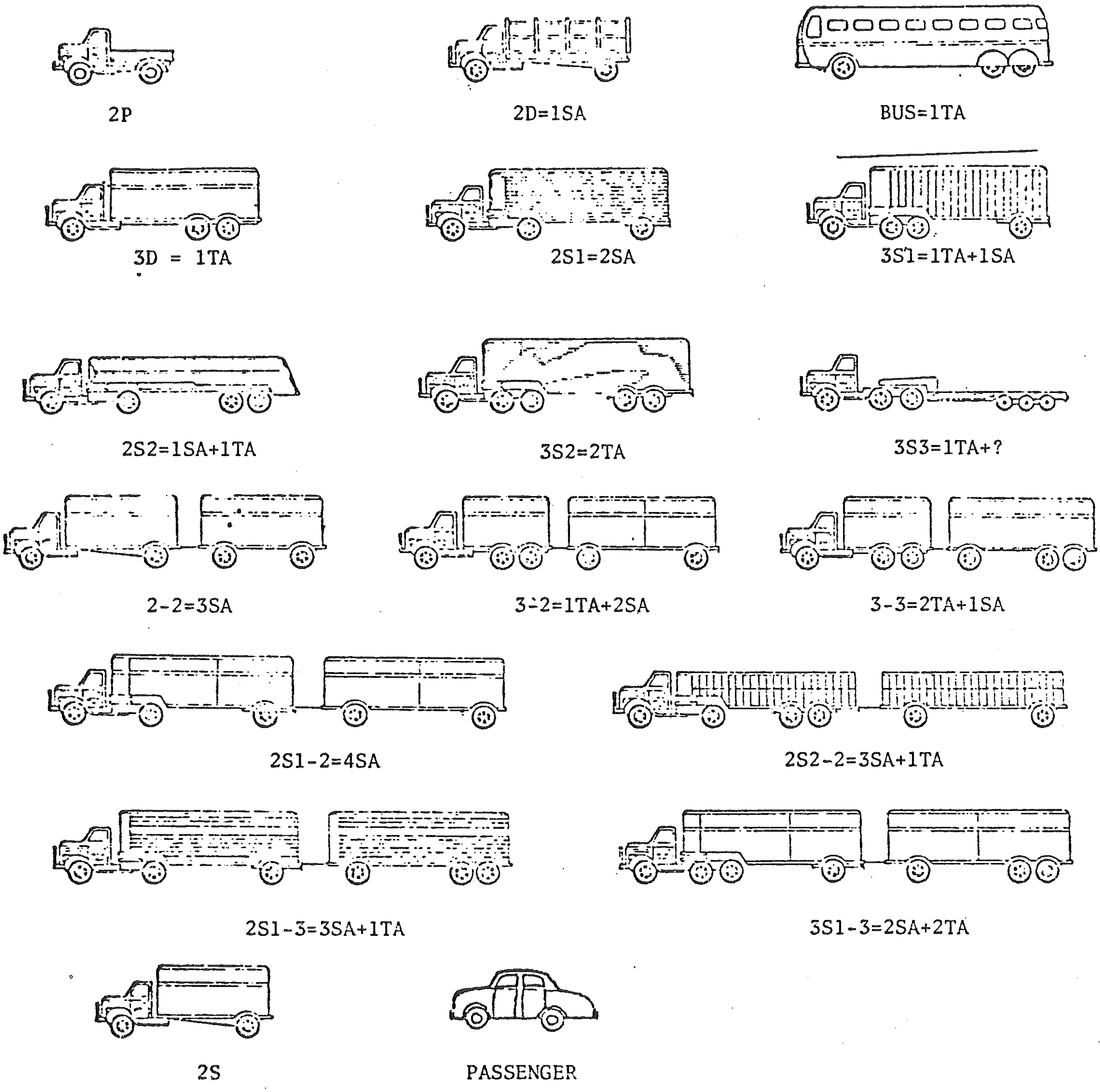


Figure 1. Illustration of Vehicle Classifications by the Arizona Highway Department and the University of Arizona

In the calculations for stresses and strains in the pavement system, it is apparent that maximum values will occur directly under the center of a tire for a two-tire axle. However, for SA and TA classifications that is not apparent and the locations for maximum values of stress and strain must be established. Since the distribution of wheel load to the subgrade is affected by the stiffness (dependent on thickness and material property) of the pavement layers, computations for locating the maximum stresses and strains were made for various pavement systems. These systems were chosen to include a range of thickness and also stiffness of pavement layers. Stresses and strains at various points within the pavement systems were calculated using the Chevron program (8). The stresses and strains examined were the radial tensile stresses at the first interface and the vertical strains at the second interface (top of subgrade) for the following conditions:

1. Loading

   (a) SA, single axle with duals of 4,500 lb. at 80 psi and spacing of 13 in. center to center

   (b) TA, tandem axles 48 in. apart with duals as in SA

2. Pavement (modulus values in psi)

   (a) E1=200,000; E2=20,000; E3=10,000; H1=3"; H2= 6"
   (b) E1=200,000; E2= 8,000; E3= 2,000; H1=6"; H2=18"
   (c) E1=100,000; E2=20,000; E3= 5,000; H1=3"; H2= 6"
   (d) E1=100,000; E2=20,000; E3= 5,000; HT=6"; H2=18"

A sketch of the loading by a TA is shown in Figure 2. The values for modulus of elasticity (E) and Poisson's ratio (μ) were assumed following a review of the literature (see Appendix A).

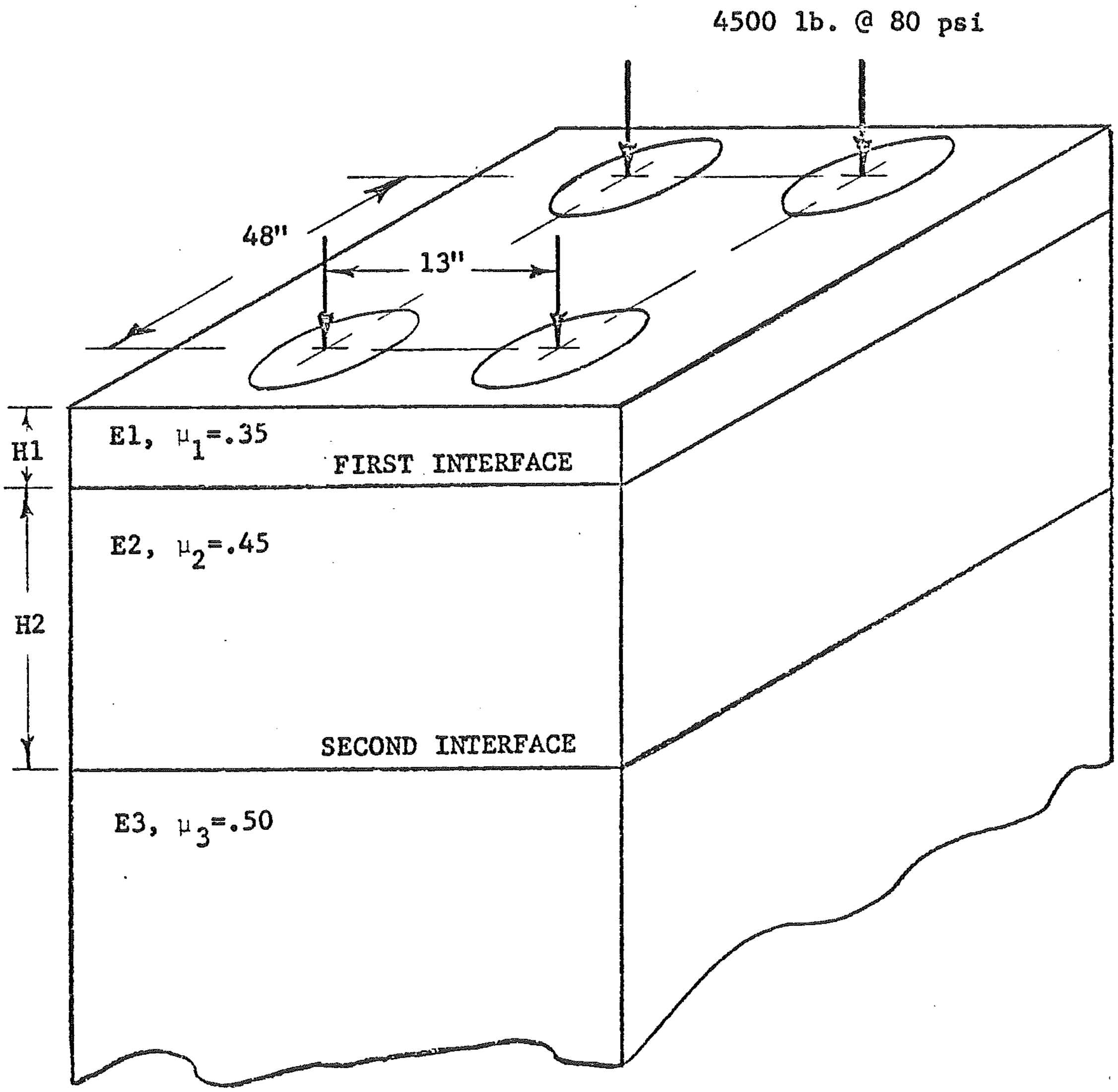


FIGURE 2. Loading of TA Vehicle on a 3-Layered Pavement System.

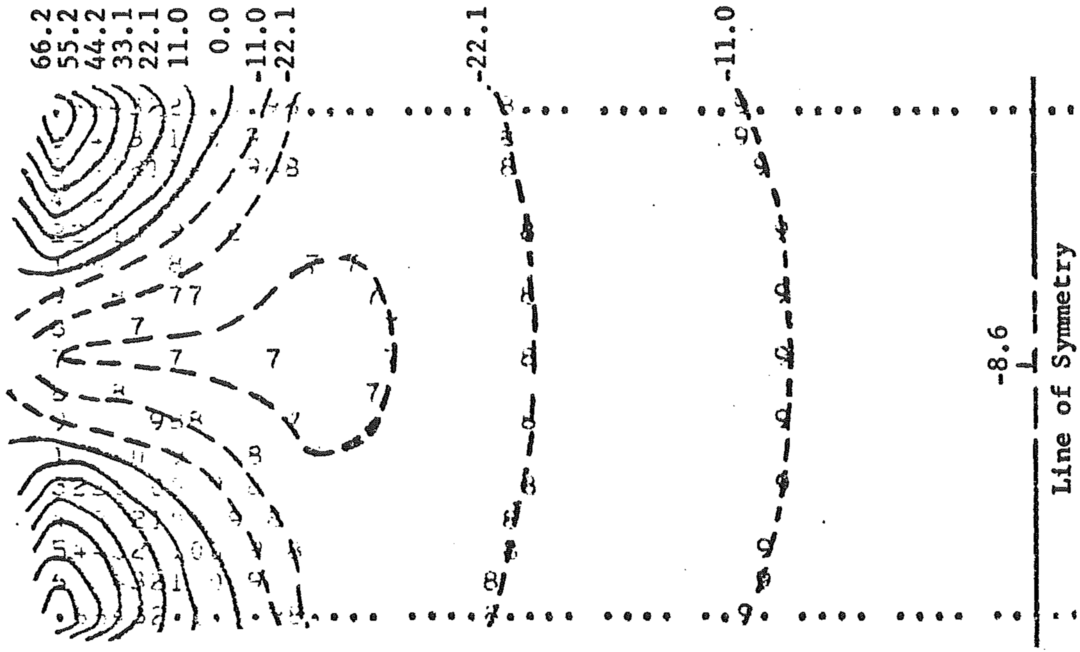
Stress and strain contours for the TA loadings are shown in Figures 3, 4, 5 and 6. Plots such as these help in analyzing the influence of load and pavement variables on the value and location of maximum stresses and strains. Examination of the data obtained from the computations indicated that the locations for maximum radial stress and vertical strain on their respective interface were as follows:

1. SA
   (a) The maximum radial stress is directly under a wheel center
   (b) The maximum vertical strain is midway between the two-wheel center

2. TA
   (a) The maximum radial stress is directly under a wheel center
   (b) The maximum vertical strain is midway between two-wheel centers of an axle most of the time but one should examine the strain under a wheel center (Figures 5a and 6a)

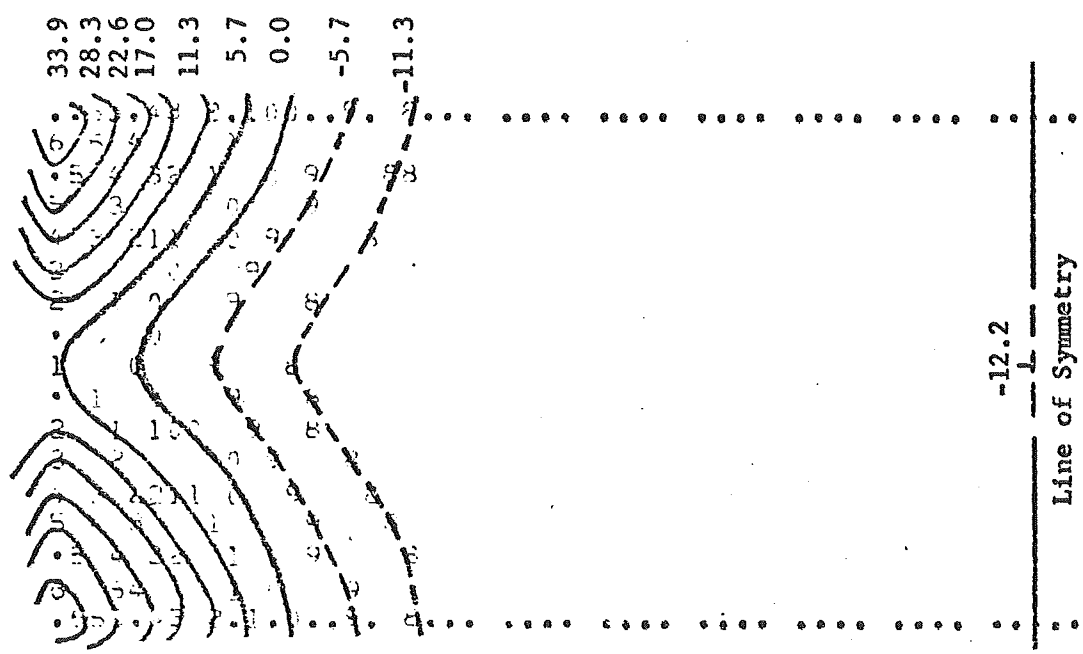
Additionally it was noted that the maximum SA stress was greater than that for the TA; however, the TA stress is applied twice for each passage. The value of the center-of-a-wheel tensile radial stress for a TA is affected by axle spacing since the wheels from the other axle would be contributing compressive stresses at that point. Figures 5a and 6a show that below the center of the loads, the vertical subgrade strain is positive; this simply means that the Poisson effect of the horizontal compressive stresses overcame the compressive strain of the vertical compressive stress.

The choice of selecting subgrade strain over subgrade or surface deflection for investigation was made on the desire to use material



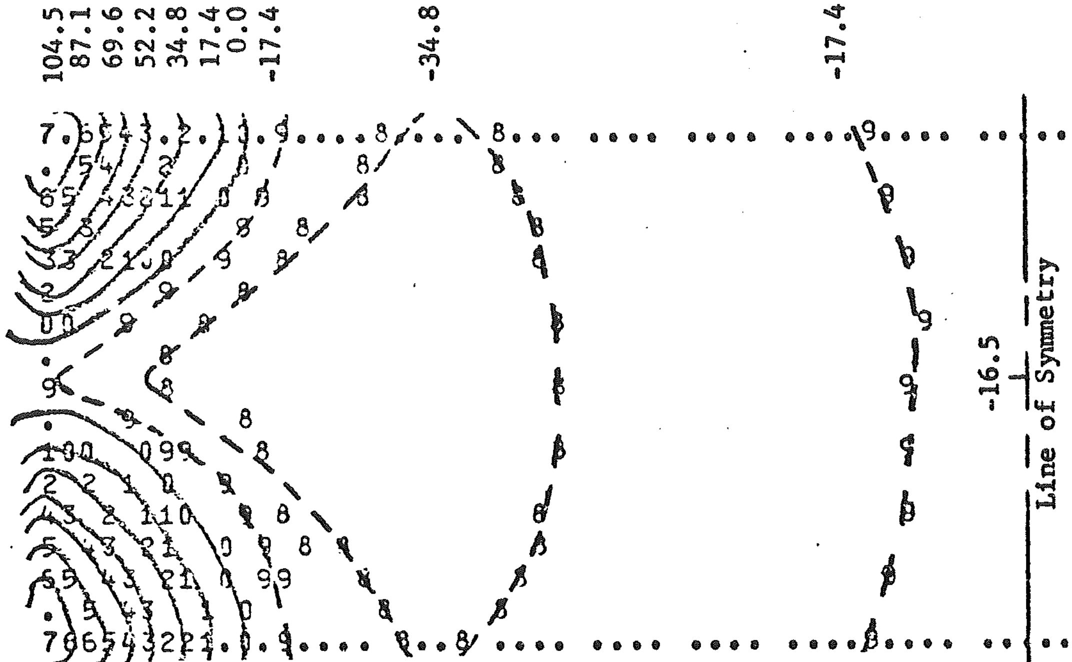
(a) E1 = 100,000 E2 = 20,000 E3 = 5,000 H1 = 3" H2 = 6"

+ Tension - Compression



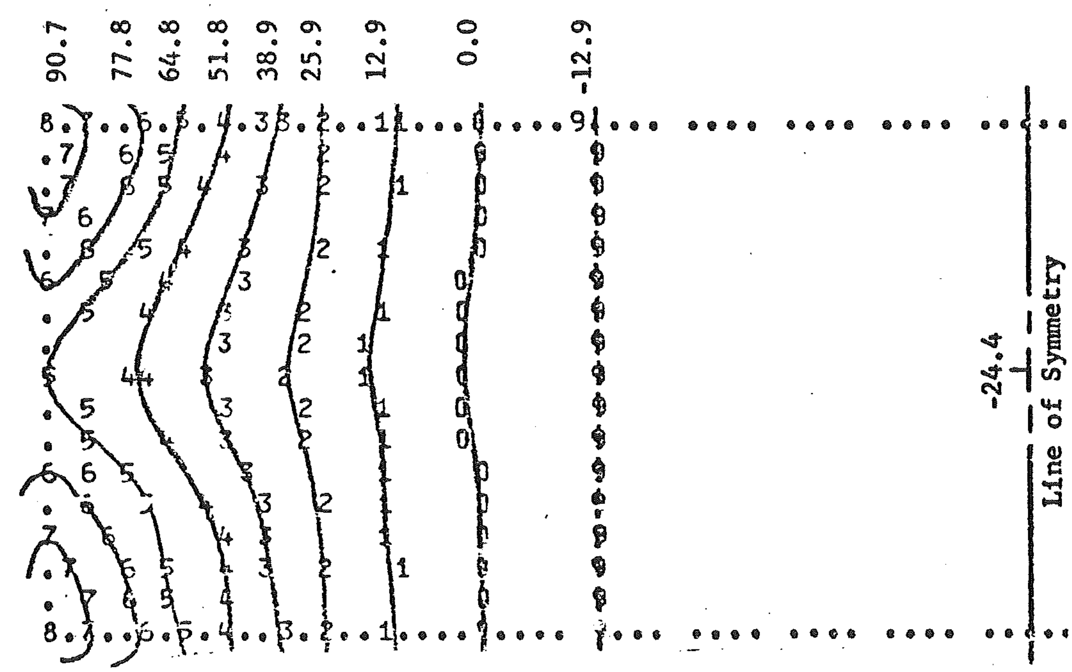
(b) E1 = 100,000 E2 = 20,000 E3 = 5,000 H1 = 6" H2 = 18"

FIGURE 3. Contours of Radial Stress at Asphalt Interface. P = 4500 lb., p = 80 psi, for dual-tandem at 13" x 48". Values are in psi.



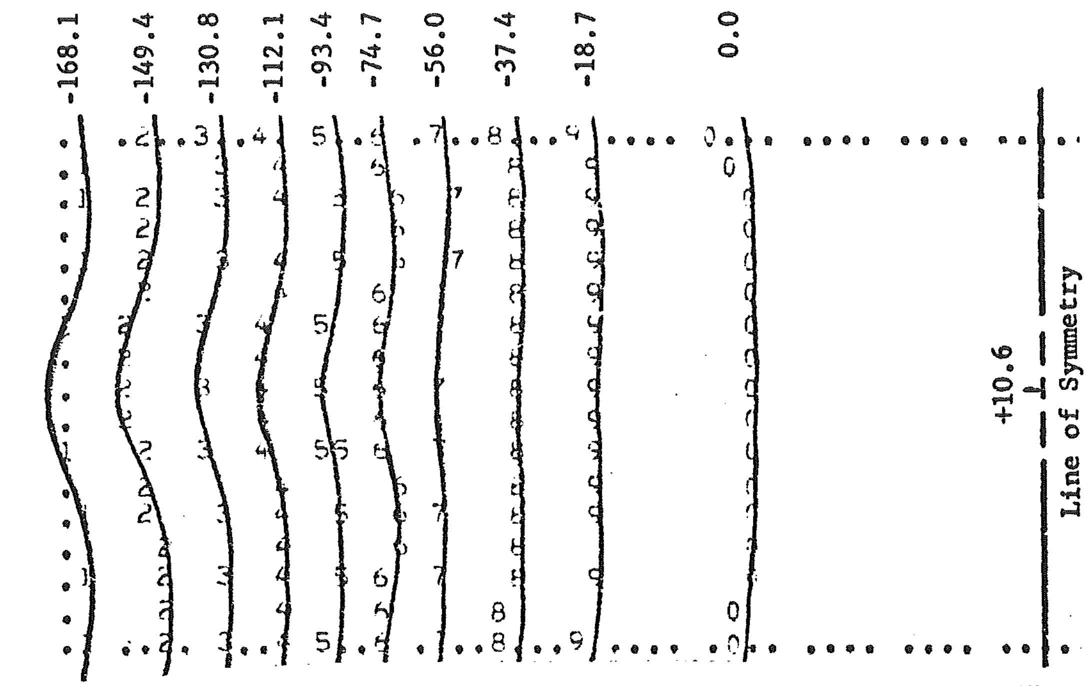
(a) E1 = 200,000 E2 = 20,000 E3 = 10,000 H1 = 3" H2 = 6"

+ Tension - Compression



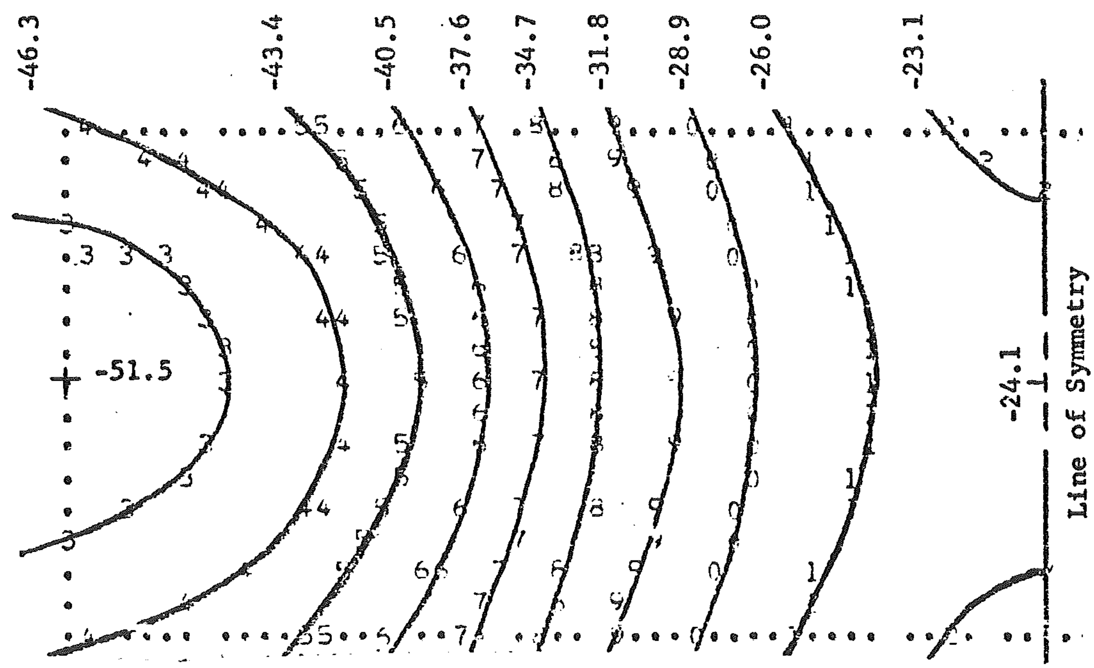
(b) E1 = 200,000 E2 = 8,000 E3 = 2,000 H1 = 6" H2 = 18"

FIGURE 4. Contours of Radial Stress at Asphalt Interface. P = 4500 lb., p = 80 psi, for dual-tandem at 13" x 48". Values are in psi.



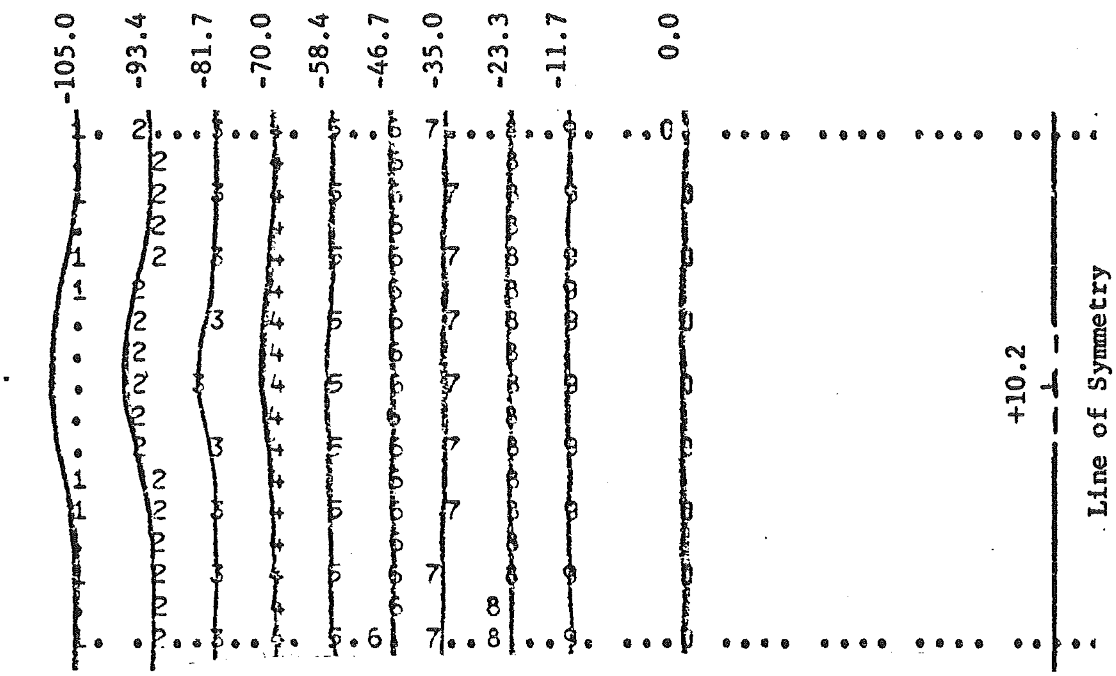
(a) E1 = 100,000 E2 = 20,000 E3 = 5,000 H1 = 3" H2 = 6"

+ Expansion - Compression



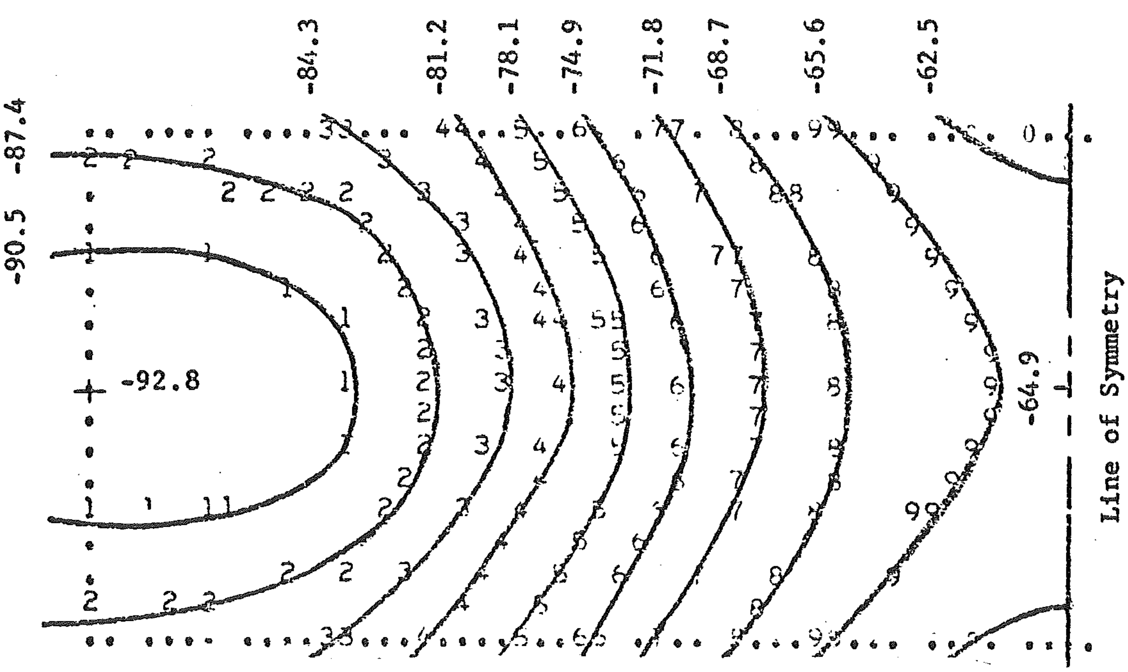
(b) E1 = 100,000 E2 = 20,000 E3 = 5,000 H1 = 6" H2 = 18"

FIGURE 5. Contours of Vertical Strain at Subgrade Interface. P = 4500 lb., p = 80 psi, for dual-tandem at 13" x 48". Values are x 10^{-5} in./in.



(a) E1 = 200,000 E2 = 20,000 E3 = 10,000 H1 = 3" H2 = 6"

\+ Expansion - Compression



(b) E1 = 200,000 E2 = 8,000 E3 = 2,000 H1 = 6" H2 = 18"

FIGURE 6. Contours of Vertical Strain at Subgrade Interface. P = 4500 lb., p = 80 psi, for dual-tandem at 13" x 48". Values are x 10^{-5} in./in.

properties rather than layer characteristics. A comparison between subgrade vertical strains with surface deflections indicated a linear relationship but the comparison with subgrade deflection was a curvilinear one. This first relationship is of interest since then the vertical subgrade strains (a design criterion) can be compared from pavement surface deflections.

From the above, it becomes evident that the locations for maximum tensile radial stress and maximum vertical compressive strain are directly under (a) the center of a wheel or (b) midway between dual wheel of an axle. These locations will be identified as WC for the first location and AC for the second.

Material Characteristics

In order to calculate stresses and strains in a 3 layered system Poisson's ratio and modulus of elasticity for each layer must be known. A review of published data indicated that Poisson's ratio for a subgrade, base, and asphaltic concrete surface could be estimated to be equal to 0.50, 0.45, and 0.35 respectively. For this preliminary report and to demonstrate the proposed design procedure the modulus of elasticity of the surface coarse and base will be fixed. The modulus for the asphaltic concrete for an average temperature of 75^{o}F will be set at 200,000 psi; this value has been obtained from dynamics testing of asphaltic concrete by the author (12). The modulus of elasticity for an unbound base material will be set at 20,000 psi. This value would approximate a minimum CBR value of 15 and yield a maximum E1/E2 = K1 ratio of 10. It is apparent that some judgement is necessary in the selection of E1 and E2; however, these are within the ranges reported for the corresponding materials. Also since these layers will be "manufactured", their characteristics will or can be specified or controlled within some predictable range of modulus of elasticity.

Subgrades have a fairly large range of variability with respect to soil type and moisture effects on stiffness. As a consequence, the subgrade soil will have to be evaluated for modulus of elasticity E3. References 1, 3, 11, 15 and 16 were used to develop the chart of Figure 7 for an indirect determination for the modulus of elasticity of soils. It is apparent that a test procedure for a direct determination of E for a soil is one of our greatest needs.

The flexural fatigue characteristic of asphaltic concrete will be represented by the following equation:

$$S = 1800N^{-0.2} \qquad \text{---- 3}$$

where:

S is the radial tensile stress, psi

N is the number of repetitions of stress S for failure.

The general form of the equation is generally accepted by asphalt paving technologists. The coefficients of 1800 and -0.2 are those found from fatigue testing of pavement specimens as reported in Reference 12. It is of interest to point out that reduction of data given by Dormon and Metcalf in the development of the Shell design procedure (10), show these coefficients to be 2070 and -0.2 respectively.

Several relationships between surface deflection, subgrade deflection, or subgrade strain and load repetitions were found in the

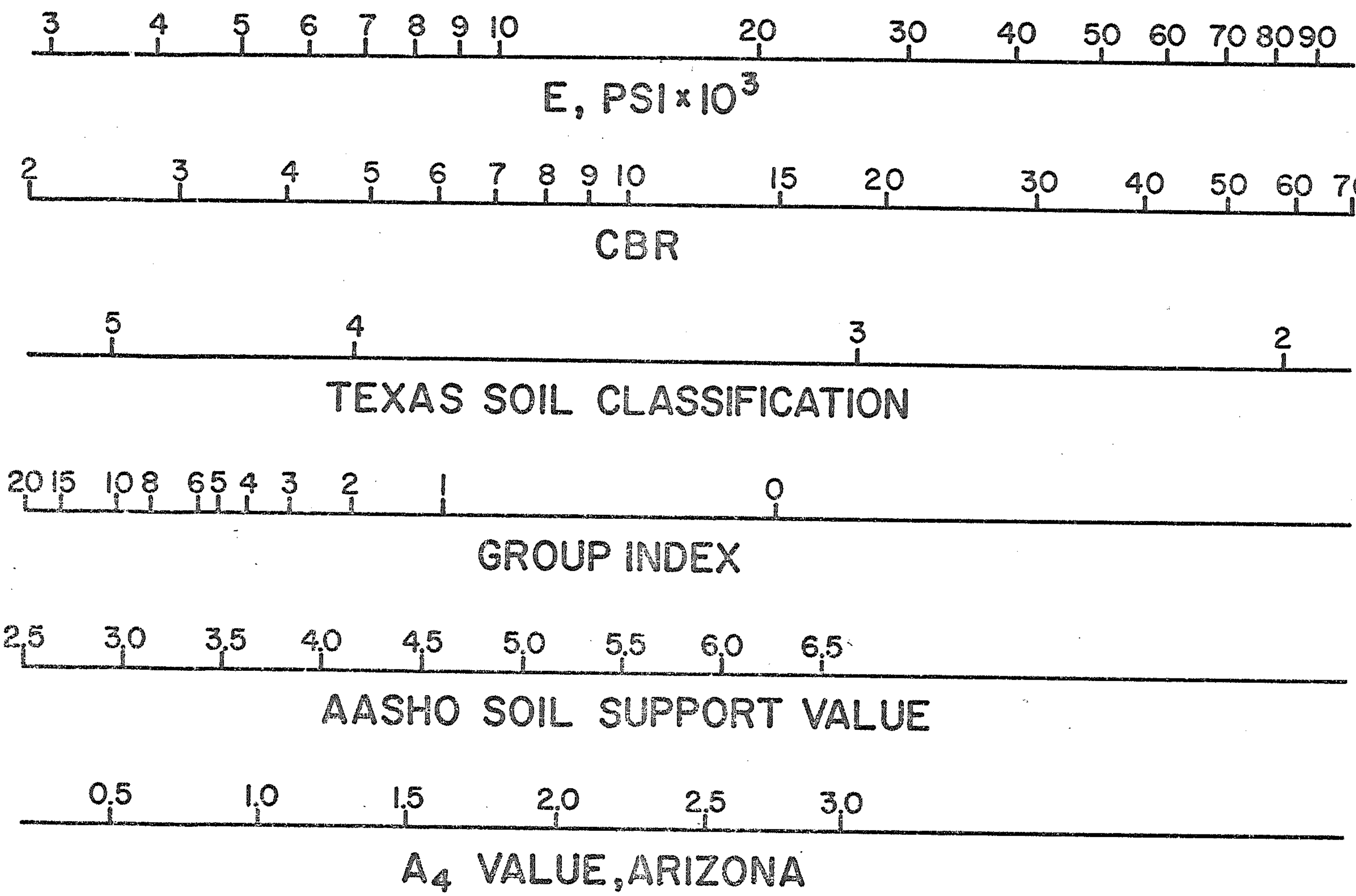


FIGURE 7. Chart for Comparing Various Soil Measurements.

literature survey (3, 5, 10, 13 and 14). Of these, the one given by Dormon and Metcalf (10) seemed the most logical one to use and is given by the following:

$$\varepsilon_c = 0.0105N^{-0.2} \qquad \text{---- 4}$$

where:

ε_c is the vertical compressive strain

N is the number of repetitions of strain ε

The general form of equation 4 is similar to that presented by the Corps of Engineers relating surface deflection and coverages to failure (14), that is, an exponential equation having repetition, N, raised to the -0.2 power.

Regional Factor

As indicated earlier, a reasonable pavement design procedure must be concerned with the influence of environmental factors upon performance. To this end, the procedure being described does recognize this influence and will adjust basic material property with a "correction" factor. This correction factor is obtained from a regional factor (RF) effect as established by AASHO (17) and we express it in term of Structural Number, SN, as follows:

$$\text{Correction Factor (CF)} = \frac{SN}{SN_c} = (RF)^{0.146} \qquad \text{---- 5}$$

A correction factor will be applied to E1 to account for the effect of temperature on the relative stiffness between the surface course and base and a different correction factor based on rainfall will be used on E3. Since the correction factors will be applied to material properties (E's), their effects on total thickness design will be interacting since our criteria depend on both H1 and H2. The following listing shows the relationship between regional factor and correction factor:

RF	0.5	1.0	2.0	3.0
CF	0.9	1.0	1.1	1.2

The correction factor for E1 is related to temperature; however, it is to be obtained from the elevation of the road since there is a linear relationship between elevation and temperature in Arizona (18). The Arizona Highway Department (16) has broken down the range of regional factor from 0.5 to 3.0 based on three zones as shown:

1. Desert Zone, below 3,500 ft. - RF = 0.5 to 1.0
2. Intermediate Zone, 3,500 ft. to 6,000 ft. - RF = 1.0 to 1.5
3. Mountainous Zone, above 6,000 ft. - RF = 1.5 to 3.0

Using the above as a guide and assuming a linear relationship between elevation and regional factor the plot of Figure 8a was drawn to obtain RF's between elevations of 150 to 7,000 ft. elevation. This correction factor will be used as a multiplier to E1 since the radial tensile stress will increase as K1 increases.

The correction factor to E3 is related to moisture in the subgrade; and as a first estimate, we are assuming a linear relationship between regional factor and annual rainfall. This relationship is shown in the

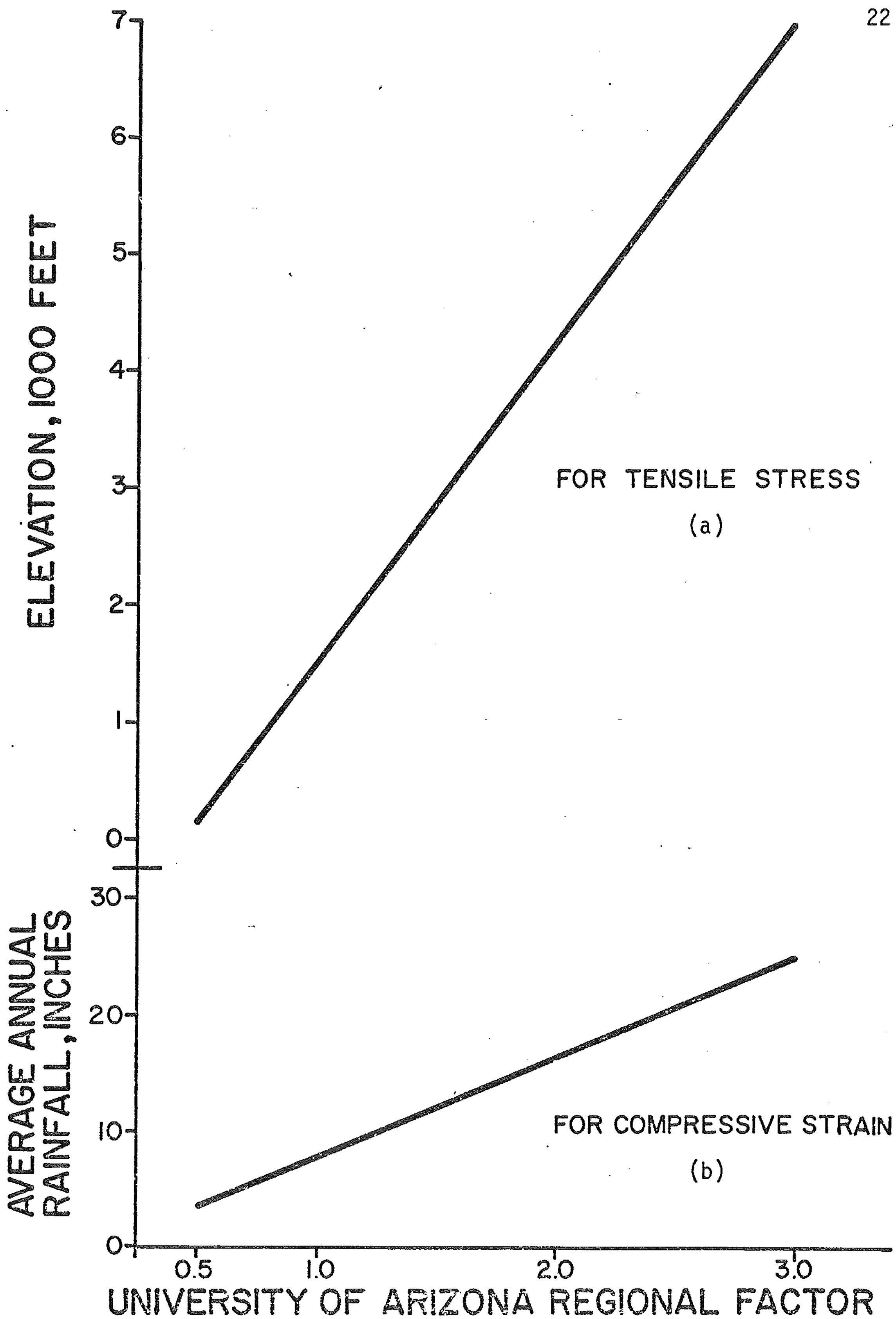


FIGURE 8. Relationships of Elevation and Rainfall with Regional Factor.

plot of Figure 8b. Temperature, elevation and rainfall data for many Arizona cities are presented in a one-page chart entitled "Climate for Arizona Cities" which is available from the Institute of Atmospheric Physics of The University of Arizona (19). This correction factor to E3 will be applied as a divider since the vertical compressive strain should increase as E3 decreases.

At the present (1972), the Arizona Highway Department is involved in modifying its method for determining the values of regional factor in Arizona. It is not our intent to describe this tentative procedure at this time; however, a comparison of regional factors obtained by The University of Arizona procedure is shown in Table B1 of Appendix B and Figure 9.

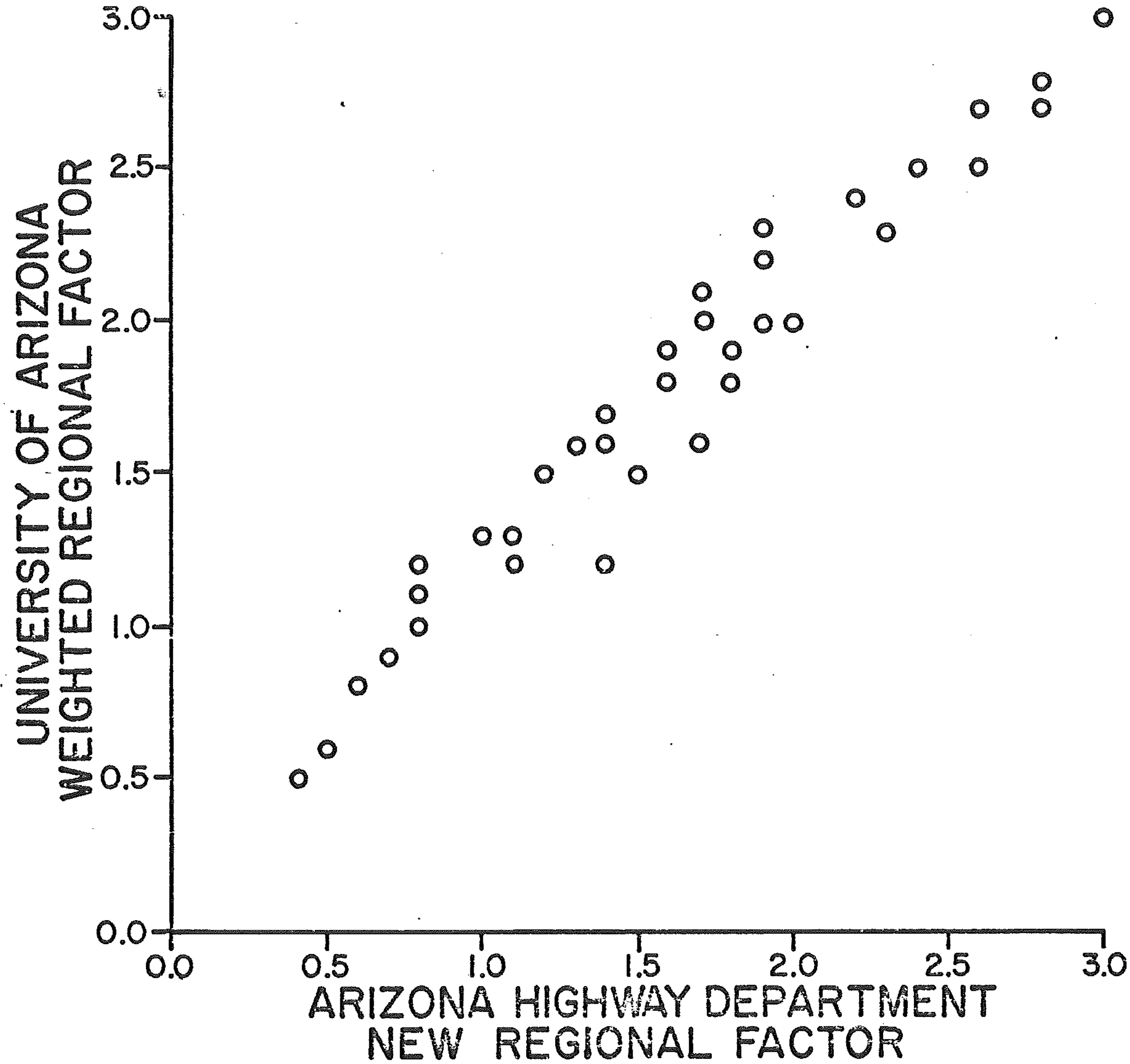


FIGURE 9. Comparison of Weighted Regional Factors of the Proposed Design with the New Method of the Arizona Highway Department.

DESIGN PROCEDURE

The following section will illustrate in detail the trial-and-error procedure for establishing the layer thickness H1 and H2 for a given condition. Also the pavement will be "designed" by various other methods.

The conditions for the example are as given below and constitute an actual situation.

1. Location - State 77 Oracle Junction East
2. Traffic - 1971 - ADT = 2255, Trucks = 620
3. Subgrade soil - P.I. = 36, Passing #200 = 36%

Proposed Design

The various parts of this procedure will be presented in detail to facilitate comparisons with other methods. Design tables to be completed in the design process are included in this discussion.

The location of the project will establish region factors and correction factors to be used. Table 1 shows the elevation to be 3600 feet and the average annual rainfall to be 11 inches; these give regional factors of 1.8 and 1.4 respectively.

Since actual loadometer data are not available for the road, we will make use of the Arizona Highway Department's break down and projections of traffic into their various vehicle categories. The break down of traffic is based on a "floating average" percent for each vehicle and is established by the Planning-Survey Division. The number of the different vehicles per day per lane is shown in Table 1. The

TABLE 1. Tabulations for the Determination of Load Repetitions.

ASPHALT PAVEMENT DESIGN

Location: ORACLE JUNCTION Station: ________ Project No.: ________

Elev.: 3600 R.F.: 1.8 Rainfall: 11 R.F.: 1.4

Traffic: ADT 1971 2255 Projected ADT: 3022

% Commercial: ________ Projection Factor: 1.17

Calculation of Axle Repetitions

Arizona Designation	U of A Designation	Present No/Day/Lane	Repetitions/Day/Lane Vehicles	SA @ W.C.	TA @ A.C.
Passenger	1.4/28	815	x1= 815		
2P	1.6/32	220	x1= 220		
2S	1.8/45	4	x1= 4		
2D	1SA	38		x1= 38	
BUS	1TA	3			x2x1= 6
3D	1TA	17			x2x1= 34
2S1	2SA	4		x2= 8	
3S1	1SA + 1TA			x1=	x2x1=
2S2	1SA + 1TA	7		x1= 7	x2x1= 14
3S2	2TA	9			x2x2= 36
3S3	1TA + ?				x2x1=
2-2	3SA	2		x3= 6	
3-2	2SA + 1TA	10		x2= 20	x2x1= 20
3-3	1SA + 2TA	1		x1= 1	x2x2= 4
2S1-2	4SA			x4=	
2S2-2	3SA + 1TA			x3=	x2x1=
2S1-3	3SA + 1TA			x3=	x2x1=
3S1-3	2SA + 2TA			x2=	x2x2=
				Σ = 80	Σ = 114

NOTE: 1. TA @ W.C. = TA @ A.C. = 114

2. For design tabulation multiply above totals by Projection Factor x Design Life (days) to get Actual Repetitions

corresponding number of repetitions for the five-wheel assemblies described is also shown for the two critical locations within the assemblies, that is, wheel center and axle center as described earlier.

The subgrade soil is given a Soil Support Value, S, through the use of the following equation which was established by the Arizona Highway Department (10).

$$S = -0.125[P.I.+0.6(\%Pass\#200)]+11.6 \quad \text{---- 6}$$

The amount passing the #200 sieve must be greater than 50 percent and the range of S for this relationship has been set between 2.1 and 6.6. The range of S has been divided equally to give A_4 values between 0 (for S = 2.1) and 3.0 (for S = 6.6). With the value of S or A_4, we enter the Chart of Figure 7 to obtain the modulus E3 for the subgrade. From the soil data given S = 3.35, A_4 = 0.85, and E3 = 4,600 psi.

The given data for design conditions are entered in a calculation form shown as Table 2. An E1 value of 200,000 psi and E2 of 20,000 psi are considered typical for asphaltic concrete and a granular unbound base. The magnitudes of E1 and E3 are modified as shown to account for environmental effects. As mentioned earlier, cooler temperatures (higher) elevations) increase radial tensile stresses; and higher rainfall should increase subgrade strains, therefore, the correction factors are applied as shown in order to effect the desired results.

The equations shown on Table 2 relate the correction factor to regional factor, tensile stress to allowable number of repetitions, and also compressive strain to allowable number of their repetitions.

TABLE 2. Pavement Design Values Based on Fatigue Considerations

Location: ORACLE JUNCTION Project No.: ______ Station: ______

Field Modulus x Correction Factor = Actual Modulus

E1 = 200,000 x 1.09 (Temp) = 218,000 (psi) C.F. = $(R.F.)^{0.146}$

E2 = 20,000 x NA = 20,000 (psi) $\sigma_t = 1800(N)^{-0.2}$

E3 = 4,600 ÷ 1.05 (H_2O) = 4,381 (psi) $\varepsilon_c = 0.0105(N)^{-0.2}$

Trial No. 1: H1 = 4 (inches) H2 = 18 (inches)

Type of Axle	Actual Reps. (x10^6) Design Life 20 Yr. @W.C.	Actual Reps. (x10^6) Design Life 20 Yr. @A.C.	σ_t (psi) @W.C.	ε_c (x10^-5) @W.C.	ε_c (x10^-5) @A.C.	Allowable Reps. (x10^6) σ_t @W.C.	Allowable Reps. (x10^6) ε_c @W.C.	Allowable Reps. (x10^6) ε_c @A.C.	%Fatigue Life Used σ_t @W.C.	%Fatigue Life Used ε_c @.W.C.	%Fatigue Life Used ε_c @A.C.
Pass. 1.4/28	6.961	NA	38.8	-10.8	NA	214.9	8726.	NA	3.24	.08	NA
2P 1.6/32	1.879	NA	44.4	-12.3	NA	110.1	4475.	NA	1.70	.04	NA
2S 1.8/45	0.034	NA	55.8	-13.9	NA	35.1	2425.	NA	.11	.00	NA
SA 4.5/80	0.683	0.683	100.9	-54.6	-60.7	1.8	26	1.5	37.16	25.53	43.33
TA 4.5/80	0.974	0.974	99.5	-55.2	-61.4	1.9	25	1.4	50.16	39.13	66.36
								Totals:	92.37	64.78	109.69

Trial No. 2: H1 = ______ (inches) H2 = ______ (inches)

Type of Axle	Actual Reps. (x10^6) Design Life ___ Yr. @W.C.	Actual Reps. (x10^6) Design Life ___ Yr. @A.C.	σ_t (psi) @W.C.	ε_c (x10^-5) @W.C.	ε_c (x10^-5) @A.C.	Allowable Reps. (x10^6) σ_t @W.C.	Allowable Reps. (x10^6) ε_c @W.C.	Allowable Reps. (x10^6) ε_c @A.C.	%Fatigue Life Used σ_t @W.C.	%Fatigue Life Used ε_c @.W.C.	%Fatigue Life Used ε_c @A.C.
Pass. 1.4/28		NA			NA			NA			NA
2P 1.6/32		NA			NA			NA			NA
2S 1.8/45		NA			NA			NA			NA
SA 4.5/80											
TA 4.5/80											
								Totals:			

The calculations for stresses, strains, allowable repetitions of these and the amount (%) of fatigue life used are based on assumed values for surface course thickness H1 and base course thickness H2.

The first column of the tabulations identifies the type of axles considered in the method.

The second and third columns show the actual repetitions of maximum stress or strain at the two locations of wheel center and axle center.

The fourth column is for the radial tensile stress under the wheel center.

The fifth and sixth columns are for the compressive strain under the wheel center and axle center.

The seventh, eighth, and ninth columns show the allowable number of repetitions for stress or strain based on the given equations.

The tenth, eleventh, and twelfth columns contain the amount of fatigue life consumed by each axle type for the design. The sum of each of these columns indicates the efficiency of the design on the basis of repeated loading.

A balanced design would be one in which one hundred percent of fatigue is used over the design life in both stress and strain considerations. However, a completely balanced design is not necessary in order to have acceptable values of H1 and H2. It can be seen that from the comparison of amount of fatigue life used for various values of H1 and H2 an indication can be obtained for the possible modification or alteration of E1, E2, and/or E3.

Table 2 presents the detailed results of computations for the design of a pavement. Since a computer program was utilized to obtain values for stress and strains, it was found convenient to develop another program for the other associated calculations of Table 2 and for printing the data. Examples of the program's output are shown in Appendix C and a program listing is included as Appendix D.

Comparison of Design

A comparison of values for H1 and H2 obtained by other pavement design procedures is included primarily for the sake of information. The characterization of materials will be made through the use of Figure 7. The general approach for each design procedure will be presented.

(a) Arizona Highway Department. The method is based on the AASHO procedure of establishing a Soil Support Value, equivalent 18 kip single axle loading and satisfying the required structural number obtained from solving the design equation (16). For the conditions given the weighted structural number for the pavement is 3.24. If we assign the highest values to coefficients $a_1 = 0.44$ and $a_2 = 0.14$ then the new design procedure would yield an SN = 0.44 x 4 + 0.14 x 18 = 4.28 which is a more conservative value than that obtained by the Arizona Highway Department. However, Arizona often uses values of $a_1 = 0.40$ and for $a_2 = 0.11$ and with these the new design yields an SN of 3.58.

(b) Texas Method. Using the chart of Figure 7 the subgrade is classified as a 4.6 soil. The Texas Flexible Pavement Design Chart (1) is entered with a wheel load of 16,000 pounds to obtain a depth of

20 inches. Under the original design concept of the Texas method, this would represent the total thickness of base needed even if the top 4 inches were replaced with asphaltic concrete. This comparison shows the new method to be more conservative than the Texas method since 4 inches of cover would be required for a class 3.0 soil (E = 26,000 psi).

(c) Kansas Method. Equation 2 is used to calculate the thickness of asphaltic concrete required to limit the subgrade deflection to 0.1 inch if the n for rainfall is 0.5 and m for traffic is 7/6. Using the given E1 and E3 yields an impossible value for T; however, if the deflection is limited to 0.05 inch, then T is equal to 2.6 inches of asphaltic concrete. It is apparent that our use of this method does not yield results comparable to the previous ones.

(d) Shell Method. The design chart for E3 equal to 3,750 psi is used since it is the closest one to our value of 4,600 psi. Reduction of the traffic data for use in this method yields a Load Distribution Factor (LDF) of 19.5 and, therefore, the number of 18 kip single axle load repetitions, N, is 1,750.000. With these two factors, E3 and N, the values of H1 and H2 are 4.5 and 21.5 inches respectively. The larger value of H2 in comparison to the proposed method is attributed to the smaller value of E3 used in the Shell procedure. The Shell method and the proposed one agree in that both show a relatively good balance design between the two criteria.

Design charts for the Arizona, Texas, and Shell methods of pavement design are shown in Figures C1, C2, and C3 of Appendix C.

CLOSURE AND RECOMMENDATIONS

We are enthused by the results obtained in this one example. We recognize that "one swallow does not a summer make" and also that various assumptions need to be examined more thoroughly and their effects verified. For example we assumed that every wheel(s) travels over the same path, that the ε_c - N relationship of subgrade soils is as shown, that the passage of the tandem axle does in fact represent two repetitions of the subgrade strains calculated, and that values of E1, E2, or E3 obtained individually in the laboratory can be used to predict their values operating jointly in the field.

Recommendations can only be made with certain reservations based on the limited data present. However, it becomes obvious that an analytical procedure such as the one demonstrated will be required for design results to be evaluated on the basis of component efficiency. Specific recommendations for better definition or design factors are listed below:

1. Methods and procedures to characterize and define the moduli of all materials but the present emphasis should be for subgrade (fine grained) soils.
2. Improve the definition for each of the design vehicles and the estimation of the number of load applications.
3. Field check existing pavements for redesign by the proposed procedure, for estimating values for the moduli of various types of materials, and for comparing design layer thickness with performance.
4. Since the design procedure is based on elastic theory, a method for optimizing proportions and character of the pavement layers should be developed.

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APPENDIX A

Summary of Material Elastic Coefficient

TABLE A1. SUMMARY OF ELASTIC COEFFICIENTS FOR SURFACES

Material	Type of Test	E (100 psi)	μ	Ref.	Remarks
Asp. Conc.					
4"	Plate Bearing	11.2		22	
1"		13.2			
2"		21.1			
2 1/2"		9.6			
3"		13.6			
6"		7.5			
3 1/2"		16.1			
3 1/2"		11.9			
Asp. Conc. (WASHO)	Plate Bearing	80-160		23	
Asp. Conc.	Unconfined compression with repeated pulse loads @ freq. 3-200 rad/sec over temp. range 20°-120°F.	40-2400		24	Size of sample affected results - shorter samples had increased E. E. not affected by orientation of specimen in pavement.
Asp. Conc.		150-1800	0.40	25	μ is an assumed value E is an assumed range to calculate strains of pavement and subgrade.
Asp. Conc.					
60-70 Pen	Unconfined compression @ 0.05"/min loading @ 80°F.	10.6		27	
85-100 Pen		10.5			
150-200 Pen		7.9			

TABLE A1 (Cont)

Material	Type of Test	E (1000 psi)	μ	Ref.	Remarks
Asp. Conc. 60-70 Pen 85-100 Pen 150-200 Pen	@ 140°F.	3.5 3.3 2.8		27	
Asp. Conc. 60-70 Pen 85-100 Pen 150-200 Pen	Triaxial compression σ_3 = 30 psi. @ 0.05"/min. loading - 80-F	17.5 18.7 13.9		27	Secant modulus @50% of maximum stress.
Asp. Conc. 60-70 Pen 85-100 Pen 150-200 Pen	@140°F	7.5 10.6 8.0		27	
Asp. Conc. 85-100 Pen	Unconfined @ 80°F " 140°F Triaxial @ 80°F " 140°F	9.5 2.8 15.8 8.8		27	Secant modulus @ 50% of maximum stress.
Asp. Conc. 85-100 Pen	Unconfined @ 80°F " 140°F Triaxial @ 80°F " 140°F	9.5 3.5 15.6 9.0		27	Secant modulus @ 50% of maximum stress.
Asp. Conc.	Flexure tests of large diameter slabs with repeated loadings.	80-100		29	Dynamic modulus - secant E increased with increased density.

TABLE A1 (Cont)

Material	Type of Test	E (1000 psi)	μ	Ref.	Remarks
Asp. Conc.	Triaxial repeated loads with strain measurements.	163-2004 98-2199	.28-.49	30	
Asp. Conc.	Flexure - repeated load.	100-250		30	Dynamic modulus E at $80^{o}F$ and 100 load applications.
Asp. Conc.	Flexure - repeated load.	175-275		30	E decreased with increased flexural stress.
Asp. Conc. 40-50 Pen 40-50 Pen 180-220 Pen 180-220 Pen	Triaxial	 470-2600 950-3080 730-3160 1350-3538		31	Modulus of rigidity - variations due to range of temperature from $14^{o}F$ to $68^{o}F$.
Asp. Conc.	Vibratory-wave	14.5		32	
Asp. Conc. 2" 8 1/2" 5 1/2" 7" 5"	Vibratory-wave	 600-1600 1150-1950 1350-1950 850-2150 1050-2100		33	Spring and fall variation
Asp. Conc. 3" 4" 5" 9 1/2"	Vibratory-wave	 660-2000 950-1950 860-1850 1950		33	

TABLE A1 (Cont)

Material	Type of Test	E (1000 psi)	μ	Ref.	Remarks
Asp. Conc.	Deflectometer, Calif.	333-1235	0.35-0.40	34	Assumed $\mu = 0.35$ @$40^{o}F$ = 0.40 $68^{o}F$ Variation in temperature.
Asp. Conc.	Unconfined - constant rate of strain		0.30-0.50	36	Variations due to test temperature and rate of loading.
Asp. Conc.	Triaxial - repeated axial loads	6-2600	0.18-0.57	37	Variations due to temperature changes $40^{o}F$ - $140^{o}F$. E decreases as temperature increases; μ increases as temperature increases.
Asp. Conc.	Plate Bearing - 30"D	135-183		38	Pavements tested after 4 years service.
Asp. Conc.	Vibratory-wave	1746-3266		39	Variations due to temperature and frequency. E increases @ temperature decreases. E decreases as frequency decreases.
Asp. Conc.	Unconfined sinusoidal stresses	90-1860		41	Variations due to temperature and frequency changes. E increases with frequency increase. E increases with temperature decrease.

TABLE A1 (Cont)

Material	Type of Test	E (1000 psi)	μ	Ref.	Remarks
Asp. Conc.	Dynaflect vibration and measured deflections			42	Calculations are based on an assumed μ = 0.5 for subgrade and pavement materials
1.2"		283			
3.7"		13.9			
7 1/2"		78.9			
Asp. Conc.	Plate Bearing	1.4-2840		44	Variation due to temperature and rate of loading. E decreases as temperature increases. E increases as rate of load increases
Asp. Conc.		8.5-1775		44	E variations as above. Comparison to above shows differences with different pen. grades.
Asp. Conc.	Plate Bearing	329-1575		44	E varies with plate D increases as D increases
Asp. Conc.	Dynamic	200-500		46	E is temperature dependent

TABLE A2. SUMMARY ELASTIC COEFFICIENTS FOR BASES

Material	Type of Test	E (1000 psi)	μ	Ref.	Remarks
20" Sand, Clay gravel	Plate Bearing	4.0		22	
Comp. Shell		9.3			
Gravel		13.6			
Crush Stone		3.0			
Crush Gravel		16.1			
Stone Sand		5.9			
Limestone	Plate Bearing	12-26		23	
Slag	Wave Velocity Meas.	45.5		26	
Rolled Brick		74.4			
Rubble		38.2			
Hand-pitched		44.7-66.5			
Gravel		105-111			
Rubble		67-95			
Gravel/sand completed in 2 layers		284			
Partially Crushed Gravel	Triaxial-repeated load			28	E Depending on relative density and degree of saturation decreases as relative density increases
3%<#200		.25-.50			
5%<#200		.32-.45			
8%<#200		.25-.50			
Crushed Gravel				28	
3%<#200		.25-.41			
5%<#200		.22-.37			
8%<#200		.23-.45			

TABLE A2 (Cont)

Material	Type of Test	E (1000 psi)	μ	Ref.	Remarks
Colorado Standard 1/2" max. Base. Course Gradation	Triaxial repeated load	12.00-60		30	
Colorado Standard 2 1/2" max. Sub-Base. Course Gradation	Triaxial repeated load	12.00-60		30	
4" Granular Base. Colorado	Vibratory-wave	110-130		33	Spring to fall variation.
14" Granular Sub-base. Colorado		72-84			
Granular Base		140-330			
Sand Gravel Sub-base		62-1800			
Compacted Gravel Base	Plate Bearing - 30"D	27-68		38	
AASHO - Base	Vibratory-wave	426		40	
Sub-base		106			
Quarry Rock	Dynamic	410		40	
Gravel		484			
Red Cinders		200			
Black Cinders		127			
Natural Gravel	Triaxial-controlled rate of strain	2-25		43	E increases with increased confining pressure.

TABLE A2 (Cont)

Material	Type of Test	E (1000 psi)	μ	Ref.	Remarks
Crushed Limestone	Triaxial-controlled rate of strain	6-22		43	
Limestone	Plate Bearing	12-303		44	E increases as plate D increases.
Cement Stab Granular	Dynamic	500-6000		46	E very low if stabilized base is broken.
Soil Cement		200-1000			E very low if stabilized base is broken.
Crushed Slag		12-600			E increases with compaction.

TABLE A3. SUMMARY ELASTIC COEFFICIENTS FOR STABILIZED BASES

Material	Type of Test	E (1000 psi)	μ	Ref.	Remarks
Gravel-50 pen. bit. 70 90 110	Wave Velocity Meas.	940 810 710 510		26	All tests at 10°C.
Gravel-90 pen. bit.	Wave Velocity Meas.	326-1175		26	Low @ 20°C; high @ 10°C.
Sand-90 pen. bit 60/40 Sand/Gravel-90 pen. bit. 40/60 Sand/Gravel-90 pen. bit.	Wave Velocity Meas.	710 995 1135		26	All tests @ 10°C.
Gravel-50 kg/cu m 75 100 125	Wave Velocity Meas.	2125 2850 3550 4260		26	All tests @ same temperature.
High stability sand, asphalt base 60-70 pen. bit.	Triaxial-repeated load	86-1262 86-1340 58-1470	0.37-0.49	30	
Low stability sand, asphalt base 60-70 pen. bit.	Triaxial-repeated load	91-1278 27-1213 54-1463	0.32-0.44	30	
High stability mix	Flexure-repeated load	110-175		30	Dynamic E_s @ 80°F. E_s decreased with increased flexural strain.
Low stabilit, mix	Flexure-repeated load	80-150		30	

TABLE A3 (Cont)

Material	Type of Test	E (1000 psi)	μ	Ref.	Remarks
6" Stabilized Base (cement)	Vibratory-wave	2500		32	Variation over one year of curing.
8" Stabilized Sub-base (lime)		196-1010			
High Stability-sand asphalt base course	Vibratory-wave	860-1750		33	Spring to fall variation.
Low Stability-sand asphalt base course		500-1500			
Asphalt treated base		1550-1650			
Emulsion treated bases	Triaxial	27-961		49	Good summation of details for each individual job.

TABLE A4. SUMMARY ELASTIC COEFFICIENTS FOR SUBGRADES

Material	Type of Test	E (1000 psi)	μ	Ref.	Remarks
Silt G = 2.76				21	Determined E_s and E_r secant moduli.
LL = 47	Triaxial-static load	.8-2.6			E_s and E_r not affected by compactive effort.
PL = 33	Triaxial-repeated load	.700-2.3			E_s and E_r decrease as % w.c. increase.
Silty Clay G = 2.74 LL = 75	Triaxial-static load	2.7-7.7		21	E_s and E_r increase with increased compactive effort;
PL = 45	Triaxial-repeated load	2.7-5.6			decrease with % w.c. increase.
Sandy Clay G = 2.65 LL = 16	Triaxial-static load	1.3-2.4		21	E_s and E_r increase with increased compactive effort;
PL = 15	Triaxial-repeated load	.7-2.3			decrease with % w.c. increase.
Lean Clay	a) Plate bearing = 30"D@ = Δ0.05"	3.6		22	
	b) Plate bearing @ surface	4.0			
Fine Sand	a)				
	b)	9.3			
Medium Sand	a)	7.8			
	b)	8.4			
Fine Sand	a)	6.5			
	b)	9.6			

TABLE A4 (Cont)

Material	Type of Test	E (1000 psi)	μ	Ref.	Remarks
Silty Fine Sand	a) Plate bearing @ surface	14.8		22	
	b)	13.6			
Sandy Clay	a)	1.9			
	b)	3.0			
Gravelly Sand	a)	7.0			
	b)	16.1			
WASHO-TEST ROAD	Plate Bearing	4500-11000		23	
LL = 34; PL = 21; PI = 13	Unconfined compression with repeated pulse loads. Over T range 20^{o}F to 120^{o}F.	.5-3.5		24	
Peat	Wave Velocity Meas.	1.0-4.8		26	
Soft Clay		8.6-10.4			
Glacial Clay-stony		33-43			
Sandy Clay		40			
Silty Clay		26-32			
Loam		12.8-32			
Sand		17-28			
Clay-gravel		47-74			
Clay (wet)	Wave Velocity Meas.	18.5-31.5	0.42	26	
Loam		12.8-31.5	0.39		
Clay (hard, dry)		61-355	0.37		
Sand		28.5-5.3	0.36		

TABLE A4 (Cont)

Material	Type of Test	E (1000 psi)	μ	Ref.	Remarks
Silty Clay (A-7-6)	Triaxial-repeated load	4.2-35.5		30	
Silty Clay (A-6)		3.7-25			
Clay-LL = 70; PL = 23; PI = 47	Wave Velocity	80.9		32	
Stabilized Clay		165-568			Cement stabilized - E increased with time.
A-7-6 A-6	Wave Velocity	2.9-4.5 2.9-4.5		33	
Plastic Clay LL = 64.5; PL = 28.0; PI = 36.5	Triaxial-repeated load	3.2-4.2		35	E @ 10,000 cycles.
Clay	Triaxial-repeated load w = 19.1% w = 21.7% w = 23.3%	40-69 6.6-18.0 4-13	0.33-0.46 0.43-0.50 0.38-0.45	37	E decreases with increased w.c.
ML	Plate Load - 30"D @Δ = 0.05" 0.10" 0.15" 0.20"	1.7-3.9 1.4-3.9 1.8-3.5 1.8-3.2	0.40	38	Calculations by Boussinesq rigid plate analysis and assumed μ = 0.40.
CL -SF-ML		2.6 2.7			

TABLE A4 (Cont)

Material	Type of Test	E (1000 psi)	μ	Ref.	Remarks
ML	@Δ = 0.05" 0.10" 0.15" 0.20"	11.2-17.0 9.0-15.6 7.5-13.0 5.6-9.8		38	Compaction had large effect on modulus @ various strain levels.
CL	@Δ = 0.05" 0.10" 0.15 0.20"	18.1 10.7 9.0 6.7			
SF-ML	@Δ = 0.05" 0.10" 0.15" 0.20"	17.8-19.7 13.4-15.2 11.2-12.7 8.4-9.5			
Silt	Wave Velocity Meas.	21.3 (before frost) 497-2130 (frozen) 8.5 (thawing)		40	
Sandy Clay (gravel) Sand Over Clay Gray Sandy Clay Stiff Clay	Wave Velocity	20 19 12.7-14.9 11.7-14.4		42	E values determined from test data using an assumed μ = 0.5.
CL	Plate Bearing - variable plate	3.6-8.1		43	E varies with applied pressure and plate size.

TABLE A4 (Cont)

Material	Type of Test	E (1000 psi)	μ	Ref.	Remarks
WASHO Road	Plate loads C.B.R. correlation	3.7-5.2 2.1-8.5		44	Variation with plate size.
AASHO Road		1.4-5.6		44	
AASHO Road	Curve fitting Dynamic	1.0-3.5		44	

APPENDIX B

Regional Factors

TABLE B1. CONTRIBUTIONS AND COMPARISONS OF REGIONAL FACTOR VALUES FOR ARIZONA

Location	U. of A. R. F. Temperature	Rainfall	Avg.	Arizona Highway Dept. New R.F.
Ajo	1.1	1.2	1.2	0.8
Alpine	3.0*	2.5	2.8	2.8
Ash Fork	2.3	1.6	2.0	1.9
Bagdad	1.8	1.7	1.8	1.8
Benson	1.8	1.4	1.6	1.3
Bisbee	2.4	2.2	2.3	1.9
Buckeye	0.8	1.0	0.9	0.7
Casa Grande	0.9	1.1	1.0	0.8
Chandler	1.0	1.1	1.1	0.8
Clifton	1.7	1.6	1.6	1.7
Cottonwood	1.7	1.3	1.5	1.5
Douglas	1.9	1.5	1.7	1.4
Flagstaff	2.9	2.5	2.7	2.6
Florence	1.0	1.2	1.1	0.8
Gila Bend	0.7	0.8	0.8	0.6
Globe	1.7	1.9	1.8	1.8
Grand Canyon	3.0	1.9	2.5	2.3
Holbrook	2.3	1.1	1.7	1.4
Kayenta	2.5	1.1	1.8	1.6
Kingman	1.7	1.3	1.5	1.2
Lees Ferry	1.6	0.8	1.2	1.1

TABLE B1. (Cont.)

Location	U. of A. F. F. Temperature	Rainfall	Avg.	Arizona Highway Dept. New R.F.
McNary	3.0*	3.0	3.0	3.0
Miami	1.8	2.3	2.0	2.0
Natural Bridge	2.1	2.9	2.5	2.6
Nogales	1.8	1.9	1.9	1.6
Parker	0.6	0.6	0.6	0.5
Pheonix	0.8	1.0	0.9	0.7
Prescott	2.4	2.4	2.4	2.2
Safford	1.5	1.2	1.3	1.1
Seligman	2.3	1.4	1.9	1.8
Sierra Vista	2.1	2.0	2.1	1.7
Snow Flake	2.5	1.5	2.0	1.7
Springerville	3.0	1.5	2.3	2.3
St. Johns	2.5	1.5	2.0	1.7
Tombstone	2.1	1.7	1.9	1.6
Tuba City	2.2	0.9	1.6	1.3
Tucson	1.3	1.4	1.4	1.2
Wickenburg	1.2	1.4	1.3	1.0
Willcox	2.0	1.5	1.7	1.4
Williams	2.9	2.6	2.7	2.8
Window Rock	2.9	1.6	2.2	1.9
Winslow	2.2	1.0	1.6	1.4
Yuma	0.5*	0.5*	0.5*	0.4

* Set by limits of regional factor.

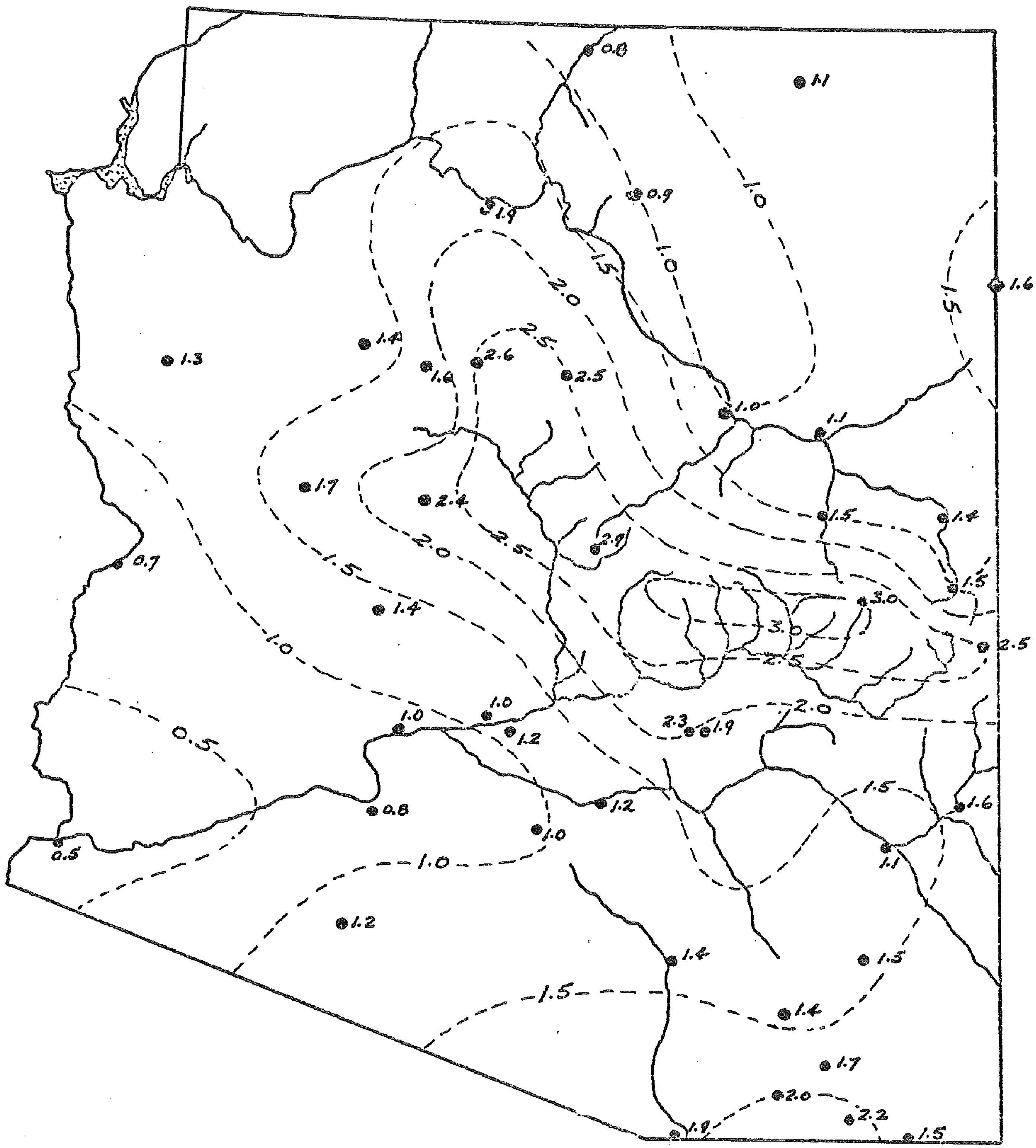


FIGURE B1

TENTATIVE ϵ_C R.F. CONTOURS - ARIZONA

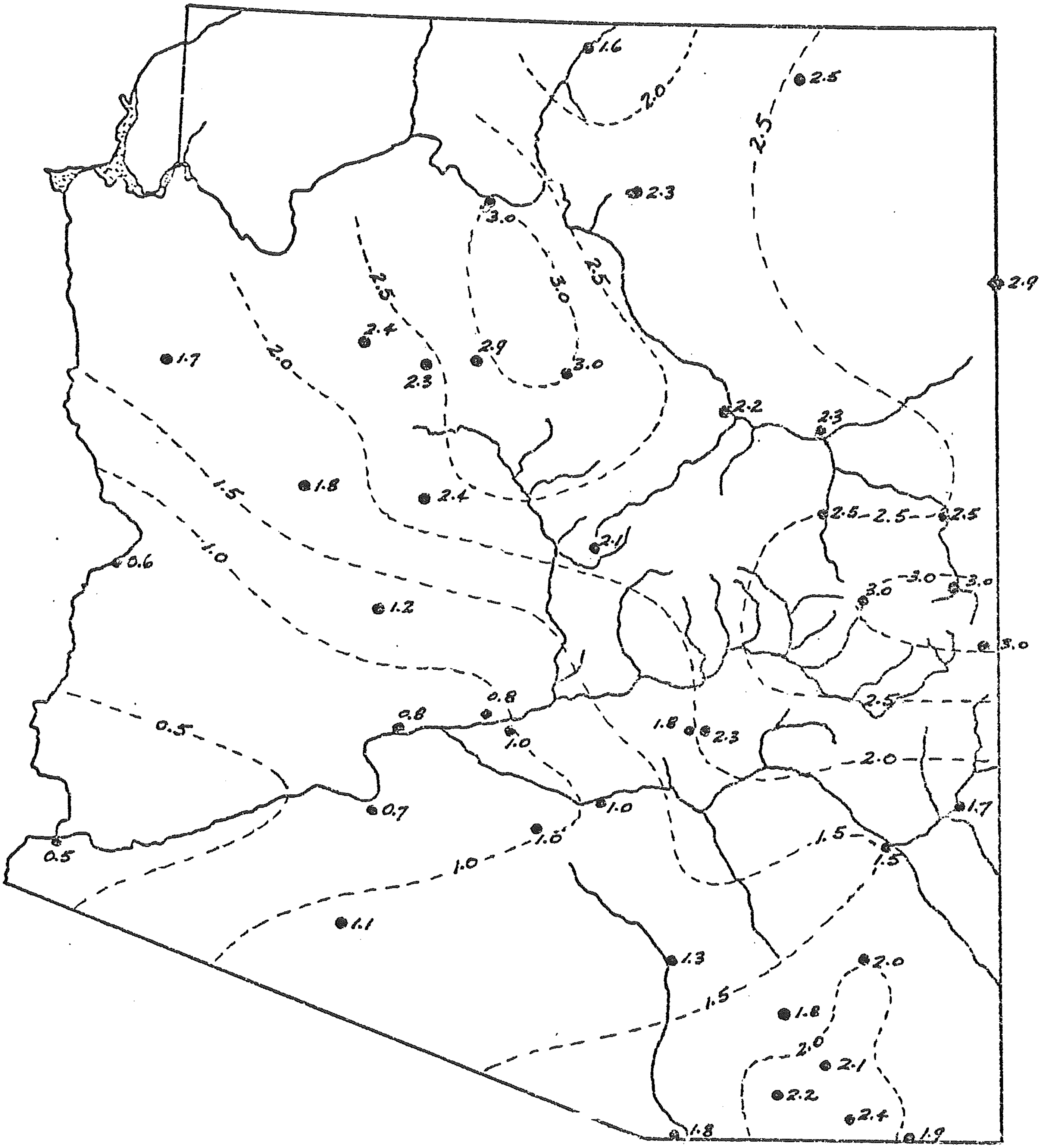


FIGURE B2
TENTATIVE σ_T R.F. CONTOURS - ARIZONA

APPENDIX C

Examples of Design Computations
and
Design Procedures

RUNER
PROGRAM TRANSFERRED TO COMPILER

WHICH TRIAL NUMBER IS THIS REPORT 1.
WHAT IS THE VALUE FOR E1 218000.
WHAT IS THE VALUE FOR E2 20000.
WHAT IS THE VALUE FOR E3 4381.
WHAT IS THE VALUE FOR H1 4.
WHAT IS THE VALUE FOR H2 18.

ASPHALT DESIGN DATA FOR TRIAL NO. 1 TABLE C1.

E1 = 218000
E2 = 20000
E3 = 4381

H1 = 4.0 IN.
H2 = 18.0 IN.

AXLE TYPE	RADIAL STRESS PSI	---VERTICAL STRAIN---- AT W.C.	AT A.C.
PASSENGER	3.880E+01	-1.079E-04	NA
2P	4.435E+01	-1.233E-04	NA
2S	5.576E+01	-1.394E-04	NA
SA	1.009E+02	-5.463E-04	-6.072E-04
TA	9.950E+01	-5.523E-04	-6.138E-04

AXLE TYPE	-------ALLOWABLE REPETITIONS-------- FOR STRESS	FOR STRAIN AT W.C.	AT A.C.
PASSENGER	2.149E+08	8.726E+09	NA
2P	1.101E+08	4.475E+09	NA
2S	3.505E+07	2.425E+09	NA
SA	1.803E+06	2.624E+06	1.546E+06
TA	1.938E+06	2.484E+06	1.465E+06

AXLE TYPE	-----PERCENT FATIGUE LIFE USED------ FOR STRESS	FOR STRAIN AT W.C.	AT A.C.
PASSENGER	3.24	.08	NA
2P	1.70	.04	NA
2S	.11	.00	NA
SA	37.16	25.53	43.33
TA	50.16	39.13	66.36
TOTALS	92.37	64.76	109.69

STOP

READY.

```
WHICH TRIAL NUMBER IS THIS REPORT  2.
WHAT IS THE VALUE FOR E1  218000.
WHAT IS THE VALUE FOR E2  20000.
WHAT IS THE VALUE FOR E3  4381.
WHAT IS THE VALUE FOR H1  5.
WHAT IS THE VALUE FOR H2  16.
```

ASPHALT DESIGN DATA FOR TRIAL NO. 2 **TABLE C2.**

```
E1 = 218000
E2 =  20000
E3 =   4381

H1 =  5.0 IN.
H2 = 16.0 IN.
```

AXLE TYPE	RADIAL STRESS PSI	---VERTICAL STRAIN---- AT W.C.	AT A.C.
PASSENGER	3.142E+01	-1.053E-04	NA
2P	3.591E+01	-1.203E-04	NA
2S	4.401E+01	-1.357E-04	NA
SA	8.623E+01	-5.375E-04	-5.976E-04
TA	8.408E+01	-5.419E-04	-6.026E-04

AXLE TYPE	-------ALLOWABLE REPETITIONS-------- FOR STRESS	FOR STRAIN AT W.C.	FOR STRAIN AT A.C.
PASSENGER	6.171E+08	9.858E+09	NA
2P	3.164E+08	5.057E+09	NA
2S	1.144E+08	2.774E+09	NA
SA	3.963E+06	2.846E+06	1.675E+06
TA	4.496E+06	2.732E+06	1.606E+06

AXLE TYPE	-----PERCENT FATIGUE LIFE USED------ FOR STRESS	FOR STRAIN AT W.C.	FOR STRAIN AT A.C.
PASSENGER	1.13	.07	NA
2P	.59	.04	NA
2S	.03	.00	NA
SA	16.91	23.55	40.01
TA	21.62	35.58	60.54
	------	------	------
TOTALS	40.28	59.23	100.55

```
STOP

READY.
```

PROGRAM TRANSFERRED TO COMPILER

WHICH TRIAL NUMBER IS THIS REPORT 3.
WHAT IS THE VALUE FOR E1 218000.
WHAT IS THE VALUE FOR E2 20000.
WHAT IS THE VALUE FOR E3 4381.
WHAT IS THE VALUE FOR H1 3.
WHAT IS THE VALUE FOR H2 20.

ASPHALT DESIGN DATA FOR TRIAL NO. 3 **TABLE C3.**

E1 = 218000
E2 = 20000
E3 = 4381

H1 = 3.0 IN.
H2 = 20.0 IN.

AXLE TYPE	RADIAL STRESS PSI	---VERTICAL STRAIN---- AT W.C.	AT A.C.
PASSENGER	4.796E+01	-1.084E-04	NA
2P	5.481E+01	-1.238E-04	NA
2S	7.127E+01	-1.402E-04	NA
SA	1.206E+02	-5.444E-04	-6.046E-04
TA	1.198E+02	-5.519E-04	-6.127E-04

AXLE TYPE	--------ALLOWABLE REPETITIONS-------- FOR STRESS	FOR STRAIN AT W.C.	AT A.C.
PASSENGER	7.447E+07	8.543E+09	NA
2P	3.820E+07	4.382E+09	NA
2S	1.028E+07	2.353E+09	NA
SA	7.394E+05	2.669E+06	1.580E+06
TA	7.669E+05	2.493E+06	1.478E+06

AXLE TYPE	-----PERCENT FATIGUE LIFE USED------ FOR STRESS	FOR STRAIN AT W.C.	AT A.C.
PASSENGER	9.35	.08	NA
2P	4.90	.04	NA
2S	.39	.00	NA
SA	90.61	25.10	42.41
TA	126.74	38.99	65.75
	------	------	------
TOTALS	231.98	64.22	108.16

STOP

READY.
REWIND,RWSF1.

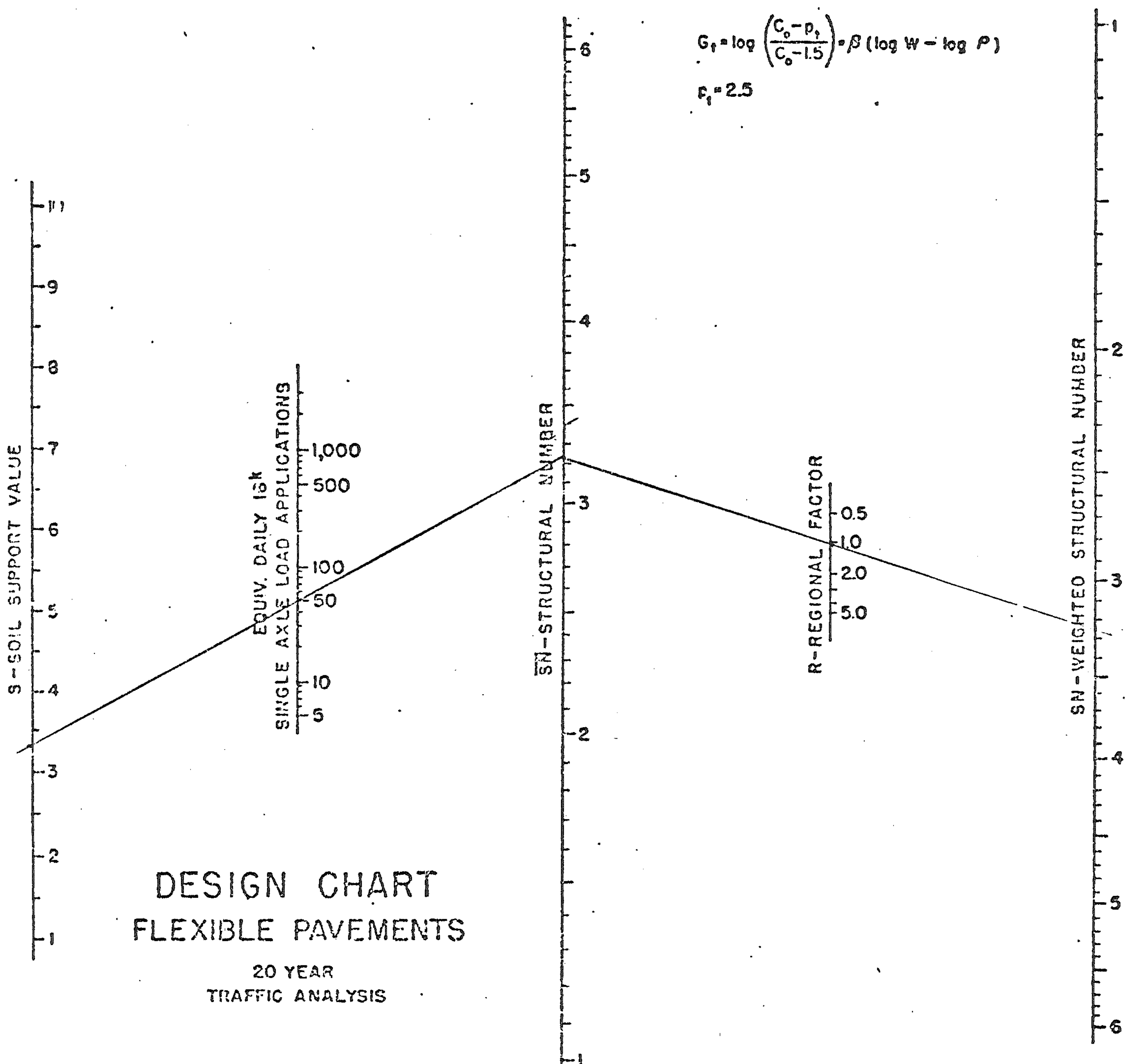


FIGURE C1 ARIZONA HIGHWAY DEPARTMENT DESIGN CHART

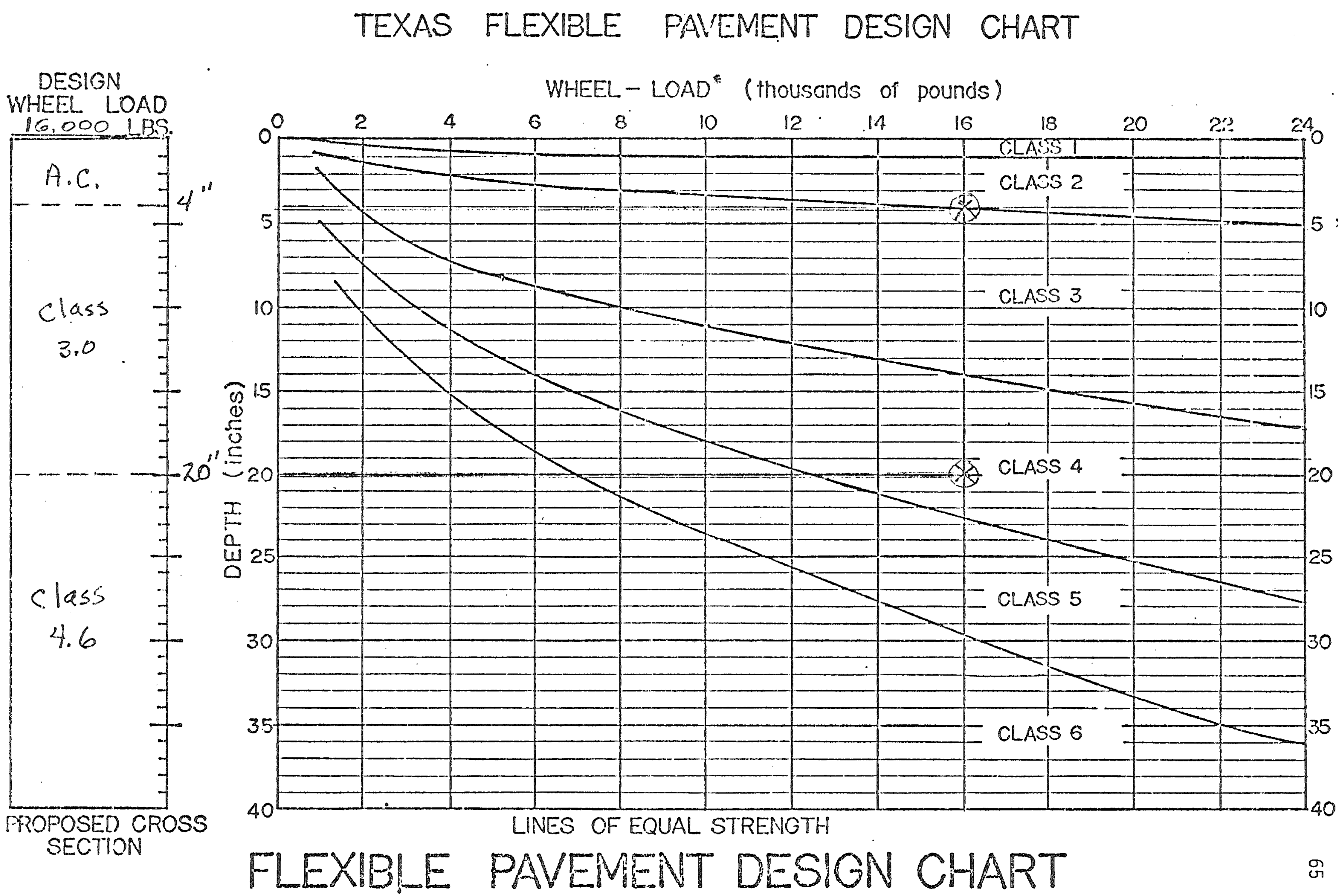


FIGURE C2 TEXAS HIGHWAY DEPARTMENT DESIGN CHART

SUBGRADE MODULUS E = 3,750 PSI
(APPROX. CBR 2.5, R-VALUE 15)

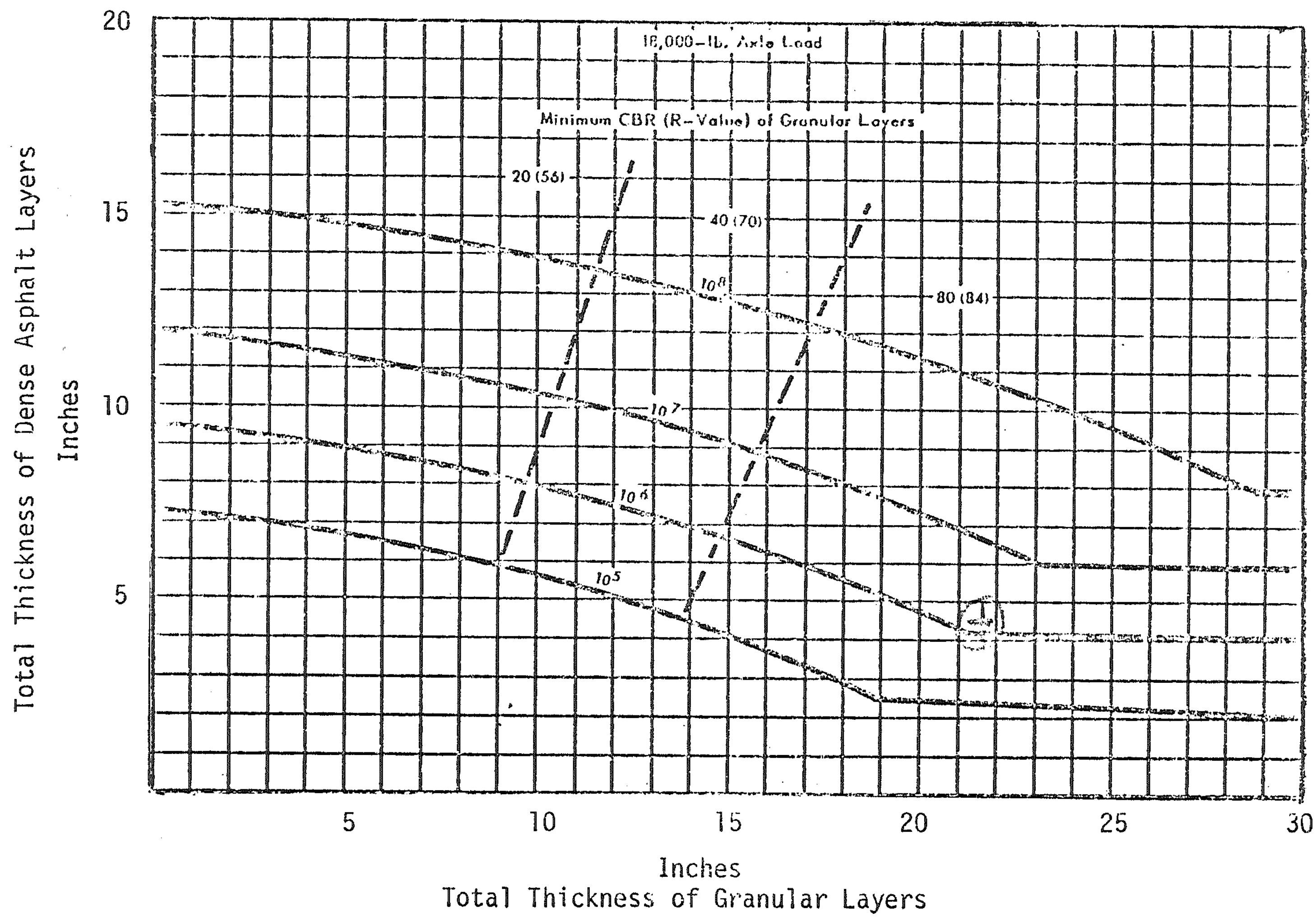


FIGURE C3 SHELL PAVEMENT DESIGN CHART

APPENDIX D

Computer Program Description and Listing

THE ASPHALT PAVEMENT DESIGN COMPUTER PROGRAM

Description

The Asphalt Pavement Design (ASPDSN) Program is a computer program written in FORTRAN IV for the CDC 6400. The program utilizes the elastic theory in determining radial stress in the asphalt layer and vertical strain in the subgrade layer for given layer characteristics. These characteristics are layer thickness and modulus of elasticity. Stresses and strains are calculated by a subroutine in the program commonly known as the CHEVRON program. This program can be used on pavement systems containing up to 15 layers, however, for the ASPDSN program only three layers are used.

Five types of loading conditions are considered by the program. They are: passenger cars, pick-up trucks, (2P); delivery trucks, (2S); single-axle trucks, (SA); and tandem-axle trucks, (TA). Stresses and strains are determined for these loading conditions at the critical points. For each loading condition, allowable repetitions are calculated and compared to the actual repetitions the pavement will experience during the design life. From this, values for percent fatigue life used are determined.

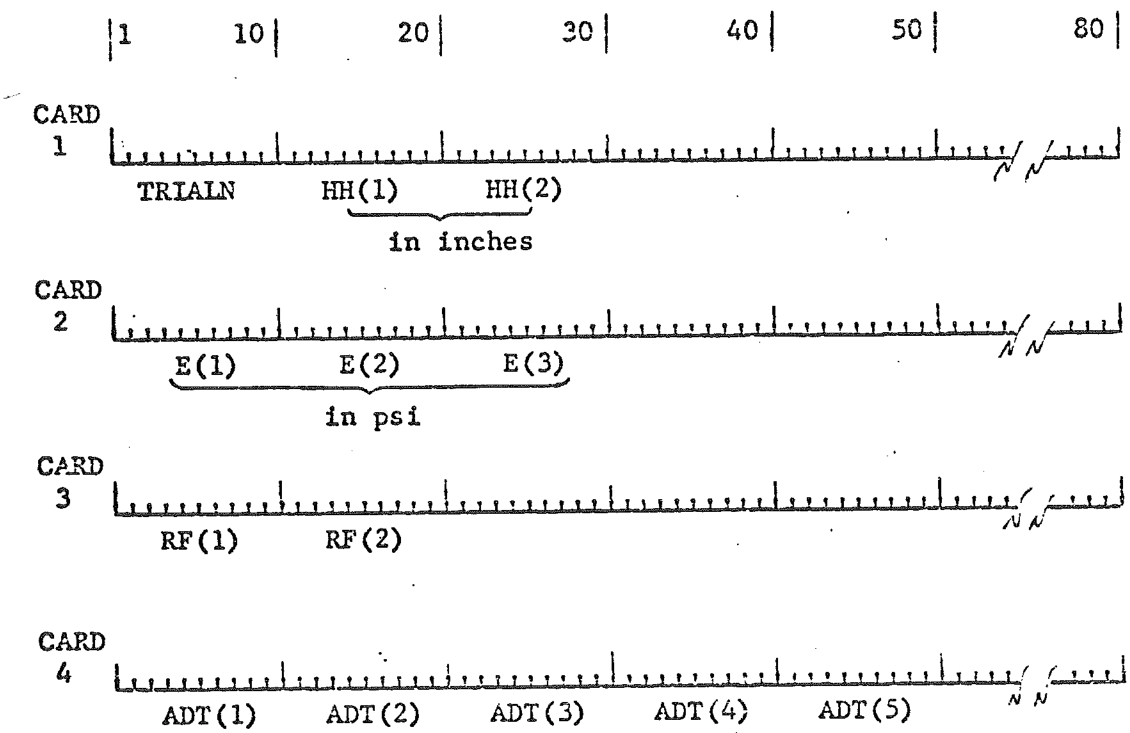
The program is presently structured so that each program run gives values for only one set of layer thicknesses and moduli. Furthermore, single-axles and tandem-axles are considered to have wheel and axle spacings of 13" and 13" x 48", respectively. Allowances are made for the environmental factors of temperature and rainfall.

Program Input Procedure

The program needs information concerning the following four areas:

1. Thicknesses for asphalt and base layers.
2. Moduli of elasticity for asphalt, base and subgrade layers.
3. Correction factors for environment.
4. Actual repetitions during the design life for the five loading conditions.

This information is input through four data cards placed at the end of the program. The format for each card is as follows:



As an illustration, consider the following design example:

Trial number (TRIALN) = 1
Asphalt layer thickness (HH(1)) = 4"
Base layer thickness (HH(2)) = 18"
Modulus of elasticity, asphalt layer (E(1)) = 200,000 psi
Modulus of elasticity, base layer (E(2)) = 20,000 psi
Modulus of elasticity, subgrade layer (E(3)) = 4,600 psi
Correction factor 1 (RF(1)) = 1.0
Correction factor 2 (RF(2)) = 1.0
Passenger car repetitions (ADT(1)) = 6,960,000
2P repetitions (ADT(2)) = 1,870,000
2S repetitions (ADT(3)) = 40,000
SA repetitions (ADT(4)) = 670,000
TA repetitions (ADT(5)) = 465,000

The correct input form for the data is:

	1 – 10	20	30	40	50	80
CARD 1	1.0	4.0	18.0			
CARD 2	200000.	20000.	4600.			
CARD 3	1.0	1.0				
CARD 4	6.96E+06	1.87E+06	0.04E+06	0.67E+06	4.65E+05	

All entries are in terms of F10.0. Note that the scientific notation form of a number can be entered in an F field as long as it is right justified. An example of this is card 4.

A copy of the program listing appears in the next several pages.

Additional Notes

As mentioned, the program is restricted to one design solution per run and 13" and 13" x 48" dual-wheel and tandem-axle spacings. The program, though, can easily be modified to solve several designs per run and allow for various dual and tandem axle spacing dimensions.

The program is ideally suited for remote terminal usage. Only minor modifications would be needed to the input/output files.

```
      PROGRAM ASPDSN
C
C      *** MAIN PROGRAM FOR ASPHALT PAVEMENT DESIGN ***
C
      COMMON   /RMCOY/RR(10),    ZZ(10),    E( 5),    V( 5),    HH( 5),
     1             H( 5),    AZ(396),   A(396, 5),B(396, 5),C(396, 5),
     2             D(396, 5),AJ(396),   RJ1(396), RJ0(396),
     3             TEST(11), BZ(100),   X( 5,4,4),SC( 5),   FM(2,2),
     4             PM( 5,4,4),R,        Z,        AR,       NS,
     5             N,        L,         ITN,      RSZ,      RSR,
     6             ROM,      RMU,       SF,       CSZ,      CST,
     7             CSR,      CTR,       COM,      CMU,      PSI,
     8             NLINE,    NOUTP,     NTEST,    I,        ITN4,
     9             K,        LC,        JT,       TZZ,      PR,
     A             PA,       P,         EP,       TIP,      TIM,
     B             T1,       T2,        T3,       T4,       T5,
     C             T6,       T2P,       T2M,      WA,       BJ1,
     D             BJ0,      ZF,        SZ1,      SZ2,      SG1,
     E             SG2,      PH,        PH2,      VK2,      VKP2,
     F             VK4,      VKP4,      VKK8,     RDT,      RDS,
     G             NCOUNT,   STRESS(20),  STRAIN(20)
      DIMENSION ADT(10), SREPS(10), EREPS(10), PERS(10), PERE(10),
     1          AT(5)
      DATA AT/10H PASSENGER,10H     2P   ,10H     2S   ,
     1     10H     SA   ,10H     TA   /
C        ** COMPUTE ZEROS OF J1(X) AND J0(X). SET UP GAUSS CONSTANTS **
      K = ITN+1
      DO 22 I=7,K,2
            T = I/2
            TD = 4.0*T - 1.0
   22       BZ(I) = 3.1415927*(T - 0.25 + 0.050661/TD
     1             -0.053041/TD**3 + 0.262051/TD**5)
      DO 23 I=8,ITN,2
            T = (I-2)/2
            TD = 4.0*T + 1.0
   23       BZ(I) = 3.1415927*(T + 0.25 - 0.151982/TD
     1             + 0.015399/TD**3 - 0.245270/TD**5)
C
      READ(5,5) TRIALN,HH(1),HH(2)
      READ(5,5) E(1),E(2),E(3)
      READ(5,1) RF1,RF2
      READ(5,2) (ADT(I),I=1,5)
    1 FORMAT(2F10.0)
    2 FORMAT(5F10.0)
    5 FORMAT(3F10.0)
      ADT(5)=2.*ADT(5)
      ADT(6)=ADT(4)      $      ADT(7)=ADT(5)
C     ADJUST FOR RAINFALL AND TEMPERATURE.
      CF1=RF1**.146
      CF2=RF2**.146
      E(1)=E(1)*CF1
      E(3)=E(3)/CF2
C     CALCULATE RADIAL STRESSES AND VERTICAL STRAINS FOR THE 5 AXLE
C     TYPES.
      CALL CHVRON
```

```
      J=1
      STRAIN(1)=STRAIN(2)
      DO 3 I=2,9
      J=J+2
      K=2*I
      STRESS(I)=STRESS(J)
    3 STRAIN(I)=STRAIN(K)
      STRESS(4)=STRESS(4)+STRESS(7)
      STRESS(5)=STRESS(4)+STRESS(5)+STRESS(6)
      STRAIN(4)=STRAIN(4)+STRAIN(7)
      STRAIN(5)=STRAIN(4)+STRAIN(5)+STRAIN(6)
      STRAIN(6)=2.*STRAIN(9)
      STRAIN(7)=STRAIN(6)+(2.*STRAIN(8))
      TPERS=0.
C     CALCULATE ALLOWABLE REPETITIONS.
      DO 30 I=1,5
      SREPS(I)=ABS((1800./STRESS(I))**5)
      PERS(I)=(ADT(I)/SREPS(I))*100.
   30 TPERS=TPERS+PERS(I)
      TPERE=0.0
      DO 31 I=1,7
      EREPS(I)=ABS((.0105/STRAIN(I))**5)
      PERE(I)=(ADT(I)/EREPS(I))*100.
   31 TPERE=TPERE+PERE(I)
      GCPER=PERE(6)+PERE(7)
      WCPER=TPERE-GCPER
      WRITE(6,41) TRIALN
   41 FORMAT(1H1,////,* ASPHALT DESIGN DATA FOR TRIAL NO.*,F3.0)
      WRITE(6,42) E(1),E(2),E(3),HH(1),HH(2)
   42 FORMAT(///,5H E1 =,F7.0,/,5H E2 =,F7.0,/,5H E3 =,F7.0,//,
     15H H1 =,F5.1,4H IN.,/,5H H2 =,F5.1,4H IN.)
      WRITE(6,43)
   43 FORMAT(///,* AXLE TYPE*,5X,*RADIAL STRESS*,3X,*---VERT*,
     1*ICAL STRAIN-----*,/,19X,* PSI*,10X,*AT W.C.*,5X,*AT A.C.*,/)
      WRITE(6,44) (AT(I),STRESS(I),STRAIN(I),I=1,3)
   44 FORMAT(A10,6X,E10.3,5X,E10.3,6X,* NA*)
      WRITE(6,45) (AT(I),STRESS(I),STRAIN(I),STRAIN(I+2),I=4,5)
   45 FORMAT(A10,6X,E10.3,5X,E10.3,2X,E10.3)
      WRITE(6,51)
      WRITE(6,52)
   51 FORMAT(///,15X,* -------ALLOWABLE  REPETITIONS*,
     1*---------*)
   52 FORMAT(10H AXLE TYPE,6X,10HFOR STRESS,11X,
     110HFOR STRAIN,/,32X,8H AT W.C.,5X,7HAT A.C.,/)
      WRITE(6,44) (AT(I),SREPS(I),EREPS(I),I=1,3)
      WRITE(6,45) (AT(I),SREPS(I),EREPS(I),EREPS(I+2),I=4,5)
      WRITE(6,53)  $  WRITE(6,52)
   53 FORMAT(///,15X,* -----PERCENT FATIGUE LIFE USED------*)
      WRITE(6,54) (AT(I),PERS(I),PERE(I),I=1,3)
      WRITE(6,55) (AT(I),PERS(I),PERE(I),PERE(I+2),I=4,5)
   54 FORMAT(A10,7X,F7.2,8X,F7.2,8X,3H NA)
   55 FORMAT(A10,7X,F7.2,8X,F7.2,5X,F7.2)
      WRITE(6,56)
   56 FORMAT(17X,7H ------,9X,6H------,6X,6H------)
      WRITE(6,57) TPERS,WCPER,GCPER
   57 FORMAT(9X,7H TOTALS,1X,F7.2,8X,F7.2,5X,F7.2)
   58 STOP
      END
```

```
      SUBROUTINE CHVRON
C
C      *** CHEVRON PROGRAM - USED TO CALCULATE STRESSES AND STRAINS ***
C
      DIMENSION DWGT(6), DPSI(6)
      COMMON    /RMCOY/RR(10),    ZZ(10),    E( 5),     V( 5),     HH( 5),
     1                H( 5),      AZ(396),   A(396, 5),B(396, 5),C(396, 5),
     2                D(396, 5),AJ(396),   RJ1(396), RJ0(396),
     3                TEST(11), BZ(100),   X( 5,4,4),SC( 5),    FM(2,2),
     4               PM( 5,4,4),R,         Z,         AR,        NS,
     5                N,          L,         ITN,       RSZ,       RSR,
     6                ROM,        RMU,       SF,        CSZ,       CST,
     7                CSR,        CTR,       COM,       CMU,       PSI,
     8                NLINE,      NOUTP,     NTEST,     I,         ITN4,
     9                K,          LC,        JT,        TZZ,       PR,
     A                PA,         P,         EP,        TIP,       TIM,
     B                T1,         T2,        T3,        T4,        T5,
     C                T6,         T2P,       T2M,       WA,        BJ1,
     D                BJ0,        ZF,        SZ1,       SZ2,       SG1,
     E                SG2,        PH,        PH2,       VK2,       VKP2,
     F                VK4,        VKP4,      VKK8,      RDT,       RDS,
     G                NCOUNT,     STRESS(20),   STRAIN(20)
      DATA DPSI/ 28., 32., 45., 80. /
      DATA DWGT/ 1400., 1600., 1800., 4500. /
      DATA RR/ 0., 48., 49.7, 13., 48.4, 6.5 /, V/ .35, .45, .5 /
      NOUTP=0    $ NPUN=0
      NCOUNT=0
      NS=3    $    IZ=2    $    N=2
      IR=1
      WRITE(6,9)
    9 FORMAT(1H1,///,37X,2H R,5X,1HZ,5X,12HRAD STRESS  ,11HRAD STRAIN ,
     112HTANG STRAIN ,12HBULK STRAIN ,//)
      DO 3636 NXN=1,4
      IF(NXN.EQ.4) IR=6
      ZZ(1)=HH(1)    $    ZZ(2)=HH(2)+HH(1)
      WGT=DWGT(NXN)
      PSI=DPSI(NXN)
      AR = SQRT (WGT/(3.14159*PSI))
C  ** ADJUST LAYER DEPTHS **
      H(1)=HH(1)
      DO 25 I=2,N
   25 H(I)=H(I-1)+HH(I)
      IRT=0
C  ** START ON A NEW R **
  100 IRT=IRT+1
      IF(IRT-IR) 105,105,3636
  105 R=RR(IRT)
      DO 31 I =1,IZ
      DO 31 J=1,N
      TZ  = ABS (H(J) - ZZ(I))
      IF(TZ - .0001) 32,32,31
   32 ZZ(I) = -H(J)
   31 CONTINUE
C  ** CALCULATE THE PARTITION **
      CALL PART
```

```
C   ** CALCULATE THE COEFFICIENTS **
      DO 125 I=1,ITN4
      P=AZ(I)
  107 CONTINUE
      CALL COE5(I)
  109   IF (R) 115,115,110
  110   PR = P*R
      CALL BESSEL (0,PR,Y)
          RJ0(I) = Y
      CALL BESSEL (1,PR,Y)
          RJ1(I) = Y
  115 PA=P*AR
      CALL BESSEL (1,PA,Y)
      AJ(I)=Y
  125 CONTINUE
  195 IZT=0
C   ** START ON A NEW Z **
  200 IZT=IZT+1
      IF(IZT-IZ) 205,205,100
  205 Z=ABS (ZZ(IZT))
  207 CONTINUE
C   ** FIND THE LAYER CONTAINING Z **
      TZZ = 0.0
      DO 210 J1=1,N
      J=NS-J1
      IF(Z-H(J)) 210,215,215
  210 CONTINUE
      L = 1
      GO TO 34
  215 L=J+1
      IF(IZT.EQ.2) ZZ(IZT)=-ZZ(IZT)
      IF(IZT.EQ.2) GO TO 34
   33 L = J
      TZZ = 1.0
   34 CONTINUE
      CALL CALCIN
      GO TO 200
 3636 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE BESSEL(NI,XI,Y)
C       ******SUBROUTINE BESSEL - N-LAYER ELASTIC SYSTEM ******
C
      DIMENSION PZ(6),QZ(6),CP1(6),Q1(6),D(20)
      DOUBLE PRECISION XYX
      DATA(PZ(I),I=1,6)/1.0,-1.125E-4,2.8710938E-7,-2.3449658E-9,
     A3.930684E-11,-1.1536133E-12/,(QZ(I),I=1,6)/-5.0E-3,4.6875E-6,
     B-2.3255859E-8,2.8307087E-10,-6.3912096E-12,2.3124704E-12/,
     C(CP1(I),I=1,6)/1.0,1.875E-4,-3.6914063E-7,2.7713232E-9,
     D-4.5114421E-11,1.2750463E-12/,(Q1(I),I=1,6)/1.5E-2,
     E-6.5625E-6,2.8423828E-8,-3.2662024E-10,7.141166E-12,
     F-2.5327056E-13/,PI/3.1415927/
    9 N = NI
      X = XI
      IF (X-7.0) 10,10,160
C
   10 X2=X/2.0
      FAC=-X2*X2
      IF (N) 11,11,14
   11 C=1.0
      Y=C
      DO 13 I=1,34
      T=I
      C=FAC*C/(T*T)
      TEST=ABS (C) - 10.0**(-8)
      IF (TEST) 17,17,12
   12 Y=Y+C
   13 CONTINUE
   14 C=X2
      Y=C
      DO 16 I=1,34
      T=I
      C=FAC*C/(T*(T+1.0))
      TEST=ABS (C) - 10.0**(-8)
      IF (TEST) 17,17,15
   15 Y=Y+C
   16 CONTINUE
   17 RETURN
  160 IF (N) 161,161,164
C
C
  161 DO 162 I=1,6
      D(I) = PZ(I)
      D(I+10) = QZ(I)
  162 CONTINUE
      GO TO 163
C
  164 DO 165 I=1,6
      D(I)=CP1(I)
      D(I+10) = Q1(I)
  165 CONTINUE
  163 CONTINUE
      T1 = 25.0/X
      T2=T1*T1
      P = D(6)*T2+D(5)
```

```
      DO 170 I=1,4
      J = 5-I
      P = P*T2+D(J)
  170 CONTINUE
      Q = D(16)*T2+D(15)
      DO 171 I=1,4
      J = 5-I
      Q = Q*T2+D(J+10)
  171 CONTINUE
      Q = Q*T1
C
      XYX=DBLE(X)*DBLE(PI)
      T4=DSQRT(XYX)
      T6 = SIN (X)
      T7 = COS (X)
C
      IF (N) 180,180,185
C
  180 T5 = ((P-Q)*T6 + (P+Q)*T7)/T4
      GO TO 99
  185 T5 = ((P+Q)*T6 - (P-Q)*T7)/T4
   99 Y = T5
 9999 RETURN
      END
```

```
      BLOCK DATA
C
      COMMON   /RMCOY/RR(10),    ZZ(10),     E( 5),      V( 5),      HH( 5),
     1                H( 5),      AZ(396),    A(396, 5),B(396, 5),C(396, 5),
     2                D(396, 5),AJ(396),    RJ1(396), RJ0(396),
     3                TEST(11), BZ(100),    X( 5,4,4),SC( 5),      FM(2,2),
     4               PM( 5,4,4),R,          Z,          AR,         NS,
     5                N,          L,          ITN,        RSZ,        RSR,
     6                ROM,        RMU,        SF,         CSZ,        CST,
     7                CSR,        CTR,        COM,        CMU,        PSI,
     8                NLINE,      NOUTP,      NTEST,      I,          ITN4,
     9                K,          LC,         JT,         TZZ,        PR,
     A                PA,         P,          EP,         TIP,        TIM,
     B                T1,         T2,         T3,         T4,         T5,
     C                T6,         T2P,        T2M,        WA,         BJ1,
     D                BJ0,        ZF,         SZ1,        SZ2,        SG1,
     E                SG2,        PH,         PH2,        VK2,        VKP2,
     F                VK4,        VKP4,       VKK8,       RDT,        RDS,
     G                NCOUNT,     STRESS(20),   STRAIN(20)
      DIMENSION ZB(100)
      EQUIVALENCE (BZ,ZB )
      DATA(ZB(I),I=1,6)/0.0,1.0,2.4048,3.8317,5.5201,7.0156/
      DATA ITN/46/,ITN4/184/
      END
```

```
      SUBROUTINE PART
C       ******SUBROUTINE PART - N-LAYER ELASTIC SYSTEM ******
C
      COMMON   /RMCOY/RR(10),   ZZ(10),   E( 5),    V( 5),    HH( 5),
     1              H( 5),    AZ(396),  A(396, 5),B(396, 5),C(396, 5),
     2              D(396, 5),AJ(396),  RJ1(396), RJ0(396),
     3              TEST(11), BZ(100),  X( 5,4,4),SC( 5),   FM(2,2),
     4             PM( 5,4,4),R,        Z,        AR,       NS,
     5              N,        L,        ITN,      RSZ,      RSR,
     6              ROM,      RMU,      SF,       CSZ,      CST,
     7              CSR,      CTR,      COM,      CMU,      PSI,
     8              NLINE,    NOUTP,    NTEST,    I,        ITN4,
     9              K,        LC,       JT,       TZZ,      PR,
     A              PA,       P,        EP,       TIP,      TIM,
     B              T1,       T2,       T3,       T4,       T5,
     C.             T6,       T2P,      T2M,      WA,       BJ1,
     D              BJ0,      ZF,       SZ1,      SZ2,      SG1,
     E              SG2,      PH,       PH2,      VK2,      VKP2,
     F              VK4,      VKP4,     VKK8,     RDT,      RDS,
     G              NCOUNT,   STRESS(20),  STRAIN(20)
      DATA G1,G2/0.86113631,0.33998104/
    4 ZF = AR
      NTEST = 2
      IF (R) 8,8,9
    9 CONTINUE
      NTEST = AR/R + .0001
      IF (NTEST) 6,6,5
    6 CONTINUE
      NTEST = R/AR + .0001
      ZF = R
    5 CONTINUE
      NTEST = NTEST + 1
      IF (NTEST-10) 8,8,7
    7 CONTINUE
      NTEST = 10
    8 CONTINUE
C          ** COMPUTE POINTS FOR LEGENDRE-GAUSS INTEGRATION **
   15 K = 1
       ZF = 2.0*ZF
       SZ2 = 0.0
      DO 28 I=1,ITN
        SZ1 = SZ2
        SZ2 = BZ(I+1)/ZF
           SF = SZ2 - SZ1
           PP = SZ2 + SZ1
      SG1=SF*G1
      SG2=SF*G2
      AZ(K)=PP-SG1
      AZ(K+1)=PP-SG2
      AZ(K+2)=PP+SG2
      AZ(K+3)=PP+SG1
         K = K + 4
   28 CONTINUE
   40 RETURN
      END
```

```
      SUBROUTINE CALCIN
C     ******SUBROUTINE CALCIN - N-LAYER ELASTIC SYSTEM ******
C
      COMMON   /RMCOY/RR(10),     ZZ(10),     E( 5),      V( 5),      HH( 5),
     1                H( 5),      AZ(396),    A(396, 5),B(396, 5),C(396, 5),
     2                D(396, 5),AJ(396),    RJ1(396), RJ0(396),
     3                TEST(11), BZ(100),    X( 5,4,4),SC( 5),     FM(2,2),
     4               PM( 5,4,4),R,          Z,          AR,         NS,
     5                N,          L,          ITN,        RSZ,        RSR,
     6                ROM,        RMU,        SF,         CSZ,        CST,
     7                CSR,        CTR,        COM,        CMU,        PSI,
     8                NLINE,      NOUTP,      NTEST,      I,          ITN4,
     9                K,          LC,         JT,         TZZ,        PR,
     A                PA,         P,          EP,         TIP,        TIM,
     B                T1,         T2,         T3,         T4,         T5,
     C                T6,         T2P,        T2M,        WA,         BJ1,
     D                BJ0,        ZF,         SZ1,        SZ2,        SG1,
     E                SG2,        PH,         PH2,        VK2,        VKP2,
     F                VK4,        VKP4,       VKK8,       RDT,        RDS,
     G                NCOUNT,     STRESS(20),  STRAIN(20)
      DIMENSION W(4)
      DATA(W(I),I=1,4)/0.34785485,0.65214515,0.65214515,0.34785485/
C
    2 VL=2.0*V(L)
      EL=(1.0+V(L))/E(L)
      VL1=1.0-VL
      CSZ=0.0
      CST=0.0
      CSR=0.0
      CTR=0.0
      COM=0.0
      CMU=0.0
      NTS1 = NTEST + 1
      ITS = 1
      JT = 0
      ARP = AR
      IF (NOUTP) 4,4,5
    4 ARP = ARP*PSI
    5 CONTINUE
   10 DO 40 I=1,ITN
C     INITIALIZE THE SUB-INTEGRALS
      RSZ=0.0
      RST=0.0
      RSR=0.0
      RTR=0.0
      ROM=0.0
      RMU=0.0
C     COMPUTE THE SUB-INTEGRALS
          K = 4*(I-1)
      DO 30 J=1,4
         J1 = K + J
      P=AZ(J1)
      EP=EXP (P*Z)
      T1=B(J1,L)*EP
      T2=D(J1,L)/EP
```

```
      T1P=T1+T2
      T1M=T1-T2
      T1=(A(J1,L)+B(J1,L)*Z)*EP
      T2=(C(J1,L)+D(J1,L)*Z)/EP
      T2P=P*(T1+T2)
      T2M=P*(T1-T2)
      WA=AJ(J1)*W(J)
      IF (R) 20,20,15
   15 BJ1=RJ1(J1)*P
      BJ0=PJ0(J1)*P
      RSZ=RSZ+WA*P*BJ0*(VL1*T1P-T2M)
      ROM=ROM+WA*EL*BJ0*(2.0*VL1*T1M-T2P)
      RTR=RTR+WA*P*BJ1*(VL*T1M+T2P)
      RMU=RMU+WA*EL*BJ1*(T1P+T2M)
      RSR=RSR+WA*(P*BJ0*((1.0+VL)*T1P+T2M)-BJ1*(T1P+T2M)/R)
      RST=RST+WA*(VL*P*BJ0*T1P+BJ1*(T1P+T2M)/R)
      GO TO 30
C     SPECIAL ROUTINE FOR R = ZERO
   20 PP=P*P
      RSZ=RSZ+WA*PP*(VL1*T1P-T2M)
      ROM=ROM+WA*EL*P*(2.0*VL1*T1M-T2P)
      RST=RST+WA*PP*((VL+0.5)*T1P+0.5*T2M)
      RSR=RST
   30 CONTINUE
C
       SF = (AZ(K+4) - AZ(K+1))/1.7222726
      CSZ=CSZ+RSZ*SF
      CST=CST+RST*SF
      CSR=CSR+RSR*SF
      CTR=CTR+RTR*SF
      COM=COM+ROM*SF
      CMU=CMU+RMU*SF
          RSZ = 2.0*RSZ*AR*SF
      TESTH = ABS (RSZ)-10.0**(-4)
      IF (ITS-NTS1 ) 31,32,32
   31 CONTINUE
      TEST(ITS) = TESTH
      ITS = ITS+1
      GO TO 40
   32 CONTINUE
      TEST(NTS1) = TESTH
      DO 33 J = 1,NTEST
      IF (TESTH-TEST(J)) 35,36,36
   35 CONTINUE
      TESTH = TEST(J)
   36 CONTINUE
      TEST(J) = TEST(J+1)
   33 CONTINUE
      IF (TESTH) 50,50,40
   40 CONTINUE
      JT = 1
   50 CSZ=CSZ*ARP
      CST=CST*ARP
      CTR=CTR*ARP
      CSR=CSR*ARP
```

```
      COM=COM*ARP
      CMU=CMU*ARP
      BSTS = CSZ+CST+CSR
      BST  = BSTS * (1.0-2.0*V(L))/E(L)
      IF (TZZ) 72,72,71
   71 Z = -Z
   72 CONTINUE
      RDS=(CSR-V(L)*(CSZ+CST))/E(L)
      SST=2.0*(1.0+V(L))*CTR/E(L)
      RDT - (CST - V(L) * (CSZ + CSR))/E(L)
      NCOUNT=NCOUNT+1
      STRESS(NCOUNT)=CSR
      STRAIN(NCOUNT)=BST-RDT-RDS
      WRITE(6,315) R, Z, CSR, RDS, RDT, BST
  315 FORMAT(    36X,F5.1,F6.1,1X,1P4E12.3)
   99 RETURN
      END
```

```
      SUBROUTINE COES (KIN)
C      USED FOR 5 OR FEWER LAYERS
C
      COMMON   /RMCOY/RR(10),   ZZ(10),    E( 5),     V( 5),     HH( 5),
     1                H( 5),    AZ(396),   A(396, 5),B(396, 5),C(396, 5),
     2                D(396, 5),AJ(396),   RJ1(396), RJ0(396),
     3                TEST(11), BZ(100),   X( 5,4,4),SC( 5),    FM(2,2),
     4               PM( 5,4,4),R,         Z,         AR,        NS,
     5                N,        L,         ITN,       RSZ,       RSR,
     6                ROM,      RMU,       SF,        CSZ,       CST,
     7                CSR,      CTP,       COM,       CMU,       PSI,
     8                NLINE,    NOUTP,     NTEST,     I,         ITN4,
     9                K,        LC,        JT,        TZZ,       PR,
     A                PA,       P,         EP,        TIP,       TIM,
     B                T1,       T2,        T3,        T4,        T5,
     C                T6,       T2P,       T2M,       WA,        BJ1,
     D                BJ0,      ZF,        SZ1,       SZ2,       SG1,
     E                SG2,      PH,        PH2,       VK2,       VKP2,
     F                VK4,      VKP4,      VKK8,      RDT,       RDS,
     G                NCOUNT,   STRESS(20),  STRAIN(20)
C
      DIMENSION SV1(4,2),CV1(2,1),SV2(4,4),CV2(2,2),SV3(4,8),CV3(2,4),
     1SV4(4,16),CV4(2,8),T(8),NT(14)
          LC = KIN
CS-MX          SET UP MATRIX X =DI*MI*KI*K*M*D
C     COMPUTE THE MATRICES X(K)
      DO 10 K=1,N
      T1=E(K)*(1.0+V(K+1))/(E(K+1)*(1.0+V(K)))
      T1M=T1-1.0
      PH=P*H(K)
      PH2=PH*2.0
      VK2=2.0*V(K)
      VKP2=2.0*V(K+1)
      VK4=2.0*VK2
      VKP4=2.0*VKP2
      VKK8=8.0*V(K)*V(K+1)
C
      X(K,1,1)=VK4-3.0-T1
      X(K,2,1)=0.0
      X(K,3,1)=T1M*(PH2-VK4+1.0)
      X(K,4,1)=-2.0*T1M*P
C
      T3=PH2*(VK2-1.0)
      T4=VKK8+1.0-3.0*VKP2
      T5=PH2*(VKP2-1.0)
      T6=VKK8+1.0-3.0*VK2
C
      X(K,1,2)=(T3+T4-T1*(T5+T6))/P
      X(K,2,2)=T1*(VKP4-3.0)-1.0
      X(K,4,2)=T1M*(1.0-PH2-VKP4)
C
      X(K,3,4)=(T3-T4-T1*(T5-T6))/P
C
      T3=PH2*PH-VKK8+1.0
      T4=PH2*(VK2-VKP2)
```

```
C
      X(K,1,4)=(T3+T4+VKP2-T1*(T3+T4+VK2))/P
      X(K,3,2)=    (-T3+T4-VKP2+T1*(T3-T4+VK2))/P
C
      X(K,1,3)=T1M*(1.0-PH2-VK4)
      X(K,2,3)=2.0*T1M*P
      X(K,3,3)=VK4-3.0-T1
      X(K,4,3)=0.0
C
      X(K,2,4)=T1M*(PH2-VKP4+1.0)
      X(K,4,4)=T1*(VKP4-3.0)-1.0
   10 CONTINUE
C     COMPUTE THE PRODUCT MATRICES PM
      SC(N)=4.0*(V(N)-1.0)
      IF (N-2) 13,11,11
   11 DO 12 K1=2,N
      M=NS-K1
      SC(M)=SC(M+1)*4.0*(V(M)-1.0)
   12 CONTINUE
   13 CONTINUE
C
      K = N
      DO 15 M=1,4
      DO 14 J=1,2
   14 SV1(M,J) = X(K,M,J+2)
   15 CONTINUE
      CV1(1,1) = -2.0*P*H(K)
      CV1(2,1) = 0.0
      K = K-1
      IF(K) 50,50,20
C
   20 CONTINUE
      DO 22 J=1,2
      J1 = J+J
      T(1) = SV1(1,J)
      T(2) = SV1(2,J)
      T(3) = SV1(3,J)
      T(4) = SV1(4,J)
      DO 21 M=1,4
      SV2(M,J1-1) = X(K,M,1)*T(1)+X(K,M,2)*T(2)
   21 SV2(M,J1) =    X(K,M,3)*T(3)+X(K,M,4)*T(4)
   22 CONTINUE
      T(1) = CV1(1,1)
      T(2) = -2.0*P*H(K)
      CV2(1,1) = T(1)
      CV2(1,2) = T(2)
      CV2(2,1) = T(1)-T(2)
      CV2(2,2) = 0.0
      K = K-1
      IF (K) 50,50,30
C
   30 CONTINUE
      DO 34 J=1,4
      J1 = J
      IF (J1-2) 32,32,31
```

```
   31 J1 = J1+2
   32 CONTINUE
      T(1) = SV2(1,J)
      T(2) = SV2(2,J)
      T(3) = SV2(3,J)
      T(4) = SV2(4,J)
      DO 33 M=1,4
      SV3(M,J1) = X(K,M,1)*T(1)+X(K,M,2)*T(2)
   33 SV3(M,J1+2) = X(K,M,3)*T(3)+X(K,M,4)*T(4)
   34 CONTINUE
      T(1) = -2.0*P*H(K)
      DO 35 J=1,2
      CV3(1,J) = CV2(1,J)
      CV3(2,J) = CV2(1,J)-T(1)
      CV3(1,J+2) = CV2(2,J)+T(1)
      CV3(2,J+2) = CV2(2,J)
   35 CONTINUE
      K = K-1
      IF (K) 50,50,40
C
   40 CONTINUE
      DO 42 J=1,4
      T(1) = SV3(1,J)
      T(2) = SV3(2,J)
      T(3) = SV3(3,J)
      T(4) = SV3(4,J)
      T(5) = SV3(1,J+4)
      T(6) = SV3(2,J+4)
      T(7) = SV3(3,J+4)
      T(8) = SV3(4,J+4)
      DO 41 M=1,4
      SV4(M,J) = X(K,M,1)*T(1)+X(K,M,2)*T(2)
      SV4(M,J+4) = X(K,M,3)*T(3)+X(K,M,4)*T(4)
      SV4(M,J+8) = X(K,M,1)*T(5)+X(K,M,2)*T(6)
   41 SV4(M,J+12) = X(K,M,3)*T(7)+X(K,M,4)*T(8)
   42 CONTINUE
      T(1) = -2.0*P*H(K)
      DO 43 J=1,4
      CV4(1,J) = CV3(1,J)
      CV4(2,J) = CV3(1,J)-T(1)
      CV4(1,J+4) = CV3(2,J)+T(1)
      CV4(2,J+4) = CV3(2,J)
   43 CONTINUE
C
   50 CONTINUE
      NT(1) = 1
      DO 51 K=2,N
   51 NT(K) = NT(K-1)+NT(K-1)
      DO 80 K=1,N
      K1 = NS-K
      DO 52 M=1,4
      PM(K1,M,1) = 0.0
      PM(K1,M,2) = 0.0
   52 CONTINUE
      I1 = NT(K)
```

```
      DO 80 M=1,I1
      I2 = M+I1
      GO TO (61,62,63,64),K
   61 CONTINUE
      T(3) = CV1(1,M)
      T(4) = CV1(2,M)
      GO TO 65
   62 CONTINUE
      T(3) = CV2(1,M)
      T(4) = CV2(2,M)
      GO TO 65
   63 CONTINUE
      T(3) = CV3(1,M)
      T(4) = CV3(2,M)
      GO TO 65
   64 CONTINUE
      T(3) = CV4(1,M)
      T(4) = CV4(2,M)
   65 CONTINUE
      T(1) = 0.0
      T(2) = 0.0
      IF (T(3)+68.0) 67,66,66
   66 T(1) = EXP(T(3))
   67 IF (T(4)+68.0) 69,68,68
   68 T(2) = EXP(T(4))
   69 CONTINUE
      DO 80 J=1,2
      GO TO (71,72,73,74),K
   71 CONTINUE
      T(3) = SV1(J,M)
      T(4) = SV1(J,I2)
      T(5) = SV1(J+2,M)
      T(6) = SV1(J+2,I2)
      GO TO 75
   72 T(3) = SV2(J,M)
      T(4) = SV2(J,I2)
      T(5) = SV2(J+2,M)
      T(6) = SV2(J+2,I2)
      GO TO 75
   73 T(3) = SV3(J,M)
      T(4) = SV3(J,I2)
      T(5) = SV3(J+2,M)
      T(6) = SV3(J+2,I2)
      GO TO 75
   74 T(3) = SV4(J,M)
      T(4) = SV4(J,I2)
      T(5) = SV4(J+2,M)
      T(6) = SV4(J+2,I2)
   75 CONTINUE
C
      PM(K1,J,1) = PM(K1,J,1)+T(1)*T(3)
      PM(K1,J,2) = PM(K1,J,2)+T(1)*T(4)
      PM(K1,J+2,1) = PM(K1,J+2,1)+T(2)*T(5)
      PM(K1,J+2,2) = PM(K1,J+2,2)+T(2)*T(6)
   80 CONTINUE
```

```
C     SOLVE FOR C(NS) AND D(NS)
      V2=2.0*V(1)
      V21=V2-1.0
      DO 90 J=1,2
      FM(1,J)=P*PM(1,1,J)+V2*PM(1,2,J)+P*PM(1,3,J)-V2*PM(1,4,J)
   90 FM(2,J)=P*PM(1,1,J)+V21*PM(1,2,J)-P*PM(1,3,J)+V21*PM(1,4,J)
      DFAC=SC(1)/((FM(1,1)*FM(2,2)-FM(2,1)*FM(1,2))*P*P)
           A(LC,NS) = 0.0
           B(LC,NS) = 0.0
           C(LC,NS) = -FM(1,2)*DFAC
           D(LC,NS) =  FM(1,1)*DFAC
C     BACKSOLVE FOR THE OTHER A,B,C,D
      DO 91 K1=1,N
      A(LC,K1)=(PM(K1,1,1)*C(LC,NS)+PM(K1,1,2)*D(LC,NS))/SC(K1)
      B(LC,K1)=(PM(K1,2,1)*C(LC,NS)+PM(K1,2,2)*D(LC,NS))/SC(K1)
      C(LC,K1)=(PM(K1,3,1)*C(LC,NS)+PM(K1,3,2)*D(LC,NS))/SC(K1)
   91 D(LC,K1)=(PM(K1,4,1)*C(LC,NS)+PM(K1,4,2)*D(LC,NS))/SC(K1)
      RETURN
      END
```